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Ambiguity Function Method Scheme for Aircraft Attitude Sensor Utilising GPS/GLONASS Carrier Phase Measurement

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ABSTRACT

When the receivers of GPS, GLONASS, COMPASS and other such systems are equipped with multiple antennas, they can give attitude information. Based on the difference carrier phase equations established in local level frame (LLF), a new algorithm is presented to resolve aircraft attitude determination problems in real-time. Presuming that the cycle integer ambiguity is known, the measurement equations have attitude analytical resolutions using single difference (SD) equations of two navigation satellites in-view. Similar with SD process, the double difference (DD) measurements are established and analysed. In addition, the SD and DD algorithms are capable of reducing the integer search space into some discrete point space and then the ambiguity function method (AFM) resolves the ambiguity function within the point solutions space. Therefore the procedures have very low computation, thus saving time. The hardware architecture has been realised using multiple GPS/GLONASS OEMs. The experimental results have demonstrated that the proposed approach is effective and can satisfy the requirement of real-time application in cases of GPS, and combined GPS, and GLONASS.

Keywords: GPS, GLONASS, ambiguity function method, single difference, double difference, attitude sensor

1. INTRODUCTION

When the receivers of GPS (USA), GLONASS (Russian) or COMPASS (China, Bei Dou II) equipped with multiple antennas are configured in the aircraft, they can give attitude information as an auxiliary attitude sensor. The phase differences between signals received by different antennas now constitute the key measurement. Since carrier phase difference measurements are ambiguous, because of the unknown number of carrier signal cycles received, the estimated attitude is, in principle, ambiguous as well. Therefore, the resolution of the signal cycle ambiguity becomes a necessary task before determining the attitude. However, due to the fact that the GLONASS satellites transmit their signals at different frequencies, processing the GPS/GLONASS carrier-phase measurement is much more complicated than processing only GPS data. In processing the GLONASS carrier-phase, one of the critical issues is that the standard double-difference (DD) procedure cannot cancel receiver clock terms in the DD carrier phase measurement [1-3].

As one of the important ambiguity resolution on-the-fly (OTF) approaches, the ambiguity function method (AFM) uses only the fractional value of the instantaneous carrier phase measurement, so the ambiguity function values are insensitive to the whole cycle change of the carrier phase or cycle slips [4-6]. In attitude determination, fixed-length baseline is used to constrain the ambiguity resolution in AFM process. Juang [5] studied the AFM formulas in double-difference measurement to determine the attitude (azimuth) and explored the competitive Hopfield neural network approach to find the attitude, which might have costed more computation load. Xu, *et al* investigated a related search algorithm to solve the GPS attitude determination based on double difference AFM [6]. The AFGA that uses an especially tailored genetic algorithm, overcomes restrictions due to computational overheads incurred by existing AFM techniques [6]. For special real time case, the genetic algorithm is complicated than some AFM methods. Wang, *et al.* studied an improved method using single-difference measurement of two antennas, which is a combination of an analytical resolution and the AFM [7]. The approach reduced the ambiguity, resolution time, and directly obtained the attitude (azimuth and pitch/roll), while some parameters (cable delay and clock bias, etc) should be estimated previously [8].

In this study, an integrated algorithm based on attitude analytical resolution and OTF ambiguity function method is proposed. It computes the analytical solutions of dual nonlinear coupled equations using observations and assumed integer ambiguities of two satellites. Rather than in the 2-D search space, some discrete points are gained, among which the maximum value is found. In addition, the GPS DD measurements are used in the above AFM process. Mathematical description of the proposed approach and practical strategies for data processing are presented and tested using field data sets collected on single- and double-baselines.

2. SINGLE DIFFERENCE MEASUREMENT EQUATION

In Fig. 1, *A* and *B* represent the location of two receiver antennas. *b* is the baseline vector (α, β are yaw and pitch/roll angle), $b = |b|(\cos\beta\sin\alpha \quad \cos\beta\cos\alpha \quad \sin\beta)$. s^i is the

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line-of-sight (LOS) unit vector (θ^i, φ^i are the azimuth and elevation angle of GPS satellite i), $S^i = \left(\cos\varphi^i \sin\theta^i \quad \cos\varphi^i \cos\theta^i \quad \sin\varphi^i\right)$.

In this study, two receivers fed a common clock reference to eliminate the clock bias. As the baseline length was short, both ionospheric delay and tropospheric delay were cancelled [7]. The single-difference carrier phase (SDCP) measurement, which is the projection of baseline in the direction of LOS, can be expressed as

$$\Delta\phi^i_{AB} = \frac{|b|}{\lambda}\left[\sin\varphi^i \sin\beta + \cos\varphi^i \cos\beta \cos\left(\theta^i - \alpha\right)\right] - \Delta N^i_{AB} \quad (1)$$

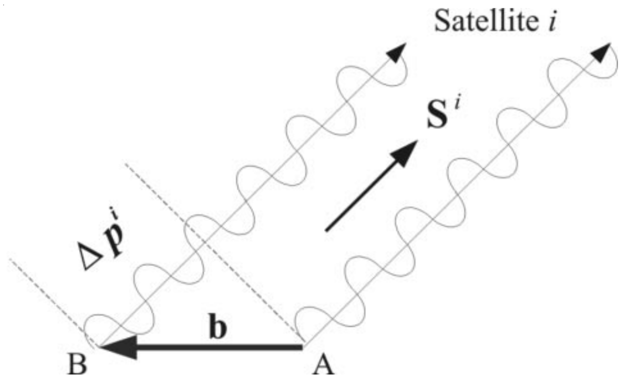

Figure 1. Carrier phase difference measurement model between two antennas.

where λ is carrier wavelength, ΔN^i_{AB} denotes the SDCP integer ambiguity. Equation (1) is a transcendental function, where α, β and ΔN^i_{AB} are unknown and ΔN^i_{AB} is an integer theoretically. Similarly, the observation equations with m_s satellites can be described and have $m_s + 2$ unknown parameters. If the satellites are kept tracking in the next epochs, integer ambiguity will remain constant. Then the real-time integer ambiguity and attitude angles, that change with the movement of the carrier, can be evaluated after many epochs.

For ΔN^i_{AB} known, Eqn (1) in the 3-D space can be demonstrated by a circle on a sphere with centre at point A |b| as the radius. The line between the circle centre and satellite i is vertical to the circle plane. The distance between the two centres of the circle and the sphere is $\lambda[\Delta\phi^i_{AB} + \Delta N^i_{AB}]$. For ΔN^i_{AB} unknown, Eqn (1) describes a series of parallel circles. The vertical distance between the two neighboring circles is λ. The projection of these circles on the 2-D space can be demonstrated by a cluster of closed curves which do not intersect each other, and represent single-differenced integer ambiguities of other satellites j and k. The circles of satellites j, k are not concentric, and there are two intersections Q_1 and Q_2. One of these should be the correct attitude angle, as shown in Fig. 2. The third circle of satellite i consequentially intersects the other two circles respectively. One of the two intersections certainly superposes the correct attitude angle. The correct attitude angle is P1. More closed curves intersect any other closed curve at two intersections, and one of the intersections should be P1[7].

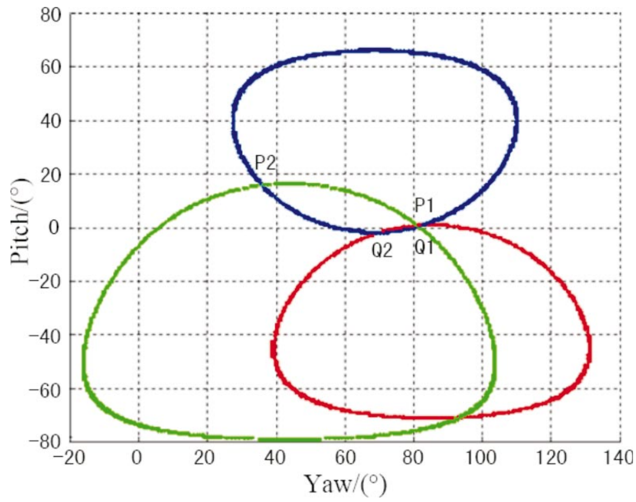

Figure 2. Intersect of three closed curves.

According to AFM approach by Counselman, *et al.*[1], the fixed-baseline ambiguity function is defined as

$$F(\alpha,\beta) = \frac{1}{m_s}\sum_{i=1}^{m_s} \cos 2\pi\left\{\left\{\Delta\phi^i_{AB} - \frac{|b|}{\lambda}\left[\begin{matrix}\sin\varphi^i \sin\beta + \\ \cos\varphi^i \cos\beta\cos(\theta^i - \alpha)\end{matrix}\right]\right]\right\} \quad (2)$$

When α, β are the correct values, the function value should theoretically have the value 1. Therefore, the solution of the adaptive function at P1 is 1, while the values of other intersections should be < 1. If the assumed integer ambiguity is false, the value of the ambiguity function at each intersection should be < 1. However, the final number of intersections passed through extreme certification is limited. If the integer ambiguities of any two satellites are known, one of the two intersections related to two closed curves is the correct attitude angle. In addition, for any observed satellite, the range of the integer ambiguity can be determined and all possible ambiguities can be listed. If the listed number is *n*, there will be *2n* intersections. Finally within these, the correct attitude can be estimated by AF approach. In contrast to the traditional all-area searching methods, the proposed approach is converted to searching among countable discrete points.

3. ATTITUDE SOLUTION FOR SINGLE DIFFERENCE

The key issue of the problem is how to get the intersections of two closed curves quickly. To do this, the following equations are obtained from Eqn (1):

$$\begin{cases} \sin\varphi^i \sin\beta + \cos\varphi^i \cos\beta\cos\left(\theta^i - \alpha\right) = \lambda^i\left(\Delta\phi^i_{AB} + \Delta N^i_{AB}\right)/|b| \\ \sin\varphi^j \sin\beta + \cos\varphi^j \cos\beta\cos\left(\theta^j - \alpha\right) = \lambda^j\left(\Delta\phi^j_{AB} + \Delta N^j_{AB}\right)/|b| \end{cases} \quad (3)$$

Note that λ^i, λ^j represent the wavelength of satellite *i* and satellite *j*, respectively, and these two satellites may belong to different global positioning systems or the same system, such as *i* for GPS and *j* for GLONASS or both for GLONASS. However, the process of Eqn (3) complicated and the final results are given as follows (for GPS $\lambda^i = \lambda^j = \lambda$).

$$\alpha = \pm \arcsin[(-g \pm \sqrt{g^2 - 4h})/2] \tag{4}$$

$$\begin{cases} g = -2(e-d)(1-d) + 4f^2/[(e-d)^2 + 4f^2] \\ h = (1-d)^2/[(e-d)^2 + 4f^2] \end{cases} \tag{5}$$

$$\begin{cases} d = a\cos^2\theta^i + b\cos^2\theta^j + c\cos\theta^i\cos\theta^j \\ e = a\sin^2\theta^i + b\sin^2\theta^j + c\sin\theta^i\sin\theta^j \\ f = \left[a\sin 2\theta^i + b\sin 2\theta^j + c\sin(\theta^i + \theta^j)\right]/2 \end{cases} \tag{6}$$

$$\begin{cases} a = \tan^2\varphi^i(1 - S^{j2})/(S^i - S^j)^2 \\ b = \tan^2\varphi^j(1 - S^{i2})/(S^i - S^j)^2 \\ c = -\tan\varphi^i\tan\varphi^j(1 - S^iS^j)/(S^i - S^j)^2 \end{cases} \tag{7}$$

$$\begin{cases} S^i = \lambda(\Delta\phi^i_{AB} + \Delta N^i_{AB})/(|\mathrm{b}|\sin\varphi^i) \\ S^j = \lambda(\Delta\phi^j_{AB} + \Delta N^j_{AB})/(|\mathrm{b}|\sin\varphi^j) \end{cases} \tag{8}$$

If α is known, β can be obtained easily. According to the descriptions above, there will be two solutions $\alpha_k, \beta_k (k = 1,2)$. It is easier to have the attitude angles solved by Eqn (3) directly than searching the maximum in the full 2-D space. Presume that there are l pairs of preliminary solutions from n pairs of integer ambiguity candidates. They are represented as (α_1, β_1), (α_2, β_2) ,... (α_l, β_l) , among which only one solution is correct which can be found via AFM approach.

4. DOUBLE DIFFERENCE MEASUREMENT EQUATION

For satellite i and j, the double difference measurement can be expressed as

$$\nabla\Delta\phi^{ij} = \frac{1}{\lambda}\mathrm{b}\bullet\left[\mathrm{s}^j - \mathrm{s}^i\right] - \nabla\Delta N^{ij} \tag{9}$$

According to Eqn (2), the double difference ambiguity function is defined as

$$F(\alpha,\beta) = \frac{1}{m_s - 1}\sum_{j=1}^{m_s-1}\cos 2\pi\left\{\nabla\Delta\phi^{ij} - \frac{|\mathrm{b}|}{\lambda}\begin{bmatrix}\sin\beta\left(\sin\varphi^j - \sin\varphi^i\right) \\ +\cos\beta\cos\varphi^j\cos\left(\theta^j - \alpha\right) \\ -\cos\beta\cos\varphi^i\cos\left(\theta^i - \alpha\right)\end{bmatrix}\right\} \tag{10}$$

For three satellites k, i, j , the azimuth and elevation angles are (φ^k, θ^k) $(\varphi^i, \theta^i), (\varphi^j, \theta^j)$ respectively.

$$\begin{cases} \mathbf{s}^i - \mathbf{s}^k = (\cos\theta^i\sin\varphi^i - \cos\theta^k\sin\varphi^k, \cos\theta^i\cos\varphi^i - \cos\theta^k\cos\varphi^k, \sin\theta^i - \sin\theta^k) \\ \mathbf{s}^j - \mathbf{s}^k = (\cos\theta^j\sin\varphi^j - \cos\theta^k\sin\varphi^k, \cos\theta^j\cos\varphi^j - \cos\theta^k\cos\varphi^k, \sin\theta^j - \sin\theta^k) \end{cases} \tag{11}$$

Like SD, the DD equations are obtained from Eqn (5),

$$\begin{cases} \sin\varphi^i\sin\beta + \cos\varphi^i\cos\beta\cos\left(\theta^i - \alpha\right) \\ \quad = \dfrac{\lambda}{|\mathrm{b}|\bullet\left|\mathrm{s}^i - \mathrm{s}^k\right|}\left(\nabla\Delta\phi^{ki} + N^{ki}\right) \\ \sin\varphi^j\sin\beta + \cos\varphi^j\cos\beta\cos\left(\theta^j - \alpha\right) \\ \quad = \dfrac{\lambda}{|\mathrm{b}|\bullet\left|\mathrm{s}^j - \mathrm{s}^k\right|}\left(\nabla\Delta\phi^{kj} + N^{kj}\right) \end{cases} \tag{12}$$

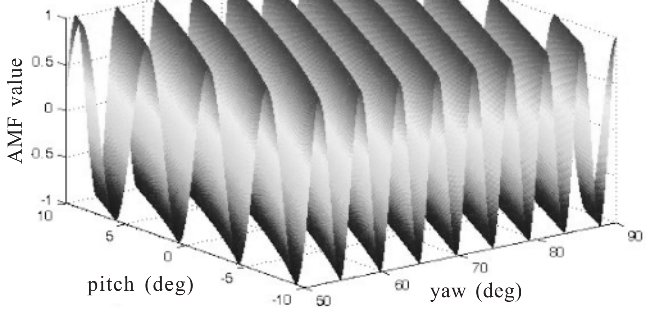

Figure 3. Double difference ambiguity function of two satellites.

Compared with Eqn (3), the yaw and pitch angles α , β of DD measurement equations can be obtained easily.

5. FIELD TEST

5.1 Hardware Architecture

GPS/GLONASS attitude sensor with a set of low-cost components is constructed (Fig. 4). The hardware board can be installed with multiple NovAtel GPS/GLONASS OEMV cards. These OEM cards are used to track and capture the constellation signals and output the original data (pseudo-range, carrier phase and almanac), and then the data are transmitted into PC-104 board. The integer ambiguity resolution and attitude determination are executed in PC-104. Beside some control functions, the PC-104 also offers the PVT (position, velocity and time) function, the initialisation for attitude determination, as well as attitude determination function.

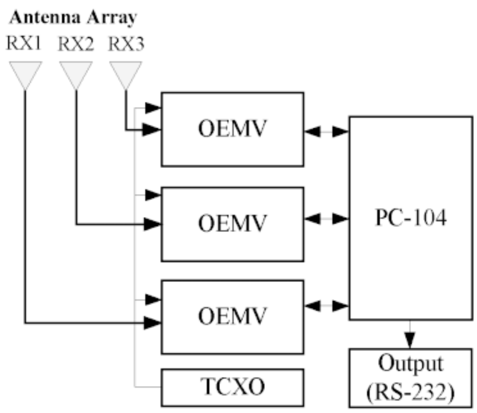

Figure 4. Architecture of GPS/GLONASS attitude sensor.

5.2 SD and DD Experiment

To verify the performance of the suggested algorithm, the whole software structure includes six modules: serial port communication, time and coordinate transformation, navigation message processing, observation data processing, attitude parameter calculation, and cycle integer ambiguity resolution. Static and kinematic experiments were conducted, and the initialisation results were analysed.

Keeping the two antennae steady based on the WDFT high-precision rotary table with the baseline of 1m, we a number of static experiments were carried out to validate the method proposed for longer times. Since the geometric distribution of GPS satellites keeps changing, it is required to detect the new integer ambiguity often.

For SD case, the average attitude angles are 44.16 ° in yaw, and 0.49 ° in pitch. The yaw and the pitch always fluctuate around the average values, as shown in Fig. 5. The standard deviations of yaw and pitch angles are 0.17 ° and 0.29 °, respectively.

For double difference case, the baseline length of two antennae is 2.88 m and the mask angle of the whole test is 10 °, as shown in Fig. 6 and Table 1. So it can be concluded that the proposed algorithm is as accurate as the traditional ones [4].

In dynamic test, integer searching time is about 10 ms while the traditional AFM approaches it is within 80 ms. Compared with the records of a high-precision IMU,

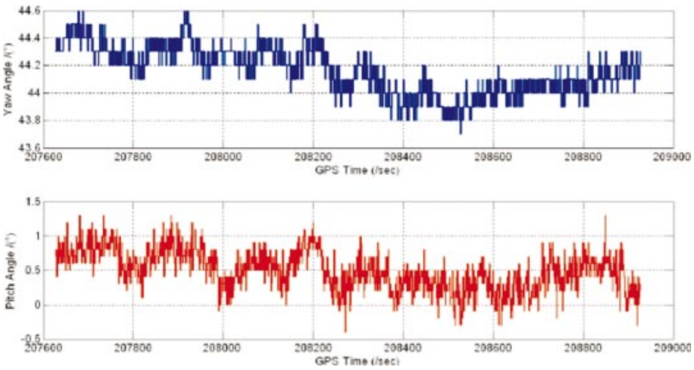

Figure 5. Single-baseline SD attitude results of static experiments.

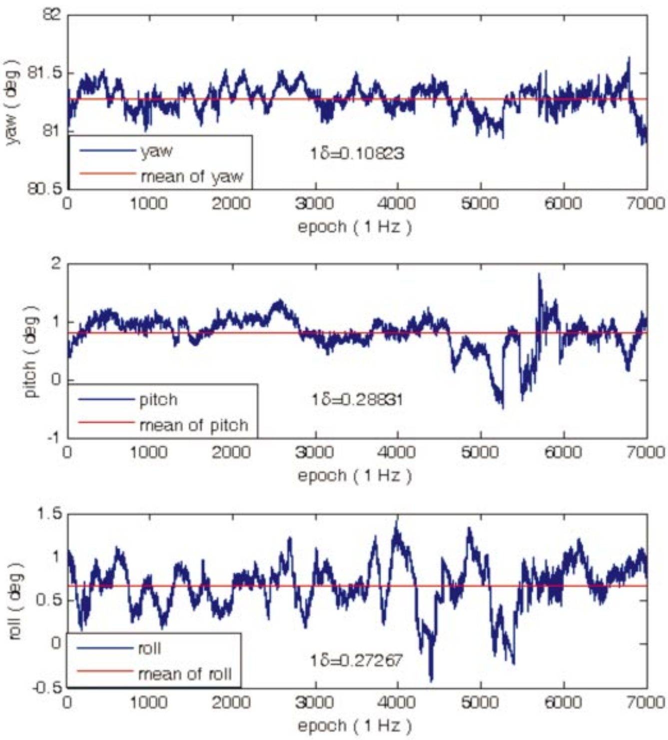

Figure 6. Double-baseline DD attitude results of static experiments.

Table 1. Results of double baseline experiments

Baseline Length (m)	Epoch (1Hz)	Mean (deg)	RMS (deg)
Master baseline = 2.88 Slave baseline = 2.84	7000	Yaw: 81.28	0.11
		Pitch: 0.80	0.29
		Roll: 0.67	0.27

the difference between the two systems is < 0.18 ° without the system error. This indicates the high accuracy and reliability of the proposed algorithm on the fly.

5.3 Combined GPS and GLONASS Experiment

Although GPS and GLONASS are different global positioning systems, the accuracy of the two systems is at the same level and furthermore single differencing method eliminates the system errors and bias largely. In addition, it is testified that the GLONASS augment attributes to improve the accuracy of the combined system. Previously when some obstructions or electronic jamming for GPS are inescapable, the final GPS signal could be lost, which means having to wait to re-initialise and then try to measure again [9]. Using the additional GLONASS signals could mean continuing to work in areas where it was previously not possible.

In Figs 7 and 8, the standard deviation of yaw angle for SD measurement of 6 GPS satellites is 0.1 ° (1σ), while with 8 GPS+GLONASS satellites (GGs), the standard deviation reaches 0.08 ° (1σ). In further investigations, the performance

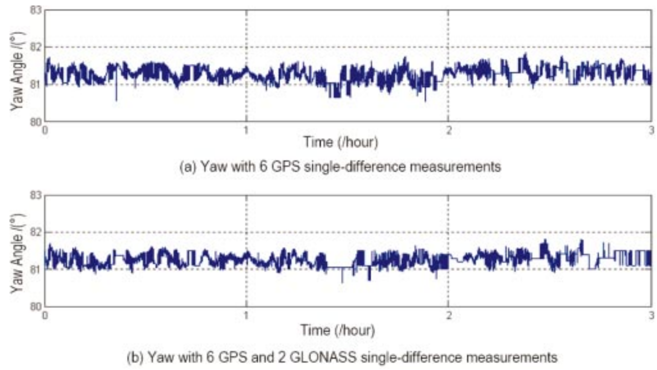

Figure 7. Yaw angle comparison between 6 GPSs and 8 GGs.

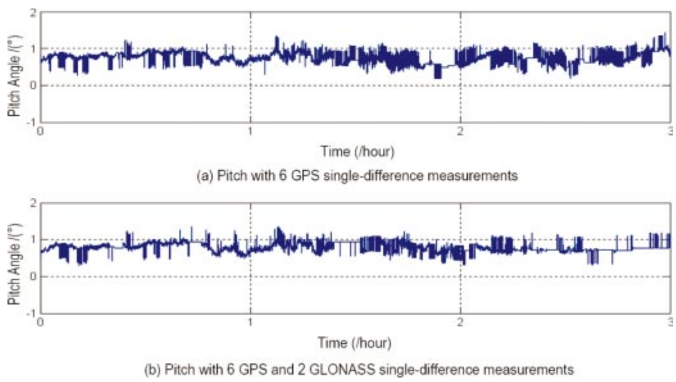

Figure 8. Pitch angle comparison between 6 GPSs and 8 GGs.

of carrier phase integer ambiguity resolution OTF resolution was improved as well. In the case of 6 GPS satellites, only 30 per cent cases could realise the initialisation in one epoch.

After adding two GLONASS satellites, 80 per cent cases above could realise the initialisation in one epoch [10].

6. CONCLUSIONS

An improved integer ambiguity function resolution using analytical resolution for GPS/GLONASS attitude sensor is presented. The proposed equations simplify the previous measurement equations under the conditions of a common reference clock for the receivers and the constraint of spherical surface. The analytical solutions can be provided by an algebraic method or direct computation method other than least-squares search [11]. As a result, the computation time for the candidate solutions reduced greatly. A number of experiments demonstrate that this improved approach significantly outperforms the traditional ones in terms of the computation load. Compared with the traditional DD approach, the improved one is faster than the traditional one on average, with equivalent performance in reliability and accuracy.

REFERENCES

1. Counselman, C.C. & Gourevitch, S.A. Miniature interferometer terminals for earth surveying: Ambiguity and mulitipath with the global positioning system. *IEEE Trans. Geo. Rem. Sens.*, 1981, **19**, 244-52.
2. Hofmann-Wellenhof, B.; Lichtenegger, H. & Collins, J. Global positioning system-theory and practice. Ed. 5, Springer Wien, New York, 2001.
3. Lu, G. Development of a GPS multi-antenna system for attitude determination, Department of Geomatics Engineering, University of Calgary, 1995. (Doctoral Dissertation)
4. Li, Y.; Zhang, K. & Grenfell, R. Improved knight method based on narrowed search space for instantaneous GPS attitude determination. *J. Inst. Navi.,* 2005, **52**(2), 111-19.
5. Juang, J.C. & Huang, G.S. Development of GPS-based attitude determination algorithm. *IEEE Trans. Aero. Elect. Syst.*, 1997, **33**(3), 968-76.
6. Xu, J.N.; Arslan, T.; Wang, Q. & Wan, D.J. An EHW architecture for real-time GPS attitude determination based on parallel genetic algorithm. *In* Proceedings of 2002 NASA/DoD Conference on Evolvable Hardware, Alexandria, VA, USA. pp. 133-141. [doi: 10.1109/EH.2002.1029877]
7. Wang, Y.Q.; Zhan, X.Q. & Zhang, Y.H. Improved ambiguity function method based on analytical resolution for GPS attitude determination. *J. Meas. Sci. Technol.,* 2007, **18**, 2985-990.
8. Li, Y.; Zhang, K.; Roberts, C. & Murata, M. On-the-fly GPS-based attitude determination using single- and double-differenced carrier phase measurements. *GPS Solutions*, 2004, **8**, 93-102.
9. Glenn, Lightsey E. & Madsen, Jared. Three-axis attitude determination using global positioning system signal strength measurement. *J. Guid. Cont. Dyn.,* 2003, **26**(2), 304-10.
10. Jin, W.R.; Zhai, C.R.; Zhang, Y.H. & Zhan, X.Q. Experimental evaluation of novel attitude determination algorithm utilising single frequency GPS/GLONASS receiver. *In* International Symposium on GPS/GNSS 2008, 11-14 November 2008, Tokyo, Japan.
11. Chen, D. & Lachapelle, G. A comparison of the FASF and least-squares search algorithms for ambiguity resolution on-the-fly. *In* KIS-94 Symposium. Banff, Canada, August 1994.

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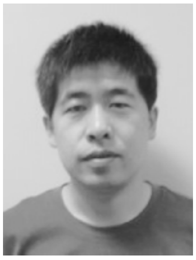

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