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SHORT COMMUNICATION

Computations of Coordinates in a Multidimensional Digital Prediction System

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ABSTRACT

Prediction of the meeting point for air targets moving in the space with a high velocity and possessing the manoeuverable ability requires considering of the advance point corresponding the shift of the target during the flight of the missile to the target. In the paper, the procedure of computing of coordinates of the moving object (target) used for prediction of the meeting point in the multidimensional digital prediction system is presented. The multidimensional system is understood as a system possessing many systems of coordinates and mutual correlations between them. This system must realize all tasks in a real time.

Keywords: Prediction system, digital system, coordinates system, resolver, real time system

1. INTRODUCTION

Faster development of industry computers, especially constantly increasing velocity of computations, enables realisation in these complex algorithms working in the real time. The algorithm of the multidimensional digital prediction system is one of these. It is applied in the specialised computing machines realising prediction of the position of the air objects. Till now, this task has been realised in the analogue systems which in original are built from the analogue electronic lamp computing machine called a resolver. To replace this resolver by a multidimensional digital prediction system called a digital resolver, the identification of the coordinates existing in the resolver and the algorithms of work of the digital resolver, considering working out all necessary control signals must be done.

One of the most important parameters which must be worked out by the digital resolver is the prediction of the position of the object target–C, i.e., computing of so called meeting point. Coordinates of the advance point are the output data of the resolver such as angles of position of the rotational-swinging system (RSS) and angles of position of the arial of the radio-location tracking head (RTH). The RSS is pointed, not in the spot where the target is at the moment of start of the missile, but in the point existing on the future trajectory of the target in which, according to computations, the missile should meet with the target. This point is called a meeting point. To aim the axis of the RSS to the meeting point, the spatial coordinates of this point must be known. On this base the settings of RSS are determined. The task of computation of the meeting point is called the meeting task. To solve the meeting task, current coordinates of the target, value, and the direction of the velocity vector, and the character of the movement of the target at the time of flight of the missile to the meeting point, must be known. Current coordinates of the target are determined as a result of its tracking by the radio-location station or optical devices. Determination of the value and direction of the vector of the velocity of the target, solving the meeting task and determination of settings of RSS are done constantly by the resolver. Worked out settings–advance azimuth and the elevation angle, are sent to the RSS and as a result, its axis is aimed in a constant way to the meeting point.

2. DEPENDENCE OF COORDINATES OF TARGET AND MEETING POINT-TRANSFORMATIONS

2.1 Algorithm of Determination of Coordinates

For uniform determination of the coordinates of the target in the tracking system of the position of the target (TSPT) and in the control systems of the rocket launcher (RL), the following systems of coordinates were introduced:

(1) The right angle coordinate system XZY, connected with TSPT (Fig. 1):

- O_S Location of TSPT
- Z Axis directed along the base line (from TSPT to RL)
- Y Axis directed vertically upwards
- X Axis directed to the left from the axis Z where:
- φ_1, φ_2 Orientation angles TPST and RL
- ψ_0 Relative azimuth of the target, $\psi_0 = \beta_0 - \varphi_1$ unless $\psi_0 < 0$ $\psi_0 = \psi_0 + 2\pi$
- X_s, X_w Centre line axes of TPST and RL
- B Base vector, a vector connecting TPST and RL
- $\varepsilon_0, \beta_0, D_0$ Spherical coordinates of the target *C* in a coordinate system of TPST

(2) The right angle coordinate system X′Z′Y′ connected with RL, which is created by a parallel shift of the right angle coordinate system X Z Y to the point O_w, location

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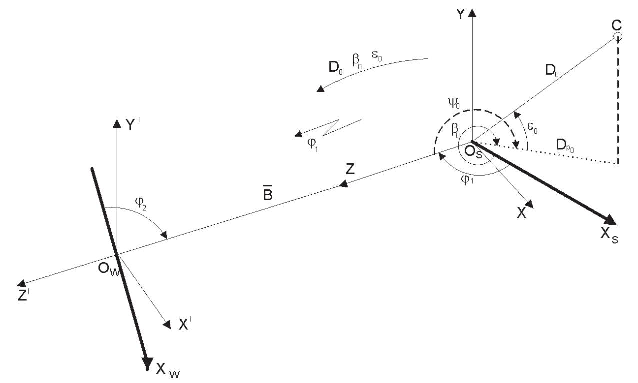

Figure 1. Mutual position of coordinates systemsTSPT and RL.

of the RL along the axis Z for the length of the vectorB:

O_w Location of RL

Z′ Axis directed along the base line in a direction of axis Z

Y′ Axis directed vertically upwards

X′ Axis directed to the left from the axis Z′

(3) Coordinate system $X_w Z_w Y_w$ – connected with RL (Fig.2)

O_w Location of RL

X_w Axis directed along with the longitudinal axis RL

Y_w Axis perpendicular to the rotational surface of the RSS of the launcher and directed vertically upwards

Z_w Axis in the surface of the rotation RSS and creating with axes X_w and Y_w a right angle coordinate system.

Transformation of the right angle coordinate system X′ Z′ Y′ into X_w Y_w Z_w is done by its three time rotation. (Fig. 2).

- Around the axis Y′ in the horizontal surface for the angle ω counterclockwise

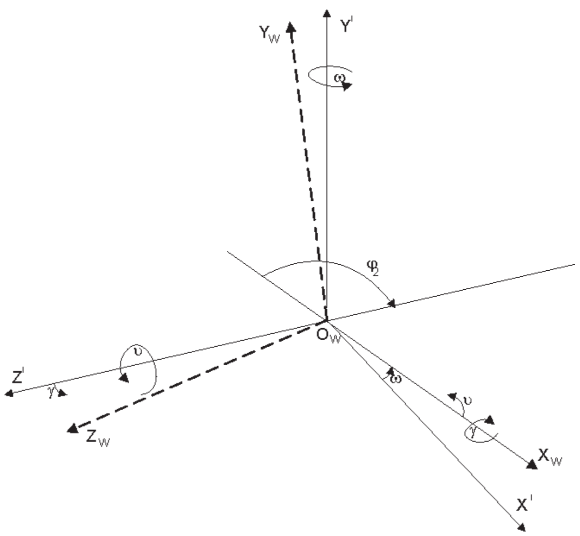

Figure 2. Transformation of the right angle coordinate system X′ Z′ Y′ into $X_w Z_w Y_w$.

- Around the axis Z′ for the angle of the inclination in a vertical surface counterclockwise
- Around the axis X′ for the angle of the inclination in the vertical surface counterclockwise

Transfomation of the right angle coordinate system X′ Z′ Y′ into $X_w Z_w Y_w$ as shown in Fig. 2 where

γ the angle of inclination around the axis X′ of the rocket launcher (RL)

υ the angle of the inclination around the axis Z′ of the rocket launcher (RL)

$$\omega = \begin{cases} \varphi_2 - \dfrac{\pi}{2} \; for\, \omega \geq 0 \\ \dfrac{3}{2}\pi + \varphi_2 \; for\, \omega < 0 \end{cases}$$

Coordinates of the target in the coordinate system TSPT are determined by the spherical coordinates D_0, b_0, e_0. These coordinates are transferred to the coordinate system of RL. Transformation of the coordinates of the target to the coordinate system of the RL $X_w Z_w Y_w$ has been determined according to the algorithm presented below:

(1) The right angle relative coordinates respective to the azimuth y_0 of the target for the TSPT, *XZY* system. These have the form:

$$x = D_0 \cos\varepsilon_0 \cos(\psi_0 + \frac{\pi}{2})$$
$$z = D_0 \cos\varepsilon_0 \sin(\psi_0 + \frac{\pi}{2}) \tag{1}$$
$$y = D_0 \sin\varepsilon_0$$

(2) The right angle coordinates of the target for the RL in a system X′ Z′ Y′- shift of the system *X Z Y* along the vector [0, *B*, 0].
They have the form:

$$X' = X$$
$$Z' = Z - B$$
$$Y' = Y \tag{2}$$

(3) The right angle coordinates of the target for RL in a system $X_w Z_w Y_w$ – taking into account angle rotations ω, υ, γ.
(a) Rotation around the axis Y′ for the angle ω (Fig.3).
After rotation for the angle ω right angle coordinates of the target take the form:

$$x'_\omega = x' \cos\omega + z' \sin\omega$$
$$z'_\omega = z' \cos\omega - x' \sin\omega \tag{3}$$
$$y'_\omega = y'$$

(b) Rotation around the axis Z'_w for the angle of inclination υ (Fig. 4). After rotation for the inclination υ right angle coordinates take the form:

$$x'_{\omega\upsilon} = x'_\omega \cos\upsilon - y'_\omega \sin\upsilon$$
$$z'_{\omega\upsilon} = z'_\omega \tag{4}$$
$$y'_{\omega\upsilon} = y'_\omega \cos\upsilon + x'_\omega \sin\upsilon$$

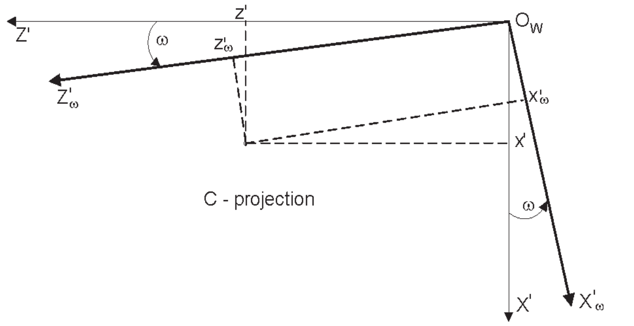

Figure 3. Transformation of the system X′ Z′ Y′ by rotation for the angle ω around the axis Y′.

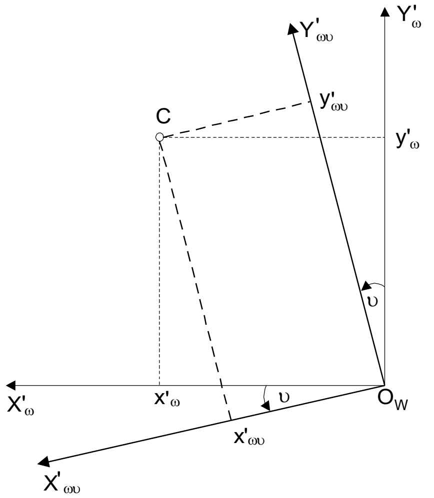

Figure 4. Transformation of the system X′ Z′ Y′ for the angle of inclination υ around the axis Z′$_w$.

(c) Rotation around the axis X′$_{wv}$ for the angle of inclination γ (Fig. 5). After rotation for the angle of inclination γ right angle coordinates of the target take the form:

$$
\begin{aligned}
X'_{\omega\upsilon\gamma} &= X'_{\omega\upsilon} \\
z'_{\omega\upsilon\gamma} &= z'_{\omega\upsilon}\cos\gamma + y'_{\omega\upsilon}\sin\gamma \\
y'_{\omega\upsilon\gamma} &= y'_{\omega\upsilon}\cos\gamma - z'_{\omega\upsilon}\sin\gamma
\end{aligned} \tag{5}
$$

Above right angle coordinates are finally right angle coordinates of the target for the RL in the system $X_w\, Z_w\, Y_w$ (Fig.6), as a result:

$$
\begin{aligned}
x^c_w &= x'_{\omega\upsilon\gamma} \\
z^c_w &= z'_{\omega\upsilon\gamma} \\
y^c_w &= y'_{\omega\upsilon\gamma}
\end{aligned} \tag{6}
$$

2.2 Algorithm of Determination of the Meeting Point

To work properly, the resolver must at first take current spherical coordinates of the target, position of the RSS, and other necessary data for computation of the meeting point, and next smooth these coordinates and work out right angle coordinates of the velocity vector of the target,

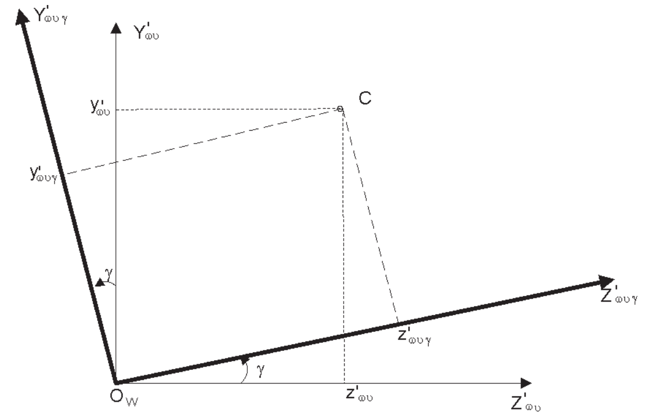

Figure 5. Transformation of the system X′ Z′ Y′ for the inclination angle γ around the axis X′$_{\omega v}$.

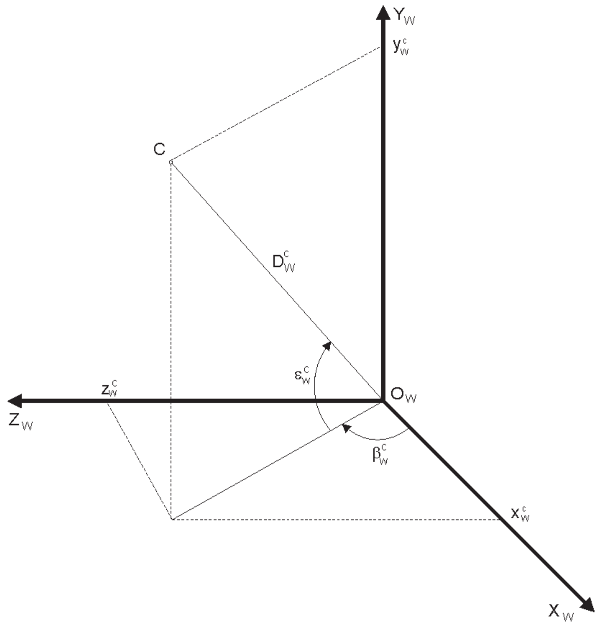

Figure 6. Right angle coordinate system $X_w Z_w Y_w$ connected with the RL.

and finally, compute coordinates of the meeting point. The full algorithm of computations of the digital resolver is the following:

(i) Taking data: spherical coordinates of the target, azimuth, and the angle of elevation of RSS, inclination angles γ, υ of RL.

(ii) Computation of the spherical coordinates of the target to right angle coordinates taking into account the base.

(iii) Smoothing of right angle coordinates of the target using a digital filter of the second order with the analogue transmittance.

$$G(s) = \frac{1}{T^2 \cdot s^2 + 2\cdot\xi\cdot T\cdot s + 1}$$

where $T = \frac{1}{3}$ for the first three seconds of the work of the algorithm and $T = 1$s later; $\xi = 0.8$.

(iv) Computation of right angle coordinates v_x, v_y, v_h of the velocity of the target with the equivalent derivative digital filter of the second order with the analogue transmittance

$$G(s) = \frac{s}{T^2 \cdot s^2 + 2 \cdot \xi \cdot T \cdot s + 1}$$

where $T = \frac{1}{2}$ for the first three seconds of the work of the algorithm and T = 1s later; ξ = 0.8.

(v) Computation of the horizontal velocity of the target

$$v_{cp} = \sqrt{v_x^2 + v_y^2}$$

(vi) Computation of the angle of the course of the target

$$q = arc\,\mathrm{tg}\frac{v_x}{v_y}$$

(vii) Computation of the parameter P of the target

$$P = x_C \cdot \cos(q) - y_C \cdot \sin(q)$$

(viii) Computation of the distance S_w of the target according to the formula

$$S_w = -x_C \cdot \sin(q) - y_C \cdot \cos(q) - v_{cp} \cdot t$$

where t is the time of the flight of the rocket to the meeting point.

(ix) Computation of the horizontal distance from the target

$$d_p = \sqrt{x_C^2 + y_C^2}$$

(x) If the time that passed from the start of the algorithm is longer than 1 s, then the time of the flight of the rocket t to the meeting point should be computed according to the dependences:

$$x_w = x_c + v_x \cdot t$$
$$y_w = y_c + v_y \cdot t$$
$$h_w = h_c + v_h \cdot t$$

(xi) $D_w = \sqrt{x_w^2 + y_w^2 + h_w^2}$

$$D_{w1} = \frac{(t-2)\cdot 1000}{1{,}44}$$
$$\Delta_D = D_w - D_{w1}$$
$$t = t + \frac{\Delta_D}{1000}$$

(xii) If $\left|\Delta_D\right| \geq \frac{D_W}{1000}$ then the former point should be repeated

(xiii) Computation of the horizontal distance to the meeting point $D_{pw} = \sqrt{x_W^2 + y_W^2}$

(xiv) Computation of the azimuth of the meeting point according to the dependence $\beta_W = arc\,\mathrm{tg}\frac{x_W}{y_W}$

(xv) Computation of the elevation angle RSS

$$\varepsilon_W = arc\,\mathrm{tg}\frac{h_W}{D_{pw}}.$$

(xvi) Updating the current time and returning to point (i).

Working according to the above algorithm one gets current needed angle coordinates of the meeting point enabling control of RSS to this meeting point and the head of the rocket to this target.

Values computed in points (vii) and (viii) are displayed on the screen of the resolver and give the basis for taking the decision about launching.

2.2.1 *Algorithm of Determination of the Velocity of the Rocket and the Mileage.*

The velocity of the rocket depends on the temperature of the gunpowder load of the starting engine and starting angles of the rocket. At the end of the starting sector the rocket reaches the velocity 600-700 m/s at the time equal to:

$$\tau_r^{(st)} = 4{,}25 - 0{,}026 T_{gl} \, [^0 C] \quad (7)$$

where $\tau_r^{(st)}$ – time when the rocket reaches the velocity 600-700 m/s at the end of the starting sector, T_{gl} is the temperature of the gunpowder load °C.

Algorithm of computation of the velocity of the rocket and the mileage:

(i) For the T_{gl} determine $\tau_r^{(st)}$ according to the dependence

$$\tau_r^{(st)} = 4{,}25 - 0{,}026 T_{lp} \, [^0 C]$$

(ii) Determine the starting angle of the rocket- (in degrees);

(iii) Determine the average velocity of the rocket on the kinematic track according to the dependence

$$v_{sr} = 673{,}5 + 0{,}63 * T_{gl} + (32{,}5 - 0{,}4 * \Theta) - 5{,}2 * (26 - \frac{D_w}{1000})$$

where D_w is the distance to the meeting point in meter.

(iv) Determine the mileage of the rocket in time $t(s)$ for its flight on the starting sector and kinematic track according to the dependence:

$$s = \frac{v_{sr}\tau_r^{(st)}}{2} + (t - \tau_r^{(st)})v_{sr}$$

2.3 Algorithm of Aiming of RSS to the Meeting Point and RTH of the Rocket to the Target

Starting notations:

$X_w Z_w Y_w$ right angle coordinate system of the rocket launcher RL

$X_R Z_R Y_R$ right angle coordinate system of the rocket

$C(x_W^c, z_W^c, y_W^c)$ coordinates of the target in a coordinate system $X_w Z_w Y_w$

D_w^w distance to the meeting point

$\varepsilon_w^c, \varepsilon_w^w$ elevation angles of the base of the rocket launcher to the meeting point and to the target in a coordinate system $X_w Y_w Z_w$.

(i) For coordinates of the target $C(x_W^c, z_W^c, y_W^c)$ in a system $X_w Z_w Y_w$, The meeting point is determined $PW(x_W^w, z_W^w, y_W^w)$.

(ii) Aim the RSS of the rocket launcher RL to the meeting point $PW\left(x_W^w, z_W^w, y_W^w\right)$ (Fig.7).

For this reason determine rotation angles of the launcher β_L, ε_L.

Elevation angle of the RSS can be computed according to the dependence:

for the height of the target hc $h_c^w \geq 1000\,m$ and

$$\begin{cases} \varepsilon_w^w > 20^0 \ i \ D_w^w \geq 7\,km \\ \text{lub } D_w^w < 7km \end{cases}$$

$$\varepsilon_L = \varepsilon_w^c + \Delta\varepsilon + 2{,}5^0$$

where $\Delta\varepsilon = \varepsilon_w^w - \varepsilon_w^c$

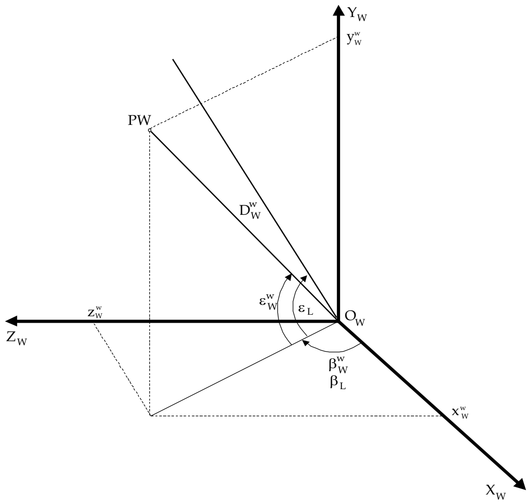

Figure 7. Notations concerning the meeting point PW in a right angle coordinate of coordinates X_w Z_w Y_w connected with a rocket launcher RL.

For the height of the meeting point $h_c^w < 1000\,m$ or $(\varepsilon_w^w \leq 20^0 \ i \ D_w^w \geq 7km)$

$$\varepsilon_L = \begin{cases} 10^o & for \ \ D_w^w \leq 7km \\ 10^o + 2{,}5(D_w^w - 7) & for \ \ 7 < D_w^w \leq 15km \\ 30^o & for \ \ D_w^w > 15km \end{cases}$$

$$\beta_L = arctg\left(\frac{z_w^w}{x_w^w}\right)$$

Determine coordinates of the target in a new coordinate system X_w^* Z_w^* Y_w^* created from the system X_w Z_w Y_w by rotations: around the axis Y_w for the angle β_L – anticlockwise (Fig. 8) around the axis Z_w for the angle ε_L – clockwise (Fig. 9) After rotation around the axis Y_w for the angle β_L coordinates of the target have the form:

$$\begin{aligned} x1r &= x_w^c \cos\beta_L + z_w^c \sin\beta_L \\ z1r &= z_w^c \cos\beta_L - x_w^c \sin\beta_L \\ y1r &= y_w^c. \end{aligned} \tag{8}$$

After rotation around the axis Z_w for the angle ε_L coordinates of the target have the form:

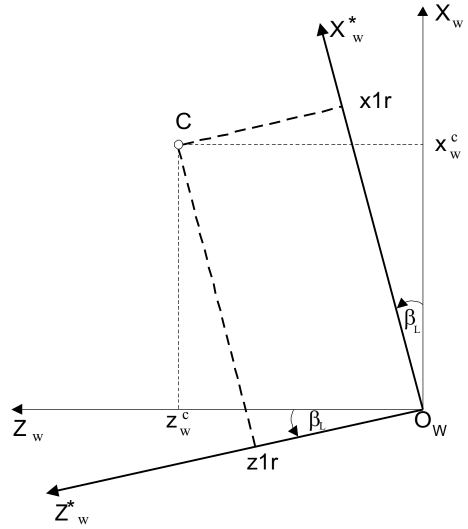

Figure 8. Transformation of the system X_w Z_w Y_w for the angle

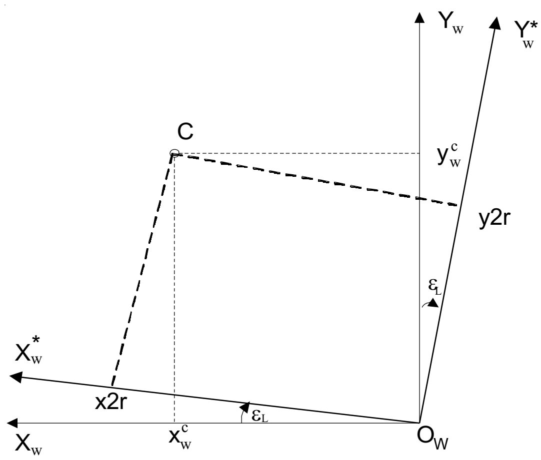

Figure 9. Transformation of the system X_w Z_w Y_w for the angle of rotation ε_L.

$$\begin{aligned} x2r &= x1r\cos\varepsilon_L + y1r\sin\varepsilon_L \\ y2r &= y1r\cos\varepsilon_L - x1r\sin\varepsilon_L \\ z2r &= z1r \end{aligned} \tag{9}$$

(iv) Control RTH to the target in a system $X_R Z_R Y_R$ connected with the rocket, which is created in the following way (Fig.10):

X_R axis directed along the longitudinal axis of the rocket (this axis covers the axis X_W^*)

Y_R, Z_R axes obtained from axes Y_w^*, Z_w^* by rotation of the system X_w^* Z_w^* Y_w^* around the axis X_w^* for the angle 45° anticlockwise (to the left).

$$\begin{aligned} x_R &= x2r \\ z_R &= \frac{\sqrt{2}}{2}\left(z2r - y2r\right) \\ y_R &= \frac{\sqrt{2}}{2}\left(y2r + z2r\right) \end{aligned} \tag{10}$$

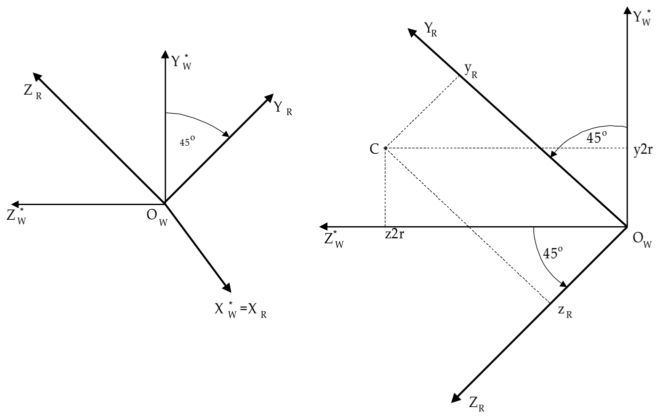

Figure 10. Right angle coordinate system $X_RZ_RY_R$ connected with the rocket.

Direct the head of the rocket to the target for the angles β_g, ε_g determined from the formulae:

$$\varepsilon_g = arctg\left(\frac{y_r}{\sqrt{x_r^2 + z_r^2}}\right)$$
$$\beta_g = -arctg\left(\frac{z_r}{x_r}\right) \tag{11}$$

3. TESTING OF THE SYSTEM

The system was tested at two places, for static and variable values of the parameters of the moving target. At first, it was done in the laboratory, and next at the rocket range.

Laboratory tests were made for static values. These values were included in the original documentation of the anti-aircraft rocket system SA-6. The data, that are right angle coordinates and coordinates of the velocity vector of the target, were introduced to the inputs of the computer. For these data, there were read out values of the angles of rotational swinging system (RSS) and the radiolocation tracking head of the rocket (RTH). The tests which were made in the laboratory required care, because reading out of the values was performed during work of the system.

At the beginning, introduced data with the value of the velocity of the target was equal to 0. The errors of angles of the RSS and RTH were not larger than 1 mrad. Afterwards the data were introduced for the value of the velocity of the target not equal to 0. The errors of angles were not larger than 2 mrad. The values which were registered give a base to the statement that the system worked correctly for static data.

The tests for variable values were made at the rocket range. The data necessary for the work of the system, i.e. spherical coordinates of the real target, were passed from the tracking system of the position of the target to the rocket launcher by the synchronous communication system. The received data were read by the computer, next the computer processed the data according to the algorithm presented in this paper. A signal about tracking of the target by the RTH and the visual observation of the system were the result of this test. Finally, the rocket shooting started. The results of rocket shooting were satisfactory and one for the few of the rockets hit the target directly.

4. SUMMARY

The procedure presented in the paper enables the proper computation of settings and aiming of the rotational-swinging system (RSS) and the tracking head of the rocket (RTH) to the meeting point and the moving target, respectively.

The algorithm of computations was implemented on the industrial computer using Turbo Pascal Language. The tests were made for data included in the documentation of the original system. The tests depended on input data of the target (right-angle coordinates and coordinates of the velocity vector) and read out values of the angles of the rotational swinging system (RSS) and the tracking head of the rocket (RTH). Accuracy of tasks made in tests was not worse that it was 1 mrad when velocity of the target was equal to 0, and 2 mrad when the velocity of the target was not equal to 0.

The dynamic tests were made for the target moving along different trajectories. Tests were realised on the rocket range.

The algorithm has been checked during practical tests and qualification tests and it did fulfill its task correctly. It can be the basis for designing of real time digital prediction systems for controlled flying objects.

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