

# Creep Response of Rotating Composite Discs having Exponential, Hyperbolic, Linear and Constant Thickness Profiles

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## ABSTRACT

The study compares the steady state creep response of rotating Al-SiC discs having constant, linear, hyperbolic and exponential thickness with different thickness profiles. All the discs are assumed to have equal volume with the same average thickness. The creep behaviour of the disc material is described by threshold stress based law while the yielding is assumed to follow Tresca criterion. The variable thickness disc is observed to have superior creep response, expressed in terms of stresses and strain rates, to a constant thickness disc. Amongst variable thickness discs, the creep response is observed to be superior for linear thickness disc, when the inner thickness of all the discs is kept the same. However, for the same outer thickness, the disc having hyperbolic thickness profile exhibits the best creep response.

**Keywords:** Creep modelling; Rotating disc; Aluminium composites; Variable thickness; Strain rates

## 1. INTRODUCTION

Rotating disc is a vital component that finds use in various structural and engineering devices like turbine rotors, ship propellers, jet engines, automotive brakes, flywheels<sup>1</sup>. In most of these applications, the disc is subjected to severe thermo-mechanical loadings and hence vulnerable to creep<sup>2-5</sup>. Under such loading conditions, the use of Al-SiC<sub>p</sub> (p refers to particle shape) composite has been suggested by several investigators, owing to their light weight, high specific strength and high temperature stability<sup>6-8</sup>. A number of studies have been performed to investigate elastic, elastic-plastic and creep response of rotating discs having different thickness profiles. The stresses and strains produced in the disk due to centrifugal loading are noticed to decrease significantly by using radially decreasing thickness profile (viz. linear or hyperbolic), as compared to constant thickness disc<sup>9-13</sup>. It has been attributed to decrease in disc weight with increasing radius, which results in lower centrifugal loading.

Deepak<sup>14-15</sup>, *et al.* analysed creep stresses and creep strain rates in uniform and functionally graded (FG) discs made of Al-SiC<sub>p</sub>. Garg<sup>16</sup>, *et al.* revealed that the creep stresses and creep rates in a linear thickness Al-SiC<sub>p</sub> FG disc are significantly lower than a similar disc of constant thickness. Khanna<sup>17</sup>, *et al.* compared creep performance of a linear thickness FG disc using Tresca and von-Mises criteria. Jalali & Shahriari<sup>18</sup>, used finite difference method to analyse elastic stress in a variable thickness rotating FG disc. Sharma<sup>19</sup>, *et al.* used Seth's transition theory to analyse creep in a rotating disc

having exponential thickness. The variable thickness disc was noticed to have less circumferential stress than the flat disk. In a similar study, Sharma & Maheshwari<sup>20</sup>, noticed that FG rotating disk made of isotropic material performs better than that made of orthotropic material.

The literature consulted reveals that studies have been undertaken to investigate creep in rotating disc having linear, hyperbolic and constant thickness profiles. The studies on rotating disc having exponential profile are rather scant. It would be interesting to estimate creep in exponential disc and compare the results with that estimated for discs having linear, hyperbolic and constant thickness profiles, so as to select suitable thickness profiles for given operating conditions. In this light, the present study attempts to analyse creep in a rotating Al-SiC<sub>p</sub> disc having exponential thickness profile. The stresses and strain rates estimated are compared with those available/estimated for similar discs having constant, linear and hyperbolic thickness profiles. The investigation is carried out for discs having the same (i) inner thickness and (ii) outer thickness.

## 2. MATHEMATICAL MODELLING

### 2.1 Disc Thickness Profiles and Distribution of Reinforcement

The creep analysis is carried out for rotating disc made of Al-20 % SiC<sub>p</sub> (by Vol.) composite having four different thickness profiles, viz. constant, linear, hyperbolic and exponential. The inner and outer radii of the discs are kept as 31.75 mm and 152.4 mm, similar to that taken in previous work<sup>11</sup>. The thickness  $h(r)$  of the exponential disc, at any radius  $r$ , is assumed to vary as,

$$h(r) = h_b e^{\beta \left( \frac{b-r}{b-a} \right)} \quad (1)$$

where  $h_b$  is disc thickness at the outer radius  $b$  and  $\beta$  is disc thickness index.

The thickness profile of the hyperbolic disc is also taken from the earlier work<sup>11</sup> as,

$$h(r) = h_b \left[ \frac{r}{b} \right]^k \quad (2)$$

where  $k$  is disc thickness index.

For linear thickness disc, the profile is given by<sup>14</sup>,

$$h(r) = h_b + 2c(b-r) \quad (3)$$

where  $c = \frac{(h_a - h_b)}{2(b-a)}$  and  $h_a$ , and  $h_b$  are disc thickness at the inner and outer radii, respectively.

The volume of all the variable thickness discs, as above, is kept equal to that of constant thickness disc. Therefore,

$$\int_a^b 2\pi r h(r) dr = \pi (b^2 - a^2) t \quad (4)$$

where  $t$  ( $=25.4$  mm) is the thickness of constant thickness disc<sup>11</sup>.

Substituting  $h(r)$  from Eqns. (1) - (3) into Eqn. (4), one gets,

$$h_b = \frac{-\beta^2 (a+b)t}{2[\beta b - a + b - e^{\beta(a\beta - a + b)}]} \quad (5)$$

$$h_b = \frac{(2+k)b^k (b^2 - a^2)t}{2(b^{k+2} - a^{k+2})} \quad (6)$$

$$h_b = \frac{3(b+a)t - h_a(b+2a)}{2b+a} \quad (7)$$

## 2.2 Creep Law

The disc material is assumed to undergo steady state creep according to the threshold stress, as described by<sup>14</sup>,

$$\dot{\epsilon} = [M \{ \bar{\sigma} - \sigma_0 \}]^n \quad (8)$$

where  $\dot{\epsilon}$ ,  $\bar{\sigma}$ ,  $\sigma_0$ ,  $M(r)$  and  $n$  ( $=5$ ) are respectively effective strain rate, effective stress, threshold stress, material dependent creep parameter and true stress exponent<sup>14</sup>.

## 2.3 Development of Governing Equations

The constitutive equations for creep in terms of polar coordinate system, described earlier<sup>1</sup>, when the principal stresses are directed along  $r$  (radial),  $\theta$  (tangential) and  $z$  (axial) directions, under plane stress condition, as observed in a thin disc, are expressed as,

$$\dot{\epsilon}_\theta = \frac{\dot{\epsilon}}{2\bar{\sigma}} [2\sigma_\theta(r) - \sigma_r(r)]$$

$$\dot{\epsilon}_r = \frac{\dot{\epsilon}}{2\bar{\sigma}} [2\sigma_r(r) - \sigma_\theta(r)] \quad (9)$$

$$\dot{\epsilon}_z = \frac{\dot{\epsilon}}{2\bar{\sigma}} [-\sigma_r(r) - \sigma_\theta(r)]$$

where  $\dot{\epsilon}$  and  $\sigma$  denote the strain rate and stress respectively.

The disc material is assumed to yield according to Tresca's criterion<sup>21</sup>,

$$\bar{\sigma} = \sigma_0 \quad (\text{since in a disc } \sigma_\theta > \sigma_r > \sigma_z) \quad (10)$$

The strain rates may be also expressed as

$$\dot{\epsilon}_r = \frac{d\dot{u}_r}{dr} = \left[ M(r) \left\{ \bar{\sigma} - \sigma_0(r) \right\} \right]^n \frac{[2x(r)-1]}{2} \quad (11)$$

where  $x(r) = \frac{\sigma_r(r)}{\sigma_\theta(r)}$  and  $\dot{u}_r = du/dt$  is the radial deformation rate.

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} = \left[ M(r) \left\{ \bar{\sigma} - \sigma_0(r) \right\} \right]^n \frac{[2-x(r)]}{2} \quad (12)$$

and

$$\sigma_\theta(\text{avg}) = \frac{\int_a^b h(r) dr - \int_a^b h(r) \sigma_\theta(r) dr}{M(r) \int_a^b \frac{h(r) \Psi_1(r)}{M(r)} dr} \Psi_1(r) + \sigma_0(r) \quad (13)$$

where the average tangential stress,

$$\sigma_\theta(\text{avg}) = \frac{\int_a^b h(r) \sigma_\theta(r) dr}{\int_a^b h(r) dr} \quad \text{and}$$

$$\Psi_1(r) = \left[ \frac{2}{r[2-x(r)]} e^{\int_a^r \frac{1}{r} \left[ \frac{2x(r)-1}{2-x(r)} \right] dr} \right]^{1/n}$$

The equilibrium equation for a variable thickness disc rotating with angular velocity  $\omega$  is<sup>11</sup>,

$$\frac{d}{dr} [rh(r)\sigma_r(r)] - h(r)\sigma_\theta(r) + \rho\omega^2 r^2 h(r) = 0 \quad (14)$$

The disc is assumed under free-free conditions<sup>22</sup>, as expressed by,

$$\sigma_r(r) = 0 \text{ at } r = a \text{ and } r = b \quad (15)$$

The integration of Eqn. (14) under the free-free conditions, Eqn. (15), yields the tangential and radial stresses as given by,

$$\sigma_\theta(\text{avg}) = \frac{\rho(r)\omega^2}{A_0} [I_1] \quad (16)$$

$$\sigma_r(r) = \frac{1}{rh(r)} \left[ \int_a^r h(r)\sigma_\theta(r) dr - \omega^2 \rho(r) I_{11} \right] \quad (17)$$

where

$$I_1 = h_b e^{-kb} \left[ e^{kb} \left( \frac{b^2}{k} - \frac{2b}{k^2} + \frac{2}{k^3} \right) - e^{ka} \left( \frac{a^2}{k} - \frac{2a}{k^2} + \frac{2}{k^3} \right) \right] \quad (18)$$

$$I_{11} = h_b e^{-kb} \left[ e^{kr} \left( \frac{r^2}{k} - \frac{2r}{k^2} + \frac{2}{k^3} \right) - e^{ka} \left( \frac{a^2}{k} - \frac{2a}{k^2} + \frac{2}{k^3} \right) \right] \quad (19)$$

$$\text{and } k = \frac{-\beta}{(a-b)}$$

The distributions of  $\sigma_\theta$  and  $\sigma_r$  in exponential disc may be obtained from Eqns (13) and (17), respectively, by following an iterative numerical scheme<sup>17</sup>. The stresses obtained are used in Eqns (11) and (12) to get the distributions of  $\dot{\epsilon}_r$  and  $\dot{\epsilon}_\theta$ , respectively, in the disc. By using similar approach, one may obtain the following relations for hyperbolic and linear discs.

(a) For hyperbolic disc

$$\sigma_\theta(avg) = \omega^2 h_b \left[ \rho(r)(b^{3+k} - a^{3+k}) / (3+k) \right] / (A_0 b^k) \quad (20)$$

$$\sigma_r(r) = \frac{1}{rh(r)} \left[ \int_a^r h(r)\sigma_\theta dr - \frac{\rho(r)\omega^2 h_b}{b^k} \left\{ \frac{r^{3+k} - a^{3+k}}{(3+k)} \right\} \right] \quad (21)$$

(b) For linear disc

$$\sigma_\theta(avg) = \frac{\rho(r)\omega^2 \left[ \frac{L}{3}(b^3 - a^3) - \frac{P}{4}(b^4 - a^4) \right]}{L(b-a) - \frac{P}{2}(b^2 - a^2)} \quad (22)$$

$$\sigma_r(r) = \frac{1}{rh(r)} \left[ \int_a^r h(r)\sigma_\theta dr - \rho(r)\omega^2 \int_a^r h(r)r^2 dr \right] \quad (23)$$

where  $L = (h_b + 2bc)$ ,  $P = 2c$ ,  $c = \frac{(h_a - h_b)}{2(b-a)}$

### 3. RESULTS AND DISCUSSION

For the purpose of computation, a code has been developed, using visual C++, to estimate the distributions of stresses and strain rates in different discs. The results are estimated for two different cases: Case 1: Variable thickness discs having the same inner thickness ( $h_a = 35$  mm), Table 1, and Case 2: Variable thickness discs having the same outer thickness ( $h_b = 19.24$  mm), Table 2. The average thickness ( $h_{avg}$ ) of all the discs is kept as 25.4 mm.

To obtain the desired thickness profiles of various discs (refer Table 1 and Table 2), the values of  $\beta$  (Eqn 1),

$k$  (Eqn 2) and  $c$  (Eqn 3) have been varied till the derived values of  $h_a$  (= 35 mm) or  $h_b$ (= 19.24 mm) are met.

#### 3.1 Validation

To validate the analysis and code developed, the tangential strain rate has been estimated in a constant thickness Al-SiC<sub>p</sub> disc, by setting  $\beta=0$  in Eqn (1). The results obtained are compared with those reported for similar disc<sup>11</sup>. A good agreement observed between the current and the reported results (Fig. 1) confirms the validity of the analysis performed and the code developed.

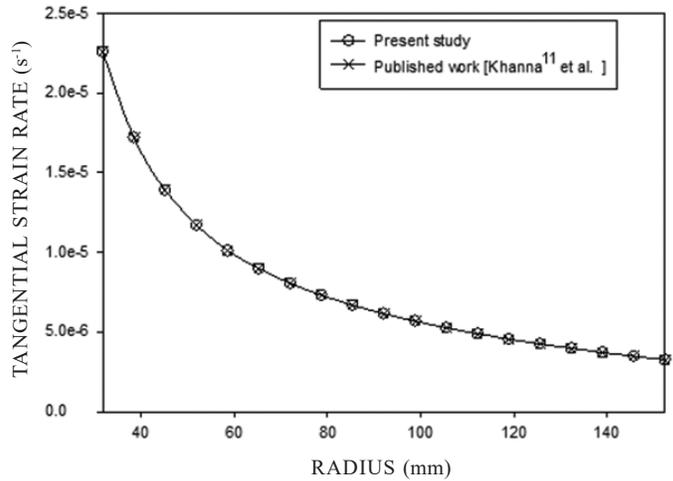


Figure 1. Comparison of tangential strain rate.

#### 3.2 Comparison of Creep Response

Case 1: Disc with same inner and average thickness

The disc profiles, shown in Fig. 2, for different discs (Table 1) indicate that near the inner radius the linear disc (D4) has the maximum thickness but towards the outer radius it has the minimum thickness. Thus, the thickness gradient ( $h_a - h_b$ ), denoted by TG, is maximum for linear disc D4 and minimum (zero) for uniform disc D1.

The radial stress, Fig. 3(a) in variable thickness discs D2-D4 is generally lower than the constant thickness disc D1, with larger variation noticed in the middle region. The radial stress in linear disc D4 is lower than the exponential (D2) and hyperbolic (D3) discs. The location of maximum radial stress in variable thickness discs (D2-D4) shifts slightly towards the outer radius than observed in uniform disc D1. The decrease noticed in the maximum radial stress in discs D2, D3 and D4 is 1.68, 0.16 and 2.35 MPa, respectively, when compared to disc D1.

Unlike radial stress, the tangential stress in variable thickness discs (D2-D4) is observed to be lower than uniform disc D1 over the entire radius (Fig. 3b). The lowest tangential stress is observed in disc D4. The decrease observed in maximum tangential stress, noticed at the inner radius, in discs D2, D3 and D4 is 9.14, 7.11 and 9.85 MPa, respectively.

Table 1. Disc notations for case-1 ( $h_a=35$  mm and  $h_{avg} = 25.4$  mm)

Disc notation	Disc thickness profile	Disc thickness (mm)		Thickness gradient (mm) = ( $h_a - h_b$ )
		$h_a$	$h_b$	
D1	Constant	25.4	25.4	0
D2	Exponential ( $\beta=0.544$ )	35	20.32	14.68
D3	Hyperbolic ( $k=-0.287$ )	35	22.33	12.67
D4	Linear ( $c= 0.0653$ )	35	19.24	15.76

Table 2. Disc notation for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm)

Disc notation	Disc thickness profile	Disc thickness (mm)		Thickness gradient (mm) = ( $h_a - h_b$ )
		$h_a$	$h_b$	
DM1	Constant	25.4	25.4	0
DM2	Exponential ( $\beta=0.669$ )	37.56	19.24	18.32
DM3	Hyperbolic ( $k=-0.59$ )	48.53	19.24	29.29
DM4	Linear ( $c = 0.0653$ )	35	19.24	15.76

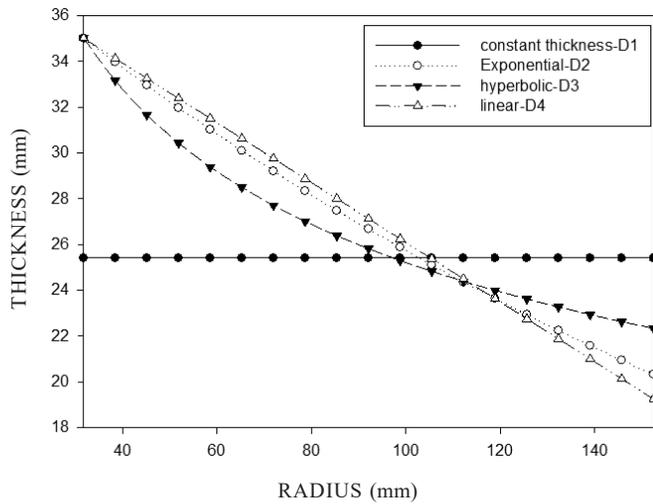


Figure 2. Thickness profiles of different discs.

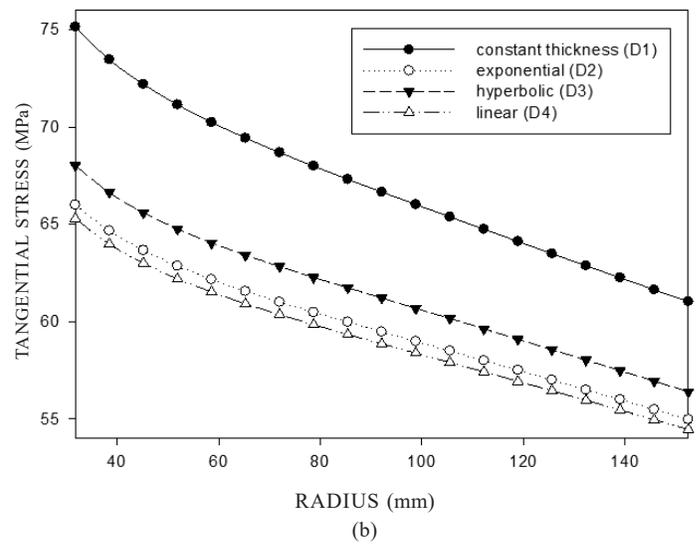
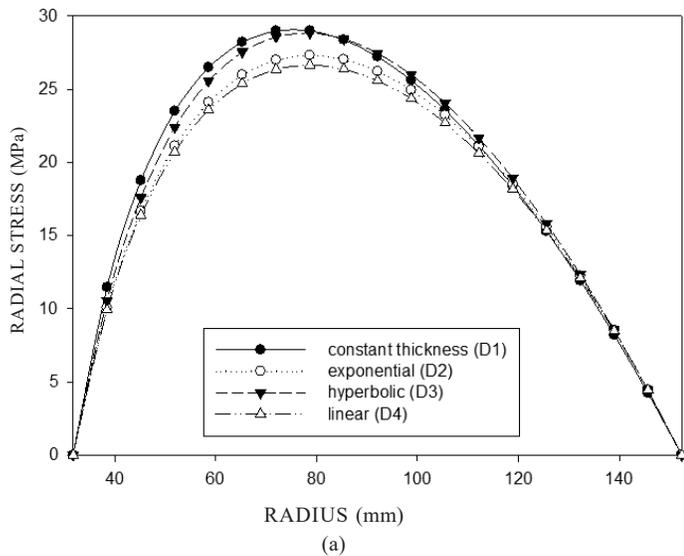


Figure 3. (a) Radial stress in different discs and (b) Tangential stress in different discs.

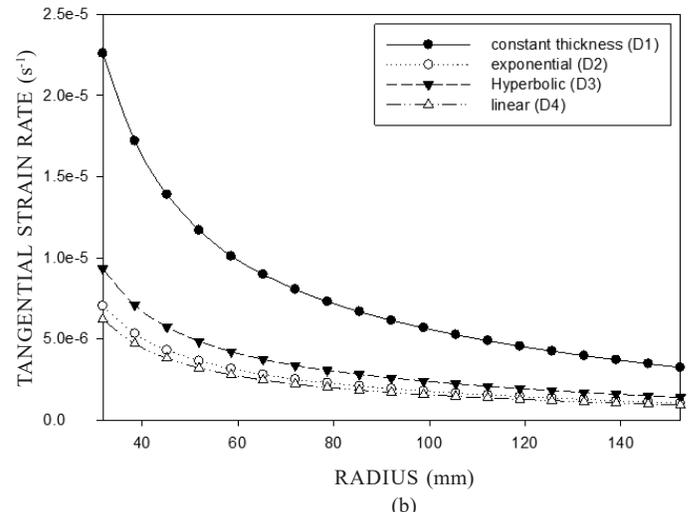
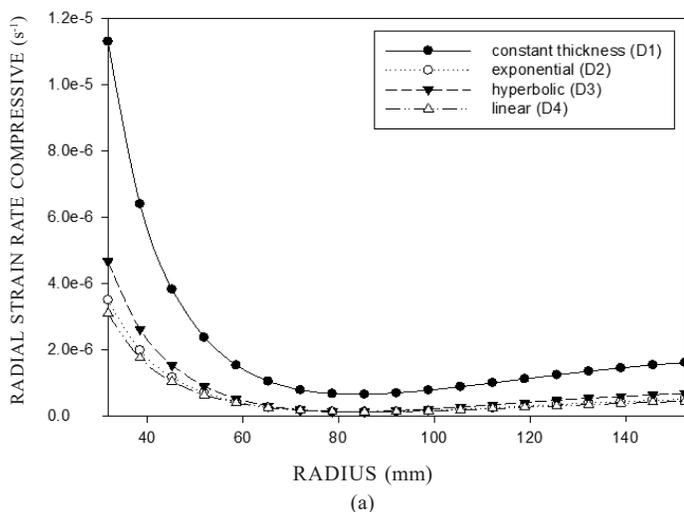


Figure 4. (a) Radial strain rate in different discs and (b) Tangential strain rate in different discs.

The radial strain rates in variable thickness discs (D2-D4) are also observed to reduce as compared to uniform disc D1 (Fig. 4(a)). The reduction observed is maximum at the inner radius and minimum near the middle of the disc. The value of maximum radial strain rate in linear disc D4, which exhibits the lowest radial strain rate, is 72.56 % lower than that noticed for uniform thickness disc D1. The radial strain rate in exponential disc D2 is slightly higher than observed in linear disc D4.

The effect of varying disc thickness profile on the tangential strain rate (Fig. 4(b)) is similar to that noticed for radial strain rate. Thus, for the same inner and average thickness, the linear disc D4 exhibits the lowest stresses and strain rates, besides having lesser possibility of distortion, owing to relatively flatter distribution of strain rates throughout the disc. Therefore, amongst variable thickness discs D2-D4, the linear disc D4 having the maximum TG, exhibits superior creep response. The hyperbolic disc D3 having the lowest TG shows inferior creep response amongst the variable thickness discs, but superior response than constant thickness disc D1.

*Case-2: Discs with same outer and average thickness*

This section compares the creep response of different variable thickness discs having the same outer and average thickness as show in Table 2. Amongst discs DM2-DM4, the hyperbolic disc DM3 has the steepest TG while the linear disc DM4 has the lowest TG as show in Fig. 5.

In comparison to uniform thickness disc DM1, the variable thickness discs DM2-DM4 exhibits lower radial stress, except for hyperbolic disc DM3, which shows slightly higher radial stress than disc DM1 towards the outer radius as show in Fig. 6(a). Similar to previous case (*i.e.* case-1), the linear disc DM4 has the lowest radial stress. The maximum radial stress noticed in linear disc DM4 is about 8% lower than the peak radial stress observed in disc DM1. The location of maximum radial stress in variable thickness discs is observed to shift slightly towards the outer radius, similar to case-1, than that observed for disc DM1. The tangential stress in variable thickness discs (DM2-DM4) are again noticed to reduce significantly than that observed in disc DM1, with a little higher decrease noticed near the inner radius as show in Fig. 6(b). Unlike case-1, amongst

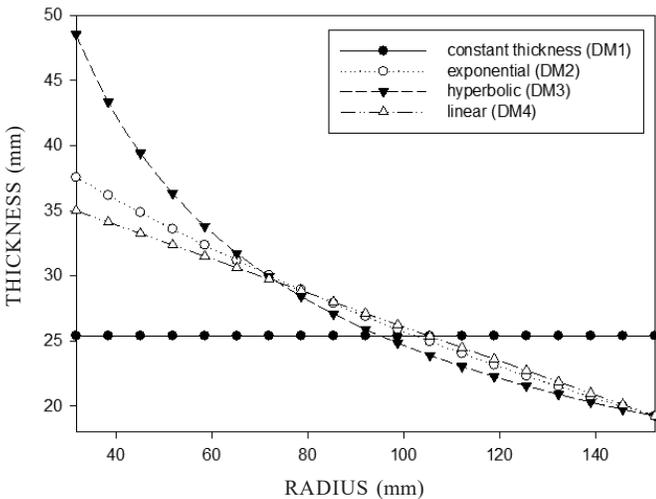
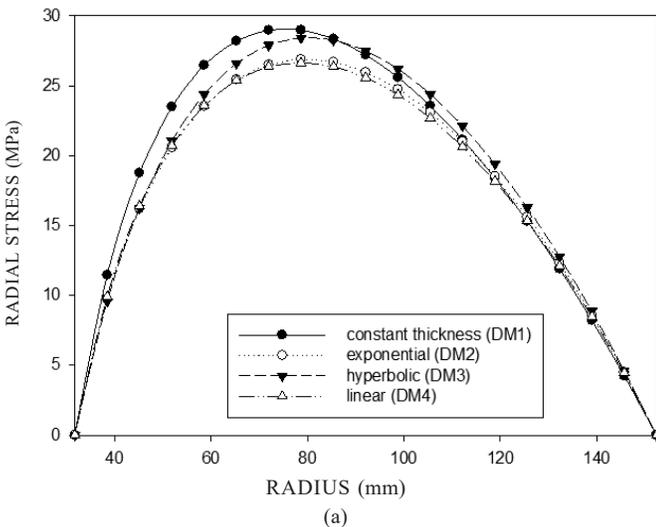


Figure 5. Thickness profile of different discs for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm).



different variable thickness discs, the tangential stress is seen to be the lowest for hyperbolic disc DM3 and the highest for linear disc DM4. The maximum value of tangential stress in discs DM2, DM3, and DM4, noticed at the inner radius, are observed to reduce by 14.7 %, 19.1 % and 13.1 %, respectively, when compared to that noted for disc DM1.

As compared to uniform disc DM1, the radial as well as tangential strain rates in variable thickness discs (DM2-DM4) are reduced significantly ((Figs. 7(a)-7(b)), besides attaining relatively flatter distribution. Unlike case-1, the lowest strain rates (radial and tangential) are noticed in hyperbolic disc DM3, having the steepest TG as show in Fig. 5. The maximum value of radial as well as tangential strain rate in hyperbolic disc DM3 is lower by about 87 % than that noticed for uniform disc DM1. Thus, for case-2, the hyperbolic disc DM3 exhibits superior creep response than any other disc.

**4. CONCLUSIONS**

The study compares the creep performance of rotating Al-SiC<sub>p</sub> discs having exponential, hyperbolic, linear and constant thickness profiles, having equal volume and subjected to similar operating conditions, for two different cases: Case 1 (variable thickness discs with the same inner thickness of 35 mm) and Case 2 (variable thickness discs with the same outer thickness of 19.24 mm). The salient conclusions drawn from the study are summarised below:

- The radial stress in variable thickness discs are lower than observed in constant thickness disc, with a little higher variation noticed in the middle region. The radial stress observed in both the cases (case 1 and case 2) is noticed to be minimum for linear thickness disc.
- The tangential stress in variable thickness discs are significantly lower throughout than noticed in a constant thickness disc. The lowest tangential stress is observed for linear disc in case-1 and for hyperbolic disc in case 2. In comparison to constant thickness disc, the maximum reduction noticed in the maximum tangential stress, noticed at the inner radius, is around 13 % for linear disc (case 1) and about 19 % for hyperbolic disc (case 2).

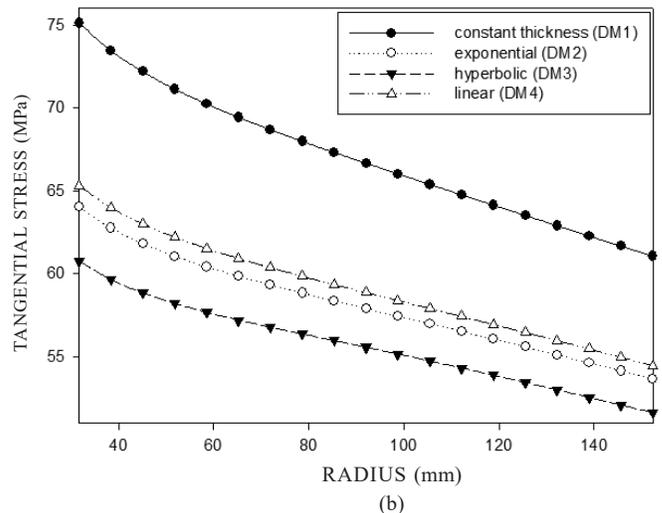


Figure 6. (a) Radial stresses in different discs for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm) and (b) Tangential stress in different discs for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm).

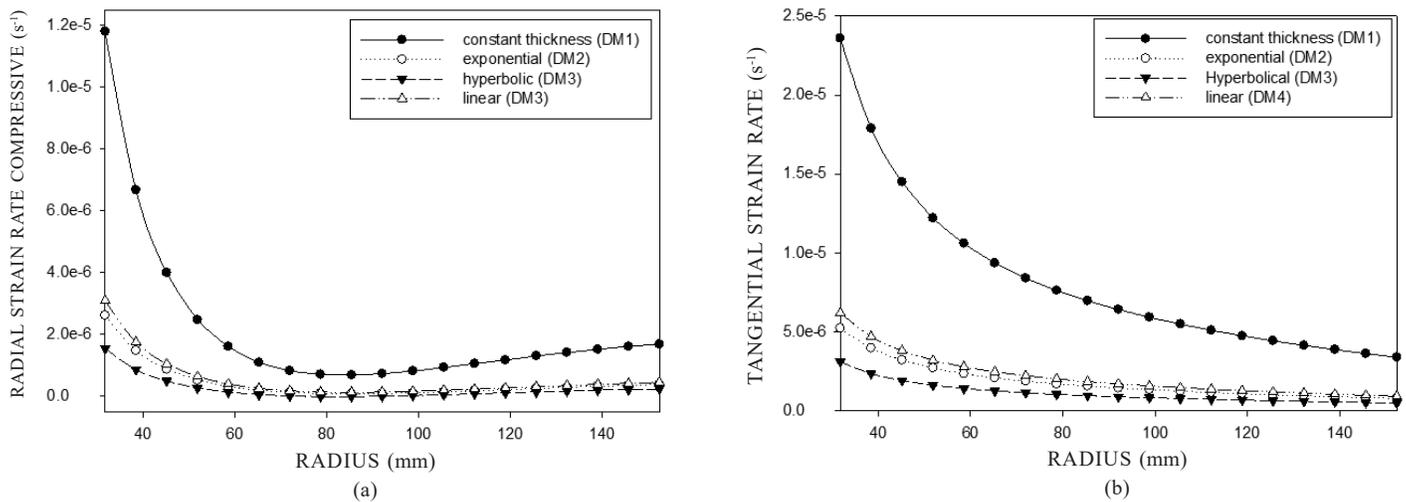


Figure 7. (a) Radial strain rates in different discs for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm) and (b) Tangential strain rates in different discs for case-2 ( $h_b = 19.24$  mm and  $h_{avg} = 25.4$  mm).

- The radial as well as tangential strain rates in variable thickness discs are significantly lower and relatively more uniform than noticed in a constant thickness disc, with the maximum reduction noticed at the inner disc radius.
- The maximum reduction noticed in the maximum strain rates, observed at the inner radius, is around 73 % for linear disc in case 1 and around 87 % for hyperbolic disc in case 2, when compared to those observed for constant thickness disc. In terms of creep rates, the exponential disc performs slightly inferior to linear disc in case 1 and hyperbolic disc in case 2.

The study reveals that the creep response of rotating composite disc depends on its thickness profile. In terms of stresses and strain rates, the use of linear thickness disc is recommended for case 1 whereas for case 2 the hyperbolic thickness disc is more suitable.

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