Application of a D Number based LBWA Model and an Interval MABAC Model in Selection of an Automatic Cannon for Integration into Combat Vehicles

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ABSTRACT

A decision making procedure for selection of a weapon system involves different, often contradictory criteria and reaching decisions under conditions of uncertainty. This paper proposes a novel multi-criteria methodology based on D numbers which enables efficient analysis of the information used for decision making. The proposed methodology has been developed in order to enable selection of an efficient weapon system under conditions when a large number of hierarchically structured evaluation criteria has to be processed. A novel D number based Level Based Weight Assessment – Multi Attributive Border Approximation area Comparison (D LBWA-MABAC) model is used for selection of an automatic cannon for integration into combat vehicles. Criteria weights are determined based on the improved LBWA-D model. The traditional MABAC method has been further developed by integration of interval numbers. A hybrid D LBWA-MABAC framework is used for evaluation of an automatic cannon for integration into combat vehicles. Nine weapon systems used worldwide have been ranked in this paper. This multi-criteria approach allows decision makers to assess options objectively and reach a rational decision regarding the selection of an optimal weapon system. Validation of the proposed methodology is performed through sensitivity analysis which studies how changes in the weights of the best criterion and the elasticity coefficient affect the ranking results.

Keywords: D numbers; LBWA; MABAC; Multi-criteria; Automatic cannon; Integration

1. INTRODUCTION

The selection of an effective weapon system is a very complex process and an important strategic issue. The decision making process is difficult and it includes a comprehensive analysis of specific characteristics. Generally, the weapon with highest grades in terms of tactical, technical and technological properties is chosen based on the research analysis. However, it often happens that different weapon systems have similar analysis results in certain categories. That is why, in addition to the use of logic and scientific models, this complex selection process requires a profound knowledge and rich experience.

This is a demanding process which includes defining variables and taking into account the influence of complex tactical and technical parameters. Such problems cannot be studied only in theory, they require engineering practice. Furthermore, the literature dealing with these problems is quite limited.

The weapon system selection involves a number of factors and it can be considered a multi-criteria decision making (MCDM) problem. Among various MCDM methods in the literature, MCDM methods with in advance expressed preferences are most commonly used1-12. Since MCDM methods cannot be divided into good and bad, it is necessary to identify the method that will yield the best results for the studied problem. Most MCDM models involve determining weight coefficients of evaluation criteria so that priorities for each of the considered alternatives can be determined13. The input parameters for MCDM models can be defined either subjectively or by measuring certain characteristics of the alternatives, or they can be calculated using a model14. Since MCDM models are sensitive to variations in weight coefficients and stabilisation parameters, validation of the obtained data is an inevitable step in all multi-criteria models. Therefore, the results validation phase is an inevitable step towards verification of the initial ranking2,15.


The selection of the optimal weapon system for integration is an important factor in the process of equipping the armed forces. The presented tools provide a possibility of impartial ranking and correct selection based on the selected indicators.
The MCDM process enables the correct ranking of available models in the design phase of complex combat systems. Based on the output parameters and rankings, a decision concerning the application of the existing system is made, namely, to design a new device or to modernise one of the offered to meet the tactical requirements. The selection of the optimal model defines the next steps in designing subassemblies and systems: fire control systems, optoelectronic and sighting devices and stabilisation systems.

This paper proposes an innovative multi-criteria methodology for weapon systems evaluation based on the use of D numbers. Nine types of automatic cannons designed for integration into combat vehicles have been studied. These weapon systems are intended for direct protection of the crew, fire support and destruction of targets located at the distance from 1500 to 2000 meters. All the automatic cannons belong to the same category and they are all described by seven technical characteristics important for their mounting on the combat vehicle. The proposed multi-criteria methodology represents a novel extension of the LBWA that enables processing of uncertain information. The extension of the LBWA model in the environment of D numbers (LBWA-D) allows rational processing of experts’ preferences while defining significance levels of the criteria. Moreover, the extension of the MABAC method using interval numbers enables taking into account the identified uncertainties. The proposed methodology offers a new multi-criteria framework for the weapon system evaluation and for complex information processing under conditions when complicated investment decisions have to be made. Efficiency of the proposed decision making framework is shown through empirical study. In brief, this paper makes the following contributions:

- It proposes a new multi-criteria methodology for selection of an automatic cannon for integration into combat vehicles using D numbers for processing information.
- The proposed multi-criteria methodology presents a novel extension of the LBWA method that enables efficient processing of linguistic information during a pairwise criteria comparison. Experts’ ambiguities in expressing their preferences in the LBWA method have additionally been processed by the application of D numbers (LBWA-D). The presented LBWA-D methodology enables objective treating of uncertainties that occur in expert preferences.
- The second segment of the presented multi-criteria framework is the novel extension of the MABAC method by application of interval numbers. The presented MABAC methodology has been developed with the aim of rationally processing uncertain information.
- The proposed multi-criteria methodology enables an efficient and objective decision making process in uncertainty, as well as processing of complex information in the conditions of a group expert evaluation of an automatic cannon for integration into combat vehicles.
- The proposed methodology offers a new framework for military management. The proposed methodology is an original methodology for reasoning and processing uncertain information.

After Introduction, the second section presents the theoretical foundation of D numbers on which the development of this new MCDM methodology is based. The third section presents development of the novel MCDM methodology using a LBWA-D algorithm to determine the criteria weight coefficients and an interval MABAC algorithm for evaluation of alternatives. The fourth section demonstrates how to implement the LBWA-MABAC framework to evaluate the alternatives. Finally, the fifth sections offers concluding discussion and guidelines for further investigations.

2. PRELIMINARIES - D NUMBERS

In recent years, there have been numerous extensions of the Dempster-Shafer (DS) evidence theory\(^{24,25}\), which used D numbers to process uncertain information\(^{26,27}\). The use of D numbers eliminates some of the weaknesses of the traditional DS theory, such as:

(i) The problem of conflict management when there is conflicting evidence. This is a problem that has been given a lot of thought in the literature\(^{28,29}\).

(ii) Exclusivity of elements in the frame of discernment\(^{26,27}\), which has greatly limited the practical application of the DS theory.

This problem will be illustrated by the following example. Let us study the formation of the matrix of experts’ preferences. The probability of experts’ preferences can be shown using the linguistic expressions: \(m(\text{low}) = 0.6\), \(m(\text{medium, high}) = 0.4\). Since the DS theory implies exclusivity of linguistic expressions, this kind of probability cannot be expressed using the DS theory (Fig. 1(a)). This significantly limits the use of this theory for solutions of real life problems in different areas.

Using D numbers, these shortcomings of the DS theory are overcome because the elements in D numbers do not have to be exclusive (Fig. 1(b)). These advantages make it possible to use D numbers for solution of various real life problems that involve processing of uncertainties in an objective way. The following definitions briefly introduce mathematical formulations of D numbers. The authors have focused on the formulations used to process uncertain information in the multi-criteria model implemented in this paper.

![Figure 1. Treating of uncertainties: a) DS theory and b) D numbers.](image-url)
3. D NUMBERS BASED LBWA-MABAC METHODOLOGY

This section of the paper proposes a new MCDM methodology which uses D numbers to process uncertainty. This multi-criteria methodology enables exploitation of uncertainty in experts’ preferences using linguistic variables and probability. The concept of D numbers is used to define the criteria weights. Now it is possible to transform dilemmas in experts’ preferences into interval criteria weights. Figure 2 shows the main phases of the hybrid MCDM methodology.

The MCDM methodology uses LBWA to determine criteria weights and the MABAC technique for the weapon system evaluation. The steps of the LBWA-D and MABAC methodology are described in details in the following section.

Step 1: Identification the best criterion. Suppose there is a group of k experts divided into two homogenous groups. Furthermore, suppose we have criteria \( C = \{C_1, C_2, \ldots, C_n\} \). The decision makers arbitrarily choose the most important criterion in the set \( C \). Suppose that the experts have arbitrarily reached a decision that \( C_i \) is the most important criterion in \( C \).

Step 2: The experts group the criteria into subsets or levels of significance (LS) as follows:

\[
LS_1: C_i \in [1,2) ;
LS_2: C_i \in [2,3) ;
\ldots
LS_k: C_i \in [k,k+1) .
\]

Step 3: Pairwise comparisons of the criteria, where each expert group compares the criteria. For example, criterion \( C_j \in S_i \) is assigned a value \( I_{jk} = \{b_{jk}(i),v_{jk}(i)\},\ldots,(b_{jk}(m),v_{jk}(m))\} \), \( b_{jk} \in [0,r] \), \( v_{jk} \leq 1 \). The upper limit on the comparison scale is defined by the expression (1).

\[
r = \max \left\{ \|S_1\|,|S_2|,|K|,|S_4| \right\}
\]

Since we have two homogenous groups of experts, for each group we get the value \( I_{jk} \), i.e. we get \( I_{jk}^1 \) and \( I_{jk}^2 \). For each position \( I_{jk}^1 \) and \( I_{jk}^2 \), a D number is defined as follows:

\[
I_{jk}^1 = \left\{b_{jk}^{1}(i),v_{jk}^{1}(i)\right\},\ldots,(b_{jk}^{1}(m),v_{jk}^{1}(m))\right\}
\]

\[
I_{jk}^2 = \left\{b_{jk}^{2}(i),v_{jk}^{2}(i)\right\},\ldots,(b_{jk}^{2}(m),v_{jk}^{2}(m))\right\}
\]

After the fusion of uncertainties, we apply the expression (2) to define the values \( I_{jk} = \left[I_{jk}^1, I_{jk}^2\right] \), given by interval numbers.

\[
I_{jk} = \left[I_{jk}^1, I_{jk}^2\right] = \begin{cases} I_{jk}^1 = \min \left\{I(D_{jk}^{(1)});I(D_{jk}^{(2)});\ldots;I(D_{jk}^{(m)})\right\} \\ I_{jk}^2 = \max \left\{I(D_{jk}^{(1)});I(D_{jk}^{(2)});\ldots;I(D_{jk}^{(m)})\right\} \end{cases}
\]

where \( I(D_{jk}) = b_{jk} v_{jk} \) stands for the integration operator for the D numbers obtained using the combination rules for D numbers*.

Step 4: Defining the elasticity coefficient. The elasticity coefficient \( r_0 \in N \) should satisfy the condition \( r_0 > r \).

Step 5. We calculate the criteria influence function, expression (3).

\[
f(C_{ij}) = \frac{r_0}{I - r_0 + I_{jk}}
\]

where \( I \) stands for the number of levels/subsets in which the criterion is classified, \( r_0 \) denotes the elasticity coefficient, while \( I_{jk} = \left[I_{jk}^1, I_{jk}^2\right] \).

Step 6: We obtain the weight coefficient of the best criterion:

\[
w_i = \left[w_i^1, w_i^2\right] = \begin{cases} w_i^1 = \frac{1}{1 + f(C_{ij})^r + f(C_{ij})^l} \\ w_i^2 = \frac{1}{1 + f(C_{ij})^l + f(C_{ij})^r} \end{cases}
\]
where \( w_i = [w_i^+, w_i^-] \) presents the weight coefficient of the best criterion, while \( f(C_a^-) \) and \( f(C_a^+) \) respectively stands for the left and right border of the interval of \( f(C_a) \). 

By applying expression (7), we can define the criteria weights of the remaining criteria.

\[
w_j = f(C_j) \cdot w_i = \left[ f(C_j) \cdot w_i^+, f(C_j) \cdot w_i^- \right]
\]

(5)

3.2 MABAC Method – Interval Numbers based Methodology

To this day, the MABAC method has undergone numerous modifications and has been used to solve many problems. The algorithm of interval MABAC methodology is as follows:

Step 1. Evaluate \( b \) alternatives according to \( n \) criteria and to form the initial decision matrix \( X = [x_{ij}]_{b \times n} \).

Step 2. Application of linear max-min normalisation.

\[
C_1, C_2, \ldots, C_n
\begin{bmatrix}
a_1 & t_{11} & t_{12} & \ldots & t_{1n} \\
a_2 & t_{21} & t_{22} & \ldots & t_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_b & t_{b1} & t_{b2} & \ldots & t_{bn}
\end{bmatrix}
\]

\( N = \left[ a_i \begin{bmatrix} t_{i1} & t_{i2} & \ldots & t_{in} \end{bmatrix}_{b \times n} \right] \)

(6)

The \( t_{ij} \) elements are calculated using linear max-min normalisation.

Step 3. Determining the weighted matrix \( V = \left[ IN(v_{ij}) \right]_{b \times n} \), where the elements of the \( V \) matrix are presented as interval numbers \( IN(v_{ij}) = [v_{ij}^+, v_{ij}^-] \), expression (7).

\[
IN(v_{ij}) = IN(w_i) \cdot t_{ij} + IN(w_j)
\]

(7)

Step 4. The matrix border approximation areas (BAA),

\[
IN(g_{ij}) = \left( \prod_{j=1}^{b} IN(v_{ij}) \right)^{1/b} = \left[ \prod_{j=1}^{b} v_{ij}^+ \right]^{1/b} \left[ \prod_{j=1}^{b} v_{ij}^- \right]^{1/b}
\]

(8)

A BAA matrix (9) is formed in the format of \( 1 \times n \).

\[ G = \left[ IN(g_{11}) \ IN(g_{12}) \ldots \ IN(g_{1n}) \right]_{1 \times n} \]

(9)

Step 5. Determining the utility criteria values matrix, \( Q = \left[ IN(q_{ij}) \right]_{b \times n} \), where \( IN(q_{ij}) = IN(v_{ij}) - IN(g_{ij}) \).

Step 6. The utility functions are determined by applying expression (10).

\[
IN(S_i) = \sum_{j=1}^{b} IN(q_{ij})
\]

(10)

Applying the expression (11), the value \( IN(S_i) = \left[ S_i^-, S_i^+ \right] \) is normalised.

\[
IN(\hat{S}_i) = \left[ \hat{S}_i^-, \hat{S}_i^+ \right] = \left\{ \begin{array}{ll}
\frac{S_i^- - \min \left\{ S_i^- \right\}}{\max \left\{ S_i^- \right\} - \min \left\{ S_i^- \right\}} & \text{if } S_i^- < S_i^+
\\
\frac{S_i^+ - \min \left\{ S_i^+ \right\}}{\max \left\{ S_i^+ \right\} - \min \left\{ S_i^+ \right\}} & \text{if } S_i^+ < S_i^-
\end{array} \right.
\]

(11)

where \( \hat{S}_i^- \) and \( \hat{S}_i^+ \) presents the normalised values of \( S_i^- \) and \( S_i^+ \).

After normalisation, applying the expression (12) and (13) we can calculate the final crisp values of the interval number \( IN(S_i) \).

\[
\beta_i = \left\{ \begin{array}{ll}
\frac{\hat{S}_i^- \cdot \left( \hat{S}_i^+ - \hat{S}_i^- \right)}{1 - \hat{S}_i^+ + \hat{S}_i^-} & \text{if } \hat{S}_i^- \cdot \left( \hat{S}_i^+ - \hat{S}_i^- \right) > 0
\\
\frac{\hat{S}_i^+ \cdot \left( \hat{S}_i^+ - \hat{S}_i^- \right)}{1 - \hat{S}_i^+ + \hat{S}_i^-} & \text{if } \hat{S}_i^- \cdot \left( \hat{S}_i^+ - \hat{S}_i^- \right) < 0
\end{array} \right.
\]

(12)

\[
S_i^{crisp} = \min \left\{ S_i^+ \right\} + \beta_i \left[ \max \left\{ S_i^- \right\} - \min \left\{ S_i^- \right\} \right]
\]

(13)

4. CASE STUDY

4.1 Problem Definition

The selection of an optimal weapon system has been performed based on important exact criteria, which were determined taking into consideration the advantages and disadvantages of the weapon systems and the significance of the criteria. That is why the criteria chosen in this study are of crucial importance when the performance of the weapon system and the technical parameters important for its integration into combat vehicles are considered. The criteria irrelevant for this study of integration into combat vehicles, such as price, are not taken into consideration. Material values of these criteria can be assessed as an advantage or disadvantage based on experience. The following seven essential criteria are studied in this paper:

- **Barrel length (C1)** – a barrel is the crucial part of any barrelled weapon, hence its length is very important. A longer barrel enables more accurate and precise shots and consequently firing at greater distances. However, a long barrel has some disadvantages and one of them is that it increases the mass of the weapon. But, since the weapon is integrated into the combat vehicle, this increase does not have significant effects, therefore, the barrel length is considered a positive criterion (B).
- **Rate of fire (C2)** is the number of fired projectiles per unit of time. The rate of fire is directly proportional to the fire power of the weapon. The rate of fire is an essential characteristic of combat vehicles because it enables faster destruction of the target, for that reason, it is considered a positive criterion (B).
- **Length of the weapon (C3)** – it is the distance between the extreme ends of the weapon. It is vital for the process of integration into a combat vehicle and for the stability of the combat vehicle. The length of the weapon is, therefore, considered a negative parameter (C).
- **Muzzle velocity (C4)** is directly dependent on the barrel length and the calibre. This parameter is essential for hitting moving targets. Since the muzzle velocity has a direct effect on the efficiency of the weapon on combat vehicles, it is considered a positive factor (B).
- **Calibre (C5)** – it gives accurate information on the efficiency of ammunition. The calibre is important because the firing efficiency depends on its parameters. The calibre size is considered a positive criterion (B).
• Effective range \((C_6)\) is the ratio of the precision parameters, i.e. the distance at which the weapon will certainly destroy the target. The longer the effective range the greater the distance at which the combat vehicle can fire and destroy the target before the enemy has fired. Therefore, the calibre is considered a positive parameter \((B)\).

• Mass \((C_7)\) – The mass of the weapon on the combat vehicle is not crucial, but it is certainly not negligible. The mass is very significant for the integration process, manoeuvre capability, rate of fire, possibility of use and maintenance. The mass is considered a handicap factor \((C)\).

4.2 Application of MCDM Methodology

4.2.1 Application of the LBWA-D Method

The expert groups have reached a consensus and defined the following seven criteria for evaluation of the alternatives, Table 1.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>B/C</th>
<th>Criteria type</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 Barrel length</td>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C2 Length of the weapon</td>
<td>C</td>
<td>Cost</td>
</tr>
<tr>
<td>C3 Rate of fire</td>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C4 Muzzle velocity</td>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C5 Calibre</td>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C6 Effective range</td>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C7 Mass</td>
<td>C</td>
<td>Cost</td>
</tr>
</tbody>
</table>

The algorithm of the LBWA-D model was used to determine the criteria weights and a detailed procedure of defining the weight coefficients using the LBWA-D model is shown as follows:

**Step 1.** The expert groups defined the criterion \(C_6\) as the best/most influential criterion.

**Step 2.** Grouping the criteria into levels:

\[
S_1 = \{C_5, C_6, C_7\},
\]

\[
S_2 = \{C_1, C_4\},
\]

\[
S_3 = \{C_1, C_2\}.
\]

**Step 3.** The upper limit of the comparisons scale is defined, expression (1).

\[
S_1 = \{C_5, C_6, C_7\},
\]

\[
S_2 = \{C_1, C_4\},
\]

\[
S_3 = \{C_1, C_2\},
\]

\[
\Rightarrow r = \max \{\|S_1\|, \|S_2\|, \|S_3\|\} = 3
\]

The criteria are compared using the values from the interval \(I_s \in [0, 3]\). The comparisons made by the expert groups \((EG1\) and \(EG2)\), by levels of significance, Table 2.

From Table 2, we can see that there are some dilemmas in defining the preferences for the criterion \(C_7\) at the first significance level (Level 1). The experts in the \(EG1\) have dilemmas about certain values on the scale \(I_s \in [0, 3]\), namely about the values 0.5, 1 and 1.5:

(i) The experts in the \(EG1\) are 30% sure that the degree of significance of the criterion is 0.5, therefore the dilemma is represented as \((0.5, 0.3)\);

(ii) The \(EG1\) is 35% sure that the significance of the criterion \(C_7\) lies between 0.5 and 1, hence that dilemma is represented as \((0.5, 1, 0.35)\), and

(iii) The \(EG1\) is 35% sure that the degree of significance is 1.5, so that dilemma is represented as \((1.5, 0.5)\).

Eventually, all the uncertainties in the \(EG1\) can be represented by \(D\) numbers \(D_7 = \{(0.5,0.3), (0.5;1,0.35), (1.5,0.35)\}\). Similarly, \(D\) numbers are formed for the remaining values in Table 2. Applying the combination rule for \(D\) numbers\(^{26}\), a synthesis of uncertainties is performed and unique \(D\) numbers within the subsets (levels) are obtained, Table 3.

In Table 2, there are two \(D\) numbers that represent experts’ preferences: \(D_7 = \{(0.5,0.3), (0.5;1,0.35), (1.5,0.35)\}\) and \(D_7 = \{(1.0,2),(1,1.5,0.35),(2,0.35)\}\). The data used for application of this combination rule for \(D\) numbers\(^{26}\), a synthesis of uncertainties is performed and unique \(D\) numbers within the subsets (levels) are obtained, Table 3.

4.2.2 Application of the LBWA-D Method

Applying the combination rule for \(D\) numbers\(^{26}\), a synthesis of uncertainties is performed and unique \(D\) numbers within the subsets (levels) are obtained, Table 3.

<table>
<thead>
<tr>
<th>Level</th>
<th>D numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS 3: C1</td>
<td>(D_1 = {(2,0.86),(3,0.09)})</td>
</tr>
<tr>
<td>LS 3: C2</td>
<td>(D_1 = {(1,0.63),(2,0.32)})</td>
</tr>
<tr>
<td>LS 2: C3</td>
<td>(D_1 = {(1,0.81),(2,0.19)})</td>
</tr>
<tr>
<td>LS 2: C4</td>
<td>(D_1 = {(0,5,0.34),(1,0.61)})</td>
</tr>
<tr>
<td>LS 1: C5</td>
<td>(D_1 = {(1,0.548),(1,1.5,0.352),(2,0.1)})</td>
</tr>
<tr>
<td>LS 1: C6</td>
<td>(D_1 = {(0,1)})</td>
</tr>
<tr>
<td>LS 1: C7</td>
<td>(D_1 = {(1,0.6),(1,5,0.35)})</td>
</tr>
</tbody>
</table>

In the Table 2 for \(C_7\), there are two \(D\) numbers that represent experts’ preferences: \(D_7 = \{(0.5,0.3), (0.5;1,0.35), (1.5,0.35)\}\) and \(D_7 = \{(0.5,0.3), (1,1.5,0.35),(2,0.35)\}\). The data used for application of this combination rule for \(D\) numbers\(^{26}\), a synthesis of uncertainties is performed and unique \(D\) numbers within the subsets (levels) are obtained, Table 3.

Then, we have:

\[
K_{D} = \frac{1}{Q_{D}} \left( D_1(0.5) \cdot D_2(1) + D_1(0.5) \cdot D_2(1;1.5) + \ldots + D_1(1.5) \cdot D_2(2) \right) = 0.65
\]

\[
Q_1 = D_1(0.5) + D_1(0.5,1) + D_1(1.5) = 1
\]

\[
Q_2 = D_2(1) + D_2(1;1.5) + D_2(2) = 0.95
\]

Table 2. Pairwise comparisons

<table>
<thead>
<tr>
<th>Level</th>
<th>EG1</th>
<th>EG2</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS 3: C1</td>
<td>(D_7 = {(2,0.5),(2,3;0,35),(3,0.15)})</td>
<td>(D_7 = {(2,0.35),(2,2.5,0.45),(3,0.15)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 3: C2</td>
<td>(D_7 = {(1,0.35),(1.2,0.3),(2,0.3)})</td>
<td>(D_7 = {(1,0.65),(2,0.35)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 2: C3</td>
<td>(D_7 = {(1,0.25),(1,2.05),(2,2)})</td>
<td>(D_7 = {(1,0.45),(1,1.5,0.35),(2,0.2)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 2: C4</td>
<td>(D_7 = {(0.5,0.2),(0.5;1,0.35),(1,0.4)})</td>
<td>(D_7 = {(0.5,0.3),(1,1.5,0.4),(1.5,0.3)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 1: C5</td>
<td>(D_7 = {(1,0.35),(1,1.5,0.4),(2,0.25)})</td>
<td>(D_7 = {(1,0.2),(1,1.5,0.55),(2,0.25)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 1: C6</td>
<td>(D_7 = {(0,1)})</td>
<td>(D_7 = {(0,1)})</td>
<td>3</td>
</tr>
<tr>
<td>LS 1: C7</td>
<td>(D_7 = {(0.5,0.3),(0.5;1,0.35),(1.5,0.35)})</td>
<td>(D_7 = {(1,0.25),(1,1.5,0.35),(2,0.35)})</td>
<td>3</td>
</tr>
</tbody>
</table>
Thus we get:

\[ D(1) = \frac{1}{1-K_d}(D(0.5;1.5) - D(0.5;1)) = 0.600; \]

\[ D(1.5) = \frac{1}{1-K_d}(D_1(1.5) - D_1(1;1.5)) = 0.350 \]

Based on the given transformations, we get that 
\( D = \{1.0, 0.6, 1.5, 0.35\} \). The rest of the values in Table 3 are obtained similarly. After fusion of uncertainties, we define interval values of the preferences 
\( i_j = \left[ i^L_j, i^R_j \right] \), Table 5.

### Table 5. Interval values of the preferences

<table>
<thead>
<tr>
<th>Level/Criteria</th>
<th>Interval numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS 3: C1</td>
<td>[0.28, 1.71]</td>
</tr>
<tr>
<td>LS 3: C2</td>
<td>[0.63, 0.63]</td>
</tr>
<tr>
<td>LS 2: C3</td>
<td>[0.38, 0.81]</td>
</tr>
<tr>
<td>LS 2: C4</td>
<td>[0.17, 0.61]</td>
</tr>
<tr>
<td>LS 1: C5</td>
<td>[0.20, 0.55]</td>
</tr>
<tr>
<td>LS 1: C6</td>
<td>[0.00, 0.00]</td>
</tr>
<tr>
<td>LS 1: C7</td>
<td>[0.53, 0.60]</td>
</tr>
<tr>
<td>LS 2: C2</td>
<td>[0.63, 0.63]</td>
</tr>
<tr>
<td>LS 1: C4</td>
<td>[0.17, 0.61]</td>
</tr>
<tr>
<td>LS 1: C6</td>
<td>[0.00, 0.00]</td>
</tr>
<tr>
<td>LS 1: C7</td>
<td>[0.53, 0.60]</td>
</tr>
</tbody>
</table>

#### 4.2.2 Interval MABAC Method

After calculation of the weight coefficients \( w_j \), \( j = 1,2,...,7 \), the alternatives \( A_i (i=1,2,...,9) \) are evaluated using the MABAC method.

**Steps 1 and 2.** In this step, the home matrix \( X = \left[ x_{ij} \right]_{9 \times 7} \) is formed (Table 7) and normalisation of the elements is performed.

### Table 7. Home matrix

<table>
<thead>
<tr>
<th>Alt.</th>
<th>C1 [mm]</th>
<th>C2 [mm]</th>
<th>C3 [mtk/min]</th>
<th>C4 [m/s]</th>
<th>C5 [mm]</th>
<th>C6 [m]</th>
<th>C7 [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2175</td>
<td>2572</td>
<td>200</td>
<td>1100</td>
<td>25</td>
<td>2500</td>
<td>119</td>
</tr>
<tr>
<td>A2</td>
<td>2416</td>
<td>3027</td>
<td>550</td>
<td>1120</td>
<td>30</td>
<td>2500</td>
<td>115</td>
</tr>
<tr>
<td>A3</td>
<td>1400</td>
<td>2210</td>
<td>450</td>
<td>820</td>
<td>20</td>
<td>1500</td>
<td>68</td>
</tr>
<tr>
<td>A4</td>
<td>1800</td>
<td>2207</td>
<td>800</td>
<td>1050</td>
<td>20</td>
<td>2000</td>
<td>47</td>
</tr>
<tr>
<td>A5</td>
<td>2160</td>
<td>2630</td>
<td>400</td>
<td>1360</td>
<td>25</td>
<td>1500</td>
<td>93</td>
</tr>
<tr>
<td>A6</td>
<td>2002</td>
<td>2612</td>
<td>1000</td>
<td>1050</td>
<td>20</td>
<td>2000</td>
<td>47</td>
</tr>
<tr>
<td>A7</td>
<td>2410</td>
<td>3405</td>
<td>200</td>
<td>1080</td>
<td>30</td>
<td>3000</td>
<td>160</td>
</tr>
<tr>
<td>A8</td>
<td>2676</td>
<td>4018</td>
<td>200</td>
<td>1010</td>
<td>35</td>
<td>3000</td>
<td>218</td>
</tr>
<tr>
<td>A9</td>
<td>1710</td>
<td>3060</td>
<td>700</td>
<td>850</td>
<td>20</td>
<td>1500</td>
<td>48</td>
</tr>
</tbody>
</table>

Applying the linear max-min normalisation technique, the home matrix is normalised, Table 8.

### Table 8. Normalised home matrix

<table>
<thead>
<tr>
<th>Alt.</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.607</td>
<td>0.798</td>
<td>0.000</td>
<td>0.519</td>
<td>0.333</td>
<td>0.667</td>
<td>0.579</td>
</tr>
<tr>
<td>A2</td>
<td>0.796</td>
<td>0.547</td>
<td>0.438</td>
<td>0.556</td>
<td>0.667</td>
<td>0.667</td>
<td>0.602</td>
</tr>
<tr>
<td>A3</td>
<td>0.000</td>
<td>0.998</td>
<td>0.313</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.877</td>
</tr>
<tr>
<td>A4</td>
<td>0.313</td>
<td>1.000</td>
<td>0.750</td>
<td>0.426</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>A5</td>
<td>0.596</td>
<td>0.766</td>
<td>0.250</td>
<td>1.000</td>
<td>0.333</td>
<td>0.000</td>
<td>0.731</td>
</tr>
<tr>
<td>A6</td>
<td>0.472</td>
<td>0.776</td>
<td>1.000</td>
<td>0.426</td>
<td>0.000</td>
<td>0.000</td>
<td>0.836</td>
</tr>
<tr>
<td>A7</td>
<td>0.792</td>
<td>0.338</td>
<td>0.000</td>
<td>0.481</td>
<td>0.667</td>
<td>1.000</td>
<td>0.339</td>
</tr>
<tr>
<td>A8</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.352</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A9</td>
<td>0.243</td>
<td>0.529</td>
<td>0.625</td>
<td>0.056</td>
<td>0.000</td>
<td>0.000</td>
<td>0.994</td>
</tr>
</tbody>
</table>
Step 3. By multiplying the criteria weights vectors by the elements of the normalised matrix, we performed the weighted matrix \( V = [IN(v_{ij})]_{n \times r} \), Table 9.

Step 4 and 5. Applying the expression (8), we can calculate the BAA matrix elements which can be used to define the distance of the alternatives from the BAA. Thus determined distances represent matrix elements \( Q = [IN(q_{ij})]_{n \times r} \) based on which the final ranking of the alternatives is done.

Step 6. Determining utility functions of the alternatives and final ranking, Table 10.

Based on the utility functions, we can select the alternative A2 as dominant. Thus we get the final ranking A2>A7>A8>A1>A4>A6>A5>A9>A3.

4.3 Sensitivity Analysis

Since data in MCDM problems often change, it is very important is to perform a sensitivity analysis and study how the changes in the input data affect the solutions\(^35\). In the literature, numerous examples of the sensitivity analysis can be found, such as linear models\(^{36-43}\), statistical models\(^{44}\) etc. The sensitivity analysis has been performed in many papers. Simanaviciene and Ustinovichius\(^{45}\), suggested verification of the obtained solution by carrying out a sensitivity analysis of changes in the criteria weights. Also, some authors\(^{46-48}\) stressed the need to conduct a sensitivity analysis in order to validate the MCDM results. Taking these recommendations into account, we have performed the sensitivity analysis in two stages. First, we analysed the effects of a change in the value of the weight coefficients on the ranking results, and then we analysed the effects of a change in the elasticity coefficient of the Level Based Weight Assessment model on the ranking results.

4.3.1 Changing Criteria Weights

In this section we analyse the effects of a change of the best criteria (C6) on the ranking results. The expression \( w'_{ij} = (1 - w_{ij}) : (1 - w_{ij}) \) is used to define the ratio that a new set of weight coefficients should satisfy in relation to the starting criteria weight values. We formed 50 scenarios in which the new vectors of the weight coefficients were defined, Fig. 3.

Table 9. Weighted home matrix

<table>
<thead>
<tr>
<th>Alt.</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.105, 0.122</td>
<td>0.128, 0.133</td>
<td>0.071, 0.112</td>
<td>0.159, 0.174</td>
<td>0.264, 0.297</td>
<td>0.375, 0.39</td>
<td>0.309, 0.326</td>
</tr>
<tr>
<td>A2</td>
<td>0.118, 0.137</td>
<td>0.11, 0.115</td>
<td>0.102, 0.16</td>
<td>0.163, 0.178</td>
<td>0.33, 0.371</td>
<td>0.375, 0.39</td>
<td>0.313, 0.331</td>
</tr>
<tr>
<td>A3</td>
<td>0.066, 0.076</td>
<td>0.142, 0.148</td>
<td>0.093, 0.147</td>
<td>0.104, 0.115</td>
<td>0.198, 0.223</td>
<td>0.225, 0.234</td>
<td>0.367, 0.388</td>
</tr>
<tr>
<td>A4</td>
<td>0.086, 0.1</td>
<td>0.142, 0.148</td>
<td>0.125, 0.195</td>
<td>0.149, 0.163</td>
<td>0.198, 0.223</td>
<td>0.3, 0.312</td>
<td>0.391, 0.413</td>
</tr>
<tr>
<td>A5</td>
<td>0.105, 0.122</td>
<td>0.126, 0.131</td>
<td>0.089, 0.14</td>
<td>0.209, 0.229</td>
<td>0.264, 0.297</td>
<td>0.225, 0.234</td>
<td>0.339, 0.358</td>
</tr>
<tr>
<td>A6</td>
<td>0.097, 0.112</td>
<td>0.126, 0.132</td>
<td>0.142, 0.223</td>
<td>0.149, 0.163</td>
<td>0.198, 0.223</td>
<td>0.3, 0.312</td>
<td>0.359, 0.38</td>
</tr>
<tr>
<td>A7</td>
<td>0.118, 0.136</td>
<td>0.095, 0.099</td>
<td>0.071, 0.112</td>
<td>0.155, 0.17</td>
<td>0.33, 0.371</td>
<td>0.45, 0.468</td>
<td>0.262, 0.277</td>
</tr>
<tr>
<td>A8</td>
<td>0.131, 0.152</td>
<td>0.071, 0.074</td>
<td>0.071, 0.112</td>
<td>0.141, 0.155</td>
<td>0.396, 0.445</td>
<td>0.45, 0.468</td>
<td>0.196, 0.207</td>
</tr>
<tr>
<td>A9</td>
<td>0.082, 0.095</td>
<td>0.109, 0.113</td>
<td>0.116, 0.181</td>
<td>0.11, 0.121</td>
<td>0.198, 0.223</td>
<td>0.225, 0.234</td>
<td>0.39, 0.412</td>
</tr>
</tbody>
</table>

Table 10. Final ranking

<table>
<thead>
<tr>
<th>Alt.</th>
<th>IN(Q)</th>
<th>Crisp Q</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>[-0.082, 0.213]</td>
<td>0.080</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>[0.019, 0.341]</td>
<td>0.233</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>[-0.297, -0.011]</td>
<td>-0.208</td>
<td>9</td>
</tr>
<tr>
<td>A4</td>
<td>[-0.101, 0.214]</td>
<td>0.067</td>
<td>5</td>
</tr>
<tr>
<td>A5</td>
<td>[-0.137, 0.168]</td>
<td>0.014</td>
<td>7</td>
</tr>
<tr>
<td>A6</td>
<td>[-0.121, 0.203]</td>
<td>0.047</td>
<td>6</td>
</tr>
<tr>
<td>A7</td>
<td>[-0.012, 0.291]</td>
<td>0.178</td>
<td>2</td>
</tr>
<tr>
<td>A8</td>
<td>[-0.036, 0.272]</td>
<td>0.149</td>
<td>3</td>
</tr>
<tr>
<td>A9</td>
<td>[-0.263, 0.038]</td>
<td>-0.156</td>
<td>8</td>
</tr>
</tbody>
</table>
After formation of 50 scenarios, new rankings and new values of the score functions are obtained, Fig. 4.

As seen in Fig. 3, a change of the criterion $C_6$ causes changes in the values of the criteria functions, however, these changes are not big enough to produce changes in the ranking of the alternatives. The analysis has revealed that the ranking of the dominant alternatives (A2 and A7) has remained unchanged despite the significant variations in $w_6$. Finally, we can select two alternatives {A2, A7} as good solutions, although the alternative A2 is slightly better solution compared to the alternative A7.

4.3.1 Analysis of the Effects of the Change in the Value of the Elasticity Coefficient

This section analyse the effects of the change of parameter $\varphi$ on selection of an automatic cannon. In this analysis, $\varphi$ is changed in the interval $\varphi \in [4, 100]$. Since higher values of parameter $\varphi$ cause negligible changes in the criteria weights, the border value $\varphi = 100$ is defined. Figure 5 shows the effects of changes $\varphi \in [4, 100]$ on the change in the criteria weights.

In the next step, we consider the influence of the generated vectors of the criteria weights on the utility functions, Fig. 6. The results depicted in Fig. 6 show that changes of parameter $\varphi$ in the interval of $\varphi \in [4, 100]$ cause changes in the values of the utility functions. Based on the shown results, the alternatives {A2, A7} represent the set of dominant alternatives, where the alternative A2 is better than the alternative A7. It should be pointed out that the parameter $\varphi$ has this kind of influence on the utility functions only for the case studied in this paper. Because of that, it is vital to perform such validation before a final decision is made.

![Figure 4. The effects of the change of the criterion $C_6$ on the change in the score function of the MABAC model.](image)

![Figure 5. The effects of parameter $\varphi$ changes on the change in the criteria weights.](image)
5. CONCLUSION

This paper presents a new way to evaluate a weapon system based on the Dempster-Shafer evidence theory (DSET). D numbers eliminate some shortcomings of the DSET and they are successfully used to process uncertain information. The proposed multi-criteria methodology utilises the advantages of D numbers and implements them into the novel algorithm for MCDM. The extension of the LBWA in the D numbers environment facilitates the rational processing of experts’ preferences while defining the significance levels of the criteria. This paper introduces an extended MABAC model, which also makes a significant contribution to this scientific field.

The main limitation of the D LBWA-MABAC framework is a calculation of probability by which the uncertainty of information in D numbers is represented. Changing the scale for evaluation of qualitative attributes and introduction of more complex functions for aggregation of expert evaluations would further complicate the application of the presented methodology. This can be overcome with a user-oriented decision making software that allows a wider application of the presented algorithm.

The proposed methodology offers a new multi-criteria framework for the weapon system evaluation and for complex information processing when complicated investment decisions have to be made. This paper is a case study that employs multi-criteria techniques. The subject of evaluation is a selection of an optimum automatic weapon system for integration into a combat vehicle. Nine types of automatic cannons intended for direct protection of the crew, fire support and destruction of targets located at the distance from 1500 to 2000 meters have been studied. All these automatic cannons belong to the same category and they are all described by seven technical characteristics important for their application and mounting on the combat vehicle. As a result, the optimal weapon system is chosen by establishing the compromise between the combat performance and limitations imposed by integration into the combat vehicle.

The proposed methodology is flexible and offers a great possibility to improve decision making based on uncertain information. This modern evaluation tool can also be used to evaluate other complex technical systems, or for reaching decisions in the fields of engineering and management. One of the further research directions is the development of a hybrid D-fuzzy and D-fuzzy type 2 methodology which would eliminate shortcomings of the traditional fuzzy approach. It would be of special interest to implement D numbers in the

Figure 6. The effects of parameter $\varphi$ changes on the change in the utility functions.
existing computational intelligence algorithms like monarch butterfly optimisation
existing computational intelligence algorithms like monarch butterfly optimisation, earthworm optimisation algorithm, elephant herding optimisation and moth search algorithm to solve various engineering problems.

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He has contributed to reviewing related literature, data collection, performing the statistical analysis, and writing the draft manuscript.

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