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Fuzzy Cognitive Maps for Identifying Critical Path in Strategic Domains

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ABSTRACT

Fuzzy cognitive maps (FCM) are hybrid tools of artificial neural network and fuzzy logic systems. One of their major uses is in decision support systems. To arrive at correct decisions, the fuzzy interconnections between attributes are either trained or assigned by domain experts. The network target may be fixed or unknown. In this paper, the FCM has been illustrated in a strategic domain in which the target is fuzzily defined and unknown. The target value is therefore estimated first by fuzzy inference rules for a collection of imprecisely defined attributes and then the fuzzy gradation to which the value belongs is used as the basis to tune the cognitive map The domain concerns a research institution. The target risk is the time by which the projects run-off beyond the stipulated time of completion. The paper shows that certain instinctively chosen membership functions to tune the cognitive map are able to reproduce the belief surrounding the criticality of the domain.

Keywords: Risk and uncertainty, strategic domain, research and development, fuzzy cognitive map, critical route, decision analysis,decision support systems

NOMENCLATURE

Decision Variables

X_R	Recruitment modes (day)
X_S	Staff exodus(%)
X_F	Fund release time(month)
X_{PME}	PME service(day)
X_{FC}	Financial concurrence(day)
X_{PP}	Payment & purchase(day)
X_{W1}	Workshop delay(%)
X_{W2}	Nature of Workshop jobs(%)
X_E	Equipment repair(month)
X_{SEM}	SEM Lab service(day)
X_{XRD}	XRD Lab service(day)
X_{Chem}	Chemical Lab service(day)
X_{Comp}	Composite Lab service(day)
X_{Ref}	Refractory Lab service(day).
Y_{Risk}	Project Risk (month)

Fuzzy Predicates

WII	Walk-in-Interview
NI	Normal interview
SH	Short
PR	Prolonged
S	Slow
F	Fast
ST	Single tender
LT	Limited tender
OT	Open tender
GT	Global tender
I	Indian
FR	Foreign
L	Low
M	Moderate
H	High
SN	Small nature
MN	Moderate nature
LN	Large nature

1. INTRODUCTION

Decision problems abound strategic domains. Operations research offers mathematical models of judgment and decision analysis in business and military settings. The real world domains of strategy are usually unstructured. These are difficult to formulate because of multiple attributes, distributed decision makers, and lack of knowledge about their interconnections. Human judgment, which suffers from vagueness and imprecision, is often applied in such situations. Fuzzy logic, which deals with imprecision, has a well developed literature[1]. In recent decades, fuzzy logic has been combined with artificial neural network (ANN) in the knowledge-based systems. This has triggered mapping of the knowledge based domains by suitable learning mechanisms. Fuzzy cognitive map (FCM) is such an offshoot technique[2-3].

2. LIMITATIONS OF AI TECHNIQUES

The intelligent decision support systems(DSS) result from the application of artificial intelligence(AI) techniques. These techniques have been used in DSS knowledge bases along with inferential procedures. The most prominent among these are the expert systems. However, such systems suffer from demerit of rule-bases that can not capture all human knowledge. To codify human expertise into an expert

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system, the latter has been combined with fuzzy inference rules. This combination has helped such systems assist the complex decision problems. But expert systems are difficult to maintain and extend, and their limitations are exposed as the decision domain and rule base expand.

The expert systems based on fuzzy inference mechanisms lack the ability to learn and adapt. The artificial neural network (ANN) has evolved as one such AI tool that learns on the basis of feedback mechanisms. The learning ability of ANN proves it superior to expert systems. Ideally, the expert needs to be endowed with such mechanisms. However, the decision trees inherent in expert systems are essentially built on a feed-forward structure. Adaptation of such structure to the ANN's feedback loop mechanisms poses problems. Kosko[2] proposed fuzzy version of cognitive maps as a way out.

3. FUZZY COGNITIVE MAPS

3.1 Scope and Utility

Cognitive map has its origin in the social sciences[4]. Basically it is a graphical representation of causal relationships as perceived by a decision maker among the attributes also called concepts of the given environment. The map is perceived mentally by a single or group of strategists who are domain experts. The attributes are cognitive units with weighted links from one node to another, which are the strength of causal relations. The domain attributes are its input variable, which correspond to the cognitive units. The interactive effect of input variables produces the output or the target value. Figure 1 shows a FCM for a strategic R&D domain. The concepts are indicated by circles while the causal relationships between them are indicated by arrows. The weighted links which reflect relationship strengths between concepts are indicated along the arrows. These links can represent positive, negative and neutral relationships. In traditional cognitive maps, the inter-links are crisp values as -1 and 1. In fuzzy version, the influences are expressed in fuzzy terms as increase, decrease, no change, etc, with fuzzy weighted links. The concepts of the domain can assume any one of three numeric data as –1 (moderately on), off (0), and 1 (on) which are input decision options. How a change in one or more concepts will influence others can be addressed by numeric data assigned to them. Prediction of the outcome of an interaction of causal concepts is possible which can be used to test the consistency of decision. Such use of a cognitive map is based on the assumption that belief system of the domain experts is accurately represented in the map. Policy studies, for example, involve cognitive information which are qualitatively evaluated. Similar situation arises in the war pursuits where the team of commanders fix up strategies on the basis of the intelligence passed on by sleuth and informers. A single or a group of investigators starts with the sleuth's inputs that are rationalised on What–If basis to support strategic decisions. The technology-based enterprises, business industries, diplomacy, security and surveillance tasks, etc, belong to such cognitive domains.

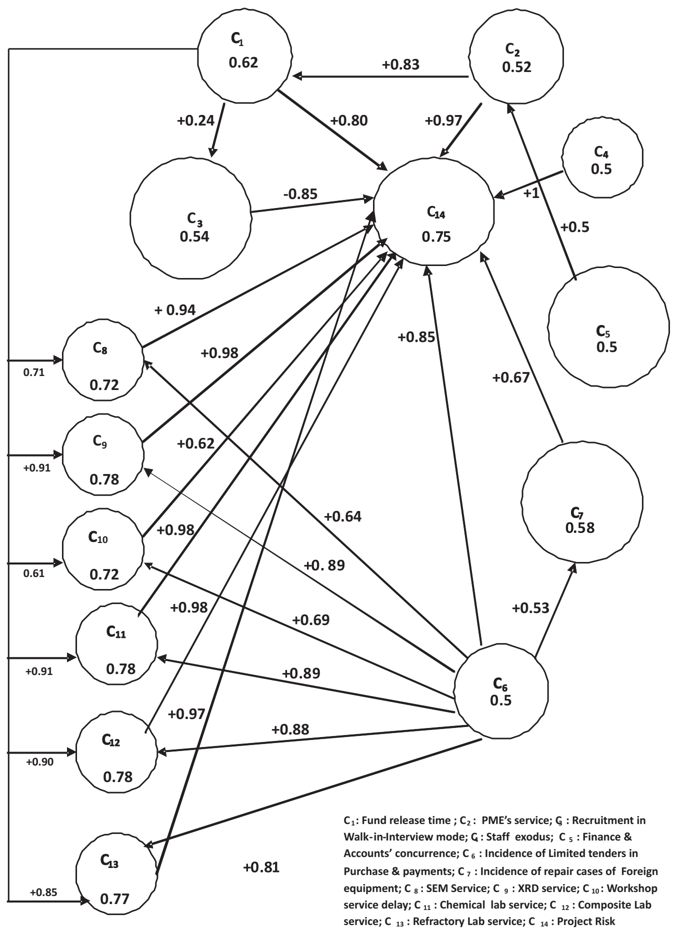

Figure 1. Fuzzy cognitive map of a R&D domain.

3.2 Limitations and Way Out

FCM is a more general representation than classical binary or fuzzy relations. The maps can learn through the feedback loops like neural network and evolve and adapt with time. The decision maker draws the FCM diagram of the given environment or a cluster of environments and interconnects these in the form of a neural network. There are well-defined procedures to process the acquired knowledge. The rule construction of expert systems can thus be avoided. However, in complex domains, the weighted links between concepts are difficult to assign by the domain experts because of highly ad-hoc nature of relations. Use of intuitively chosen membership functions is an alternative. However, such functions are instinctively set and their efficiencies depends on the output behaviour. A qualitative knowledge of the output or target is therefore necessary. Fuzzy inference rules can be used to estimate the target value. The associated gradation of the target can form the basis for FCM analysis.

In the present paper, a couple of intuitively defined functions have been used to connect a set of fuzzy inference rules to the fuzzy cognitive map. The problem domain has been broadly focused in two interconnected perspectives. The fuzzy risk is first estimated and then its gradation is used as the basis to perform a decision analysis by fuzzy cognitive map. This helps to identify the critical path of the domain. The paper shows that the intuitively chosen functions discern the inherent features of the strategic domain.

4. FUZZY INFERENCE RULES

In decision problems, one often faces facts and relationships which have variable degrees of truth or opinion among decision makers. The binary logic of categorical assertions as Yes or No, True or False, etc., can not be employed in such contexts to support decision. Fuzzy logic[5] allows natural language statements to be expressed and operated in a list of rules which, for example, ask IF-THEN questions as

IF *(X_1 is slow* AND *X_2 is Fast ……………OR X_n is medium)* THEN *Y is Moderate*

where X_i's and *Y* are input and output variables of the decision domain. Because of non-repetitive and uncertain nature of variables the input-output can not be defined objectively. This kind of uncertainty does not fit into the axioms of probability theory. It emanates from imprecise domain where lexical or language like words slow, fast, medium and moderate type of gradations on variables work. The defined words have truth as $\mu_A(x) = 1$ if x belongs to *A* and 0 if *x* does not. Here *A* is the lexical word used to describe *x*. This arrangement helps to deal with the inherent imprecision of language. Whether a job can be accomplished in a certain time is not important but the truth that it would be accomplished in the stated time or percentage matters.

To cite an instance, the output from a certain project may get delayed if it receives slack service from its support functional units. The service is a vague word and can be regarded as slow, moderate, and fast based on the experience. The occupancy or business of a functional unit can be linguistically labelled as large natured jobs exist in small percentage or moderate natured jobs exist in moderate percentage or small natured jobs exist in large percentage, etc. Here large, moderate and small are obscure terms. Instead of job completion times, different ranges of percentage volume of jobs completed can be assigned to the lexical words. The percentage volumes in the discourse may overlap and their non-distinct gradations would make 'service' a fuzzy term. To clarify the point further, Fig. 2 displays a membership function of a fuzzy variable. This concerns the output project risk. The risk is decomposed into three lexical gradations of Low(0-6 months), Moderate (3-9 months) and High (6-12 months). The gradation ranges within bracket and the figure reveals that .the gradations overlap and there is no sharp boundary. The change from one level of gradation to another is gradual. A rule may be laid down as: IF the functional unit Workshop serves Slow (Moderate or Fast) THEN the Project risk is High (Moderate or Low). A still more complex statement can be: If in the Workshop Small (Medium or Large) nature of jobs are in Large (Medium or Small) percentage THEN the Project risk is Low (Moderate or High).

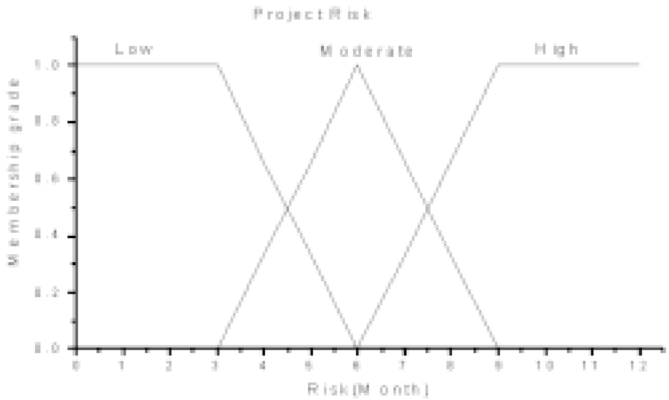

Figure 2. Membership function of project risk.

In the absence of clear boundary, the risk or input service is regarded fuzzy and requires unconventional treatment. The truth designated as μ value lies between 0 and 1. The input variables assume their conventional numerical values. These can be fuzzified by membership functions[6]. These aspects will be discussed in detail in the subsequent sections.

4.1 Fuzzy Operations and Defuzzification

The list of IF-THEN rules after fuzzification are combined and processed by fuzzy mathematical operations. The OR and AND operations are indicated by symbols U and $\cap$ respectively. If X and Y are two conventional sets whose lexical gradations are A and B then OR operator implies union operation as $\mu_{A \cup B}(X, Y) = \text{Max}\{\mu_A, \mu_B\}$ while AND operator for the same sets would be $\mu_{A \cap B}(X, Y) = \text{Min}\{\mu_A, \mu_B\}$. By fuzzy operations, the list of IF-THEN rules applied on input fuzzy decision variables are combined to obtain the fuzzy output which is further defuzzified to get the target value in its conventional numerical terms.

Data and the experts knowledge are used to construct the fuzzy rule base. If there are *n* input variables lexically represented by *m* gradations, there can be n^m possible rules. Such a system construction is not simple so far as the generalisation and applicability are concerned. Both approaches compress the data or knowledge so that general assertions are possible on the basis of simple, small and comprehensible rule bases. Of the two approaches, the experts knowledge is a preferred option in case the data base is scanty.

4.2 Joint Membership Functions

After the target value has been estimated, the FCM has to be tuned with the weighted link values between concepts. In this paper, the weighted links which represent causal influences between concepts are obtained from joint membership functions which justify the degree of belief about relationship between input-output concepts. The input-output relationship in Fig.1 is represented by directed arrows As discussed, the domain attributes are its input-output variables which correspond to the concepts. The values of variables can be used to compute the weighted links. Let the variable *X* represent an input concept, which aims another variable Y that represents an output concept in the cognitive map. The conventional values of $X=x$ and $Y=y$ where either $x \leq y$ or $y \leq x$ can be chosen and the weighted link function can be defined by a couple of joint fuzzy membership functions as

$$\mu_{XY}(x, y) = 1 - x/y \text{ when } x \leq y \quad (1)$$

$$\mu_{YX}(y, x) = 1 - y/x \text{ when } y \leq x \quad (2)$$

In Eqn (1) x represents the chosen value of a decision variable which is much smaller than the variable y and in Eqn (2) y is much smaller than the variable x. In other words, to aim y, x must be complete and hence $x \leq y$ or the vice versa. In both (a) and (b) the denominator can not exceed numerator because to aim an output concept the input concept must be ready. In case x or y =0, $\mu_{XY}(x, y)$ or $\mu_{YX}(y, x)$ =1 and if $x = y$, $\mu_{XY}(x, y)$ and $\mu_{YX}(y, x)$ =0. In neither case the value of input can exceed the output value. To vary the weighted links, in functions Eqns (1) and (2), the conventional values of decision variables are used in a manner that do not violate µ's range of [0,1]. In decision domains, the weighted links between concepts are assigned by experts. The assignment is generally fixed and suffers from experts' personal bias. The defined functions facilitate continuous changes in the weighted links as the values of decision variables change. With changed weighted links, new and newer FCM's can be tuned and used to test the consistency of input options of decision.

5. EVALUATION OF STRATEGY

FCM can be used in both supervised and unsupervised modes. In supervised mode[7], the target output of the domain is fixed in advance while in the unsupervised mode the domain target is not fixed. In the present research, the target output is fixed and estimated. Hence, FCM is used in supervised mode. A critical path is thought of as a chain of concepts which if allowed to slacken will affect the target output.

What-If scenarios are constructed to examine whether increase or decrease in a concept will increase or decrease the target and thereby make it critical. If the concept is assigned a unit value, it is considered active: the belief that the concept would increase or decrease target concept is On. If the concept is assigned a 0 value, the belief is Off.

Let C_i for $i=1,\ldots,n$ be the concepts of the FCM. The input state can be represented by the state vector [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] which implies that only the second element C_2 in the vector has a value 1. To model the effect of I_0 = [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] on the overall map a new state is identified in N^{th} discreet step for each concept C_i at each time t_{N+1} an input state emits. Signal function S indicates if C_i is turned Off (0) or On (1) as

$$C_i(t_{N+1}) = S\left[\sum_{k=1}^{n} e_{ki}(t_N)C_k(t_N)\right]$$

The function involves a matrix-vector multiplication to transform the weighted input to each node C_i. The initial input state I_0 = [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,0] is called and a weighted link matrix E_c of concepts and their product for n=14 concepts is written as

$$I_0E_c = \left[\sum_{k=1}^{14} I_{0k}e_{kl}, \sum_{k=1}^{14} I_{0k}e_{k2}, \ldots\ldots \ldots\ldots ., \sum_{k=1}^{14} I_{0k}e_{k14}\right]$$

where I_{0k} refers to the kth element in the state vector I_0, the weighted link e_{ki} for i, k= 1,…,n refers to entry in the kth row in the ith column of E_c. In case any element in the product matrix $I_0 E_c$ exceeds the threshold value of its concept, the value of concept concerning that element is turned one in I_i for i=.1,…, n. The threshold values are cutoff points which decide if the belief about the kind of relationship between any two concepts should be put On(active) or Off(inactive) as new states emerge. FCM can confirm the belief related to the nature of such relationships. To calculate threshold value, a sigmoid function is used as

$$Y_j = \frac{1}{1 + e^{\sum_{i=1}^{k} e_{ij}x'_i}} \quad (3)$$

where Y_j is the threshold value of the jth concept, x'_i 's are the normalised values of the domain variables x'_i s, which represent the C_i's in the cognitive map. Normalised values are computed as $x'_i = x_i$ /Max{ x_i } where Max{ x_i } are the gradation limits of variables x_i for i=1,…, n. The e_{ij}'s are the weighted links between the concerned concepts indicated by the subscripts for i=1,…,$k \leq n$ and j=1,…, $m \leq n$, respectively. Threshold value indicates how much active are the concepts at any time. These values lie in the range [0,1].

6. ILLUSTRATION

The limitations in knowledge of technological uncertainty compel the research and development managers to address their perspectives in a manner that often overlooks actual pitfalls. The domain of research and development therefore demands strategic decisions. Due to new and non-repetitive tasks, there can be disagreements over degree of human involvement. Hence, tasks can not be objectively scheduled. Also, the project leaders have to be instinctive about the contingent developments such as unexpected blockade of fund, sudden reduction in workforce, typical breakdown of equipment, etc. In regular matters, the disparate strategic missions, variant research perspectives, tedious purchase negotiations for scientific consignments, organisational constraints, etc, can cause undesirable delay and uncertainties. The project risk, which normally concerns time duration, is a complex term. A clear definition of the risk is difficult to perceive. However, research and development activity offers many common templates in terms of timeliness, common managerial guidance, similar review mechanisms, identical technology domain and the common facilities for technical and administrative support functions. Hence, the term project has a generic connotation and does not refer to any specific activity. The delay in interconnected functional units can prolong the project duration. Obviously, the risk assessment in such an environment can be rationalised if the human or non-statistical aspects of uncertainty are properly perceived and utilized. The interconnected functional units can be subjectively classified and defined in linguistic terms. This kind of perception can be modeled by fuzzy logic and correlated to the various units to trace the critical path that can affect the timely completion of projects.

For the purpose of illustration, the research and development system of the Central Glass and Ceramics Research Institute (CGCRI), a CSIR laboratory in Kolkata has been used as a case. Figure 1 presents a FCM description of functional units of the Institute, the project has to interact with during execution. The figure shows that there are internal and external causal variables designated as concepts. These concepts interact with each other and influence the project risk. There are input concepts represented by internal causal variables: administrative unit engages itself in staff recruitment and other functions related to the project; planning, monitoring, and evaluation (PME) cell processes and clears the project proposals; finance and accounts department credits and debits the project's funds and expenditures; the purchase department in phased manner floats tenders, analyses the parties' responses, negotiates with parties, places orders, and finally procures equipment and materials for the projects.

The technical support to the project is provided by an array of common facility units. These are namely the Institute's Workshop unit, Scanning Electron Microscopy (SEM) unit, X-ray Diffraction (XRD) unit, Chemical lab, Composite lab, and Refractory lab respectively. Their functions vary and any delay in work schedules can affect the project risk. Figure 1 further display by arrows that these facilities like many others depend on PME and purchase and payment activities for fund and consignments to provide support to the project. The input concepts represented by external variables are: staff exodus from the projects, fund release time by sponsor and the maintenance and repair time of equipment in the event of failures. These variables are external because they depend on the state of affairs of external world that are beyond control of project leaders but can be managed by perception.

The application of fuzzy inference rules in expert domains is a common knowledge and FCM has been applied to a wide variety of strategic domains[8-11]. In this illustration, fuzzy inference and cognitive systems have been presented in interconnected perspective to trace the critical path of a research organisation. Fuzzy logic tools to perceive research and development uncertainty are of recent origin[12-15].

7. MODEL ASPECTS

To quantify the perception, a list of fuzzy rules is used to estimate the project risk. It connects risks to the input decision variables of the domain. These variables are lexically expressed as that of low, moderate and high gradations. The target variable, project risk grows from low to high if the input decision units slacken. Basically, risk is assessed by the time delays in progress. The delay may be similarly decomposed into three gradations of low, medium and high. Figure 2 displays the project risk. The figure reveals that the risk is initially low but turns high as time progresses with a gradual transition through the medium region in which risk is both low and high. Because the definition of risk is lexical, where exactly low risk becomes medium or the latter becomes high is difficult to distinguish. Experienced scientists can define on the basis of their perceptions the risk gradations and the accompanied range. The non-availability of history of business of input variables can be approximated by guess or membership values computed from certain functions.

7.1 Model Inputs

Table 1 presents the key decision variables or concepts, which have a causal relation with project risk. The abbreviation and units of these variables are presented in the nomenclature. The fifth column in the table indicates that these variables have a definite range of conventional values. Fuzzy predicates, which specify lexical characteristics of these variables, are provided in abbreviated terms in the same column. An approximate numerical range of parameter is offered by the domain experts. Intuitively a value is selected from the range that the expert deems instinctively best under specific situation. The sixth column in Table 1 provides the chosen values of variables. These are posted in the membership functions chosen on the subjective judgment and experience of the expert as given in the second column of the table. The chosen value is thus transformed into fuzzy value. These values are inserted in the last column of Table 1 for each respective variable and its lexical gradations.

7.2 Model Outputs

To perform fuzzy operations on the fuzzy values from the last column of Table 1, three IF-THEN fuzzy rules from Table 2 are used to connect the input causal variables to the target project risk. In Table 2, the OR and AND refer to the fuzzy union and intersection operations carried on the input variables. The variables represent the service times of various functional units that are fuzzified. The OR operation is employed if the work progress of a unit can be judged by either its percentage of job volume or the percentage involvement as per the nature of the jobs. For example for the functional unit workshop, the progress of distinct jobs can be judged in terms of its size or nature while there can be phases when this functional unit is occupied by preponderance of small and non distinctive jobs and therefore the delay can be expressed in percentage. Work progress can not be anticipated in time terms as the jobs are of non-repetitive nature. This reflects the complexity of research and development domain. For the progress of the project in such a case, the membership or truth levels of either job size or its percentage involvement is good enough. This is incorporated in all three rules in Table 2. Similarly OR operation can also be applied to the case of purchase tenders. The rule 3 in Table 2 shows that

Table 1. Membership functions with lexical gradations and variable range

No	Variable/ Concept	Membership function	Function type	Gradation with limit + (a,b,c,d, e)	Value chosen+ (x)	Membership value $\mu_A(x)$
1	X_F/C_1	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	SH : 15,25,35,-,- M : 25,40,55,-,- P : 50,70,90,-,-	20 35 65	0.5 0.66 0.75
2	X_{PME}/C_2	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤c	Triangular	F : 2,3, 4,-,- M: 3.5, 7, 10.5,-,- S : 10. 15, 20,-	2.5 6 13	0.5 0.71 0.6
3	X_R/C_3	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤c	Triangular	WII: 25, , 27.5, 30, -,- WII : 25, 27.5, 30, -,- NI : 30, 37.5, 45, -,-	26.5 26.5 36	0.6 0.60 0.80
4	X_S/C_4	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b	Straight Line	L : 0, 30, -, -,- M: 0 , 60, -, -,- H: 0, 90,-, -,-	20 40 90	0.66 0.67 0.9
5	X_{FC}/C_5	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=1$ where b≤ x ≤ c $\mu_A(x)=\frac{c-x}{c-d}$ where c≤ x ≤d	Triangle cum Trapezoidal	F: 0,7.14,-,- M: 0,3,15,18,- S :7,10,,30, 33,-	10 3 3	0.43 1 1
6	X_{PP}/C_6	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=1$ where b≤ x ≤ c	Semi–trapezoidal	ST: 0,7,12,-,- LT: 10, 15, 20.-,- OT : 10,30,35,-,- GT: 25,30,40,-,-	4 14 27 30	0.57 0.8 0.7 1
7	X_E/C_7	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=1$ where b≤ x ≤ c	Semi–trapezoidal	I : 0,2, 3,-,- FR: 0, 3, 6,-,- FR: 0,3,6,-,-	1 2 2	0.66 1 1
8.	X_{SEM}/C_8	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	F : 0,7,14,-,- M: 6,12,18,-,- S: 10,20,30,-,-	5 10 24	0.71 0.66 0.6
9.	X_{XRD}/C_9	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	F : 0, 2, 4, -,- M : 1.5, 3, 4.5, -,- S : 4, 7, 10, -,-	1.5 3 6	0.75 1 0.6
10	X_{W1}/C_{10}	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	L: 0, 10, 20, -,- M:17..25,31..25,45, -,- H:42.5, 46.25, 50, -,-	5 25 44	0.5 0.54 0.4
11	X_{W2}/C_{10}	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b	Straight Line	SN : 0, 60, -, -,- MN : 0, 30, -, -,- LN : 0, 10, -, -,-	42 15 3	0.7 0.5 0.3
12	X_{Chem}/C_{11}	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	F : 0,3,6,- ,- M : 4,10,16,-,- S : 15,30,45,-,-	4 3 20	0.66 0.50 0.33

Table 1. Membership functions with lexical gradations and variable range (contd.)

13.	X_{Comp}/C_{12}	$\mu_A(x)=\frac{x-a}{b-a}$ where a ≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c	Triangular	F : 0,1, 2, -,- M : 1.5, 3, ,4.5, -,- S : 3.5, 7, 10.5,-,-	1.5 3.5 5	0.5 0.66 0.43
14.	X_{Ref}/C_{13}	$\mu_A(x)=\frac{x-a}{b-a}$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b ≤ x ≤ c $\mu_A(x)=1$ where b≤ x ≤ d	Triangular cum Trapezoidal	F : 0,2,4,-,- M: 3.5,7, 10.5,- S: 7, 10,12,15,-	1 5.25 9	0.5 0.5 0.66
15.	Y_{Risk}/C_{14}	$\mu_A(x)=1$ where a≤ x ≤ b $\mu_A(x)=\frac{c-x}{c-b}$ where b≤ x ≤ c $\mu_A(x)=\frac{x-b}{c-b}$ where b≤ x ≤c $\mu_A(x)=\frac{d-x}{d-c}$ where c ≤ x ≤d $\mu_A(x)=\frac{x-c}{d-c}$ where c≤ x ≤ d $\mu_A(x)=1$ where d≤ x ≤ e	Semi–trapezoidal Triangle Semi–trapezoidal	L: 0,3,6,-,- M: 3,6,9,-,- H: 6,9,12,-,-		

\+ Units of the chosen and limit values along with the abbreviations of the fuzzy predicates are referred in the Nomenclature.

Table 2. IF-Then fuzzy rules for project risk estimation

Rule	Condition	Risk
1	IF the Recruitment is done in walk-in-interview mode AND staff exodus is low AND fund release time is short AND PME clears fast AND financial concurrence is given fast AND payment and purchase are done in single tender mode AND workshop delay is low OR small nature jobs are of large percentage AND equipment to be repaired is Indian AND SEM serves fast AND XRD serves fast AND chemical lab serves fast AND composite lab serves fast AND refractory lab serves fast	THEN project risk is low.
2	IF the recruitment is done in walk-in-interview mode AND staff exodus is moderate AND fund release time is moderate time AND PME clears in moderate time AND financial concurrence is given in moderate time AND payment and purchase are done in limited tender mode AND workshop delay is moderate OR moderate nature jobs are of moderate percentage AND equipment to be repaired is foreign AND SEM serves in moderate time AND XRD serves in moderate time AND chemical lab serves in moderate time AND composite lab serves in moderate time AND refractory lab serves in moderate time	THEN project risk is moderate.
3.	IF the recruitment is done in normal interview mode AND staff exodus is low AND fund release time is prolonged AND PME clears slowly AND financial concurrence is given slow AND payment and purchase are done in either open tender OR in global tender mode AND workshop delay is high OR large nature jobs are of small percentage AND equipment to be repaired is foreign AND SEM serves slow AND XRD serves slow AND chemical lab serves slow AND composite lab serves slow AND refractory lab serves slow	THEN project risk is high.

the purchase and payment can follow either Open or Global tenders as these modes are in effect same. So, truth associated with either of these can work. The AND operation is employed in case progress of the project depends on progress of more than one functional units. The truth levels of all the variables that represent these units have to be taken into account. Table 3 presents the results of fuzzy operations.

The three risk values in Table 3 are in fuzzy terms, which have to be combined and defuzzified to conventional numerical value for further use. The centre of maximum method is employed. The resultant fuzzy values obtained by fuzzy rules are combined by the union operation of fuzzy sets and the combined value is obtained as Max {0.43, 0.5, 0.33}= 0.5. The fuzzy explanation of project risk in Fig. 2 is now used. The combined fuzzy value of 0.5 in Fig. 2 by centre of maximum method corresponds to the moderate risk predicate. Figure 3 shows the defuzzification and the centre of risk plateau 0.5 corresponds to 6 months delay on the time axis. Hence the estimated project risk is 6 months. This value belongs to the moderate risk grade. If membership values are continuously updated, with changes in input values of decision variables the result will also

Table 3. Fuzzy operations on input variables for estimation of fuzzy risk

Rule	Risk type	Fuzzy operations
1	Low	(i) Union operation: $\mu_{\mathrm{L\,U\,SN}}(x_{W1}, x_{W2}) = \mathrm{Max}\{0.5, 0.7\} = 0.7$ (ii) Intersection Operation $\mu_{IT}(x_R, x_S, x_F, x_{PME}, x_{FC}, x_{PP}, x_E, x_{SEM}, x_{XRD}, x_{Chem}, x_{Comp.}, x_{Ref.})$ $= \mathrm{Min}\{0.6, 0.66, 0.5, 0.5, 0.43, 0.57, 0.66, 0.71, 0.75, 0.66, 0.50, 0.50\}$ $= 0.43$ $\mathrm{Min}\{\mu_{\mathrm{L\,U\,SN}}, \mu_{IT}\} = \mathrm{Min}\{0.7, 0.43\} = 0.43$ where $IT = WII \cap L \cap SH \cap F \cap F \cap ST \cap I \cap F \cap F \cap F \cap F \cap F$
2	Moderate	(i) Union operation: $\mu_{\mathrm{M\,U\,MN}}(x_{W1}, x_{W2}) = \mathrm{Max}\{0.54, 0.5\} = 0.54$ (ii) Intersection Operation $\mu_{IT}(x_R, x_S, x_F, x_{PME}, x_{FC}, x_{PP}, x_E, x_{SEM}, x_{XRD}, x_{Chem}, x_{Comp.}, x_{Ref.})$ $= \mathrm{Min}\{0.6, 0.67, 0.66, 0.71, 1, 0.8, 1, 0.66, 1, 0.5, 0.66, 0.5\}$ $= 0.5$ $\mathrm{Min}\{\mu_{\mathrm{M\,U\,MN}}, \mu_{IT}\} = \mathrm{Min}\{0.54, 0.5\} = 0.50$ where $IT = WII \cap M \cap M \cap M \cap M \cap LT \cap FR \cap M \cap M \cap M \cap M \cap M$
3	High	(i) Union operation: (a) $\mu_{\mathrm{H\,U\,LN}}(x_{W1}, x_{W2}) = \mathrm{Max}\{0.4, 0.3\} = 0.4$ (b) $\mu_{\mathrm{OT\,U\,GT}}(x_{PP1}, x_{PP2}) = \mathrm{Max}\{0.7, 1\} = 1$ (ii) Intersection Operation* $\mu_{IT}(x_R, x_S, x_F, x_{PME}, x_{FC}, x_E, x_{SEM}, x_{XRD}, x_{Chem}, x_{Comp.}, x_{Ref.})$ $= \mathrm{Min}\{0.8, 0.9, 0.75, 0.6, 1, 1, 0.6, 0.6, 0.33, 0.43, 0.66\}$ $= 0.33$ $\mathrm{Min}\{\mu_{\mathrm{H\,U\,LN}}, \mu_{\mathrm{OT\,U\,GT}}, \mu_{IT}\} = \mathrm{Min}\{0.40, 1, 0.33\} = 0.33$ where $IT = NI$ Ç $H \cap PR \cap S \cap S \cap FR \cap S \cap S \cap S \cap S \cap S$

be updated. The moderate risk grade thus forms the basis for decision analysis.

8. DECISION ANALYSIS

To examine the influence of various decision variables on the target risk , a simulation is performed on the FCM under case. For this purpose, the decision variables will now represent the concepts. The nature and extent of causal influences between different concepts are reflected in Fig. 1 by the sign and magnitude of weighted links values. The matrix E_C as shown in Table 4 consists of all the weighted links e_{ij}'s. In the absence of links, the e_{ij}'s

Table 4. Weighted link value matrix with thresholds

Concepts	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
C_1	0	0	0.24	0	0	0	0	0.71	0.91	0.61	0.91	0.90	0.85	0.80
C_2	0.83	0	0	0	0	0	0	0	0	0	0	0	0	0.97
C_3	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.85
C_4	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C_5	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0
C_6	0	0	0	0	0	0	0.53	0.64	0.89	0.69	0.89	0.88	0.81	0.85
C_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0.67
C_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.94
C_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0.98
C_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0.62
C_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0.98
C_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0.98
C_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0.97
C_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Threshold Value	0.62	0.52	0.54	0.50	0.50	0.50	0.58	0.72	0.78	0.72	0.78	0.78	0.77	0.75

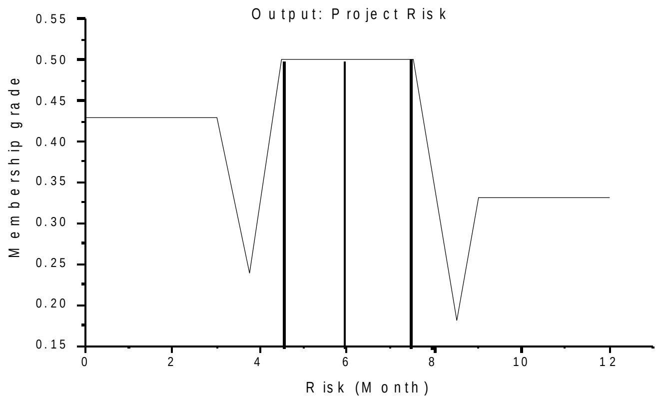

Figure 3. Membership function of output project risk (defuzzified).

represents the causal strength of the belief. Figure 1 also reveals that the concept C_{14} has a threshold value of 0.75. Increased service time by PME indicates that a delayed service would inflate the project risk. Set C_2= 1 in I_0 implies that C_2 is 'on'. By on it is meant that the belief that a delayed PME service time will increase the project risk is made active by the decision maker. The effect of concept C_2 on the project risk is examined. The elements in the product I_0E_c that exceed the threshold strength of the respective concepts are made active with placement of the value 1 in the concept position in new state vector I_1. The Table 5 shows the state and product matrices. The concept C_{14} is turned on as its element value of 0.97 in the product matrix exceeds its threshold strength. This means increased service time by PME will increase the project risk. The next input state I_1 is passed through E_C to get a new product matrix as I_1 E_C. Now C_1: Fund release time is turned on in I_2 and the latter is passed through E_C again. The elemental value 0.83 › 0.62 implies that the element of C_1 in the product matrix has exceeded C_1's threshold strength. This reveals that the sustained increase in service time of PME will also delay the fund release time by the sponsor. The new state vector is now I_3. One can proceed as before and finally see that the state vectors I_3 and I_4 are equivalent which means the simulation process has converged. The FCM has reached the limit state I_4. Table 5 shows how new concepts are turned on. It may be noticed that in the state vector I_4, the concepts C_9 = 0.91 › 0.78, C_{11} = 0.91 › 0.78, C_{12} = 0.90 › 0.78, C_{13}= 0.85 ›0.77 and C_{14} = 1.77 › 0.75 are turned to 1 because their elemental values in the product matrix are found greater than their threshold values.

Table 5. Results of FCM simulation

State i	State vectors I_i	Product matrix $I_i E_c$	Concepts Set On	Concepts Turned on
0	I_0 = {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	[0, 0.83, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.97]	C_2	---
1	I_1 = [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]	[0.83, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.97]	C_2	C_{14}
2	I_2 = [1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]	[0.83, 0, 0.24, 0, 0, 0, 0, 0.71, 0.91, 0.61, 0.91, 0.90, 0.85, 1.77]	C_2 , C_{14}	C_1
3	I_3 = [1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1]	[0.83, 0, 0.24, 0, 0, 0, 0, 0.71, 0.91, 0.61, 0.91, 0.90, 0.85, 1.77]	C_1, C_2 , C_{14}	C_9, C_{11} , C_{12}, C_{13}
4	I_4 = [1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1]	[0.83, 0, 0.24, 0, 0, 0, 0, 0.71, 0.91, 0.61, 0.91, 0.90, 0.85, 1.77]	C_1, C_2 , C_9, C_{11} C_{12}, C_{13} ,C_{14}	C_1, C_2 , C_9, C_{11} C_{12}, C_{13} ,C_{14}

are treated as 0. All weighted links are indicated in Fig. 1 along arrows. Equations (1) and (2) from Section 4.2 are used to obtain the weighted links between concepts while Eqn. (3) from Section 5 is used to compute the threshold values of concepts. For example, to compute the e_{21}= 0.83 between concepts C_2 and C_1 in Fig. 1 we use the input values x = 6 , y = 35 from the fifth column of Table 1 for variables X_{PME} and X_{FC} in Eqn. (a) and to compute threshold value C_2 we use the concept C_5 which has a link with the former. The normalised value $X'_{FC} = X_{FC}/\max\{X_{FC}\}$=3/18 are for X_{FC} from the fifth and sixth columns of Table 1 for moderate risk. This normalised value is multiplied by weighted link, e_{52} = 0.5 from Fig. 1. The product is raised to negative exponent in the threshold value Eqn. (3) and the corresponding value of 0.52 is obtained. For all calculations, the values of input decision variables which correspond to the moderate grade are used as the latter forms the basis of analysis. The threshold value of concepts C_4 , C_5 and C_6 are taken as 0.5, the middle of concept values [0,1] as no arrow targets them. The weighted link $e_{4\ 14}$ = 1 is a crisp value.

The belief of an increased service time of concept C_2: PME will increase the concept C_{14}: project risk can be tested as an example. In Fig. 1 the value of $e_{2\ 14}$ = 0.97 along the arrow directed from concept C_2 to concept C_{14}

8.1 Critical Path

The analysis reveals that the sustained increase in the fund release time due to slackened service by PME will prolong the activities of a set of concepts that are common facility units. These are C_9: XRD lab, C_{11}: chemical lab, C_{12} : composite lab and C_{13} : refractory lab respectively. Delay in PME service will delay the fund release process.

Consequently these facilities will suffer as their service times will slacken. These results are discerned with the nature of causal relationships described in FCM by these critical concepts. Thus, the belief of experts incorporated in the map is reproduced. PME is the root critical concept while the common facility units constitute its offshoots. Finally, critical path consists of input concepts {C_2, C_1, C_9, C_{11}, C_{12}, C_{13},} which are believed to be risky because the project risk, $C_{14} = 1$, is activated during simulation in all the state vectors. The decision variables that correspond to these concepts are therefore critical and their chain constitutes the critical path of the organization.

9. CONCLUSIONS

A list of fuzzy inference rules has been constructed to estimate target risk in a public funded R&D domain. Fuzzy concept proves flexible as they can operate on grey definitions and events created by technological uncertainty. The fuzzy rules are connected to the FCM of the domain by a couple of intuitive membership functions. Decision options have been tested to identify the critical path of the domain. The FCM can be extended to facilitate knowledge flows between network of several domains, technology agencies and industries. Trans-disciplinary factors such as trust, communication, shared values, cooperation and conflict can be examined[16] for evaluation of strategic and global alliances. The future studies can explore the more complex issues.

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