# New Geo-location Approach Based on Camera Coordinates and Common Points on Multiple Images 

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#### Abstract

The accuracy of traditional unmanned aerial vehicle (UAV) geo-location based on single image is too low to meet the needs of precise strike. In this paper, a new UAV geo-location method is presented. The mathematical models are constructed by linearization of the collinearity equations to iteratively compute the pose angles and focal length of the camera. At least three images of the target, along with at least three identifiable common points among the images, are needed for reckoning camera pose angles and focal length. The three dimensional (3D) coordinates of ground target are calculated using forward intersection. The new method can get the target coordinates with no dependence on digital elevation model (DEM) and the measured values of camera pose angles, therefore two of the three primary error sources in the traditional UAV target location approach are eliminated. Simulation and real image experiment results show that the accuracy of the estimated target location is close to that of the UAV position, and that target location error is within 5 m circular error probable (CEP) on condition that the UAV is navigated by differential global positioning systems (DGPS).


Keywords: Geo-location, camera coordinates, common points, unmanned aerial vehicle, UAV

## 1. INTRODUCTION

The traditional UAV geo-location methods ${ }^{1,2}$ use UAV position data, digital elevation model (DEM), and pose angles of the onboard camera to estimate the 3D coordinates of ground target. The largest contributors to the error in this method are poor knowledge of the three inputs. Coordinates derived in this fashion are accurate to 100 m CEP at best ${ }^{3,4}$, which is inadequate for precision strike. Therefore, some new accurate UAV geo-location methods have been reported in literatures ${ }^{5-7}$.

Gibbins, et al. ${ }^{5}$ use low-cost GPS, video and attitude sensors to estimate the position of ground targets. This method cannot work with a pan, tilt and zooming camera mount. Xie and $\mathrm{Li}^{6}$ calculate geographical coordinates of a target by computing correlation between real time image and digital terrain map of local target area. His method has the accuracy of 50 m CEP. Duan ${ }^{7}$ presents a geo-location approach using sequential aerial images. The approach is composed of two stages: relative position estimation and absolute position estimation, which are similar to those used in ${ }^{8,9}$. The former is based on stereo modeling of two successive image frames, whereas the latter is accomplished by image matching with reference images. The most serious shortcoming of Duan's method is the inability to work without ground control points.

Inspired by the work of Agouris and Schenk ${ }^{10}$, this paper provides a new UAV geolocation method. In this
method, at least three images of the target from nonlinear viewpoints, along with at least three identifiable common points among the images, will allow the calculation of the 3D coordinates of the target and all common points among the images. The new method doesn't depend on DEM and the measured values of camera pose angles, therefore two of the three primary error sources in the traditional UAV target location are eliminated.

## 2. CAMERA MODEL

In this paper, the pinhole camera model, i.e. the central projection model, is used as the projection model. We denote the image taken from the $i^{\text {th }}$ viewpoint by $\mathrm{I}_{i}$, and denote the $\mathrm{j}^{\text {th }}$ point in the 3D world coordinate system by $\mathrm{P}_{i}$. Assume that point $\mathrm{P}_{j}$ is mapped to on image $\mathrm{I}_{i}$, and the projection can be governed by the following collinearity equations ${ }^{11}$ :

$$
\left.\begin{array}{l}
x_{i j}=-f_{i} \frac{a_{i 1}\left(X_{j}-X_{s_{i}}\right)+b_{i 1}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 1}\left(Z_{j}-Z_{s_{i}}\right)}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)} \\
y_{i j}=-f_{i} \frac{a_{i 2}\left(X_{j}-X_{s_{i}}\right)+b_{i 2}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 2}\left(Z_{j}-Z_{s_{i}}\right)}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)} \tag{1}
\end{array}\right\}
$$

Here, $f_{i}$ and ( $X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}$ ) are the effective focal length and camera position corresponding to image $I$, respectively, $\left(X_{j}, Y_{j}, Z_{j}\right)$ are the 3D coordinates of $P_{j},\left(x_{i j}, y_{i j}\right)$ are the measured 2D coordinates of $p_{i j}, a_{i j}, b_{i j}$ and $c_{i j}$, where $i$
$=1,2,3$ and $j=1,2,3$, are the elements of the rotation matrix $\mathbf{R}_{i}$ which governs the transformation between the coordinate systems of the world and the camera coordinates system.

The matrix $\mathbf{R}_{i}$ can be represented by the exterior angle parameters of image $I_{i}$, i.e. the pitch angle $\varphi_{i}$, yaw angle $\omega_{i}$, and roll angle $\kappa_{i}$ of the camera, as follows ${ }^{12,13}$ :

$$
\mathbf{R}_{i}=\left[\begin{array}{ccc}
\mathrm{c}_{\kappa_{i}} & \mathrm{~s}_{\kappa_{i}} & 0  \tag{2}\\
-\mathrm{s}_{\kappa_{i}} & \mathrm{c}_{\kappa_{i}} & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \mathrm{c}_{\omega_{i}} & \mathrm{~s}_{\omega_{i}} \\
0 & -\mathrm{s}_{\omega_{i}} & \mathrm{c}_{\omega_{i}}
\end{array}\right]\left[\begin{array}{ccc}
\mathrm{c}_{\varphi_{i}} & 0 & \mathrm{~s}_{\varphi_{i}} \\
0 & 1 & 0 \\
-\mathrm{s}_{\varphi_{i}} & 0 & \mathrm{c}_{\varphi_{i}}
\end{array}\right]
$$

where $\mathrm{c}_{\kappa_{i}}$ stands for $\cos \kappa_{i}, s_{\kappa_{i}}$ for $\sin \kappa_{i}, \mathrm{c}_{\omega_{i}}$ for $\cos$ $\omega_{i}$, for $s_{\omega_{i}} \sin \omega_{i}, c_{\phi_{i}}$ for $\cos \varphi_{i}$, and $s_{\varphi_{i}}$ for $\sin \varphi_{i}$.

In this study, $n$ images $\mathrm{I}_{i}(i=1,2 \ldots \mathrm{n})$ taken from different camera positions and geographical common points $\mathrm{P}_{j}(j$ $=1,2 \ldots \mathrm{~m}$ ) with unknown coordinates ( $X_{j}, Y_{j}, Z_{j}$ ) are used for geo-location. The camera coordinates ( $X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}$ ), which can be easily calculated from UAV navigation system, are assumed to be known for geo-location. The coordinates $\left(x_{i j}, y_{i j}\right)$ of $\mathrm{p}_{i j}$ can be extracted from image whenever needed, so they are supposed to be already known, too. The coordinates $\left(X_{j}, Y_{j}, Z_{j}\right)$ of $\mathrm{P}_{j}$, as well as the focal length $f_{i}$ and the angle parameters $\varphi_{i}, \omega_{i}$, and $\kappa_{i}$ of the camera on the $i^{\text {th }}$ viewpoint, are unknown vari-ables that need to be estimated for geolocation.

It can be easily found that each image $I_{i}$ introduces four unknown variables, i.e. $f_{i}, \varphi_{i}, \omega_{i}$, and $\kappa_{i}$, and each common point $\mathrm{P}_{i}$ introduces three variables, i.e. $X_{i}, Y_{i}$, and $Z_{i}$. Consequently, $n$ images along with $m$ geographical common points produce $(4 n+3 m)$ unknown variables all together.

The next section will give an approach for estimating the $(4 n+3 m)$ unknown variables mentioned above. At least three images from nonlinear viewpoints, along with at least three identifiable common points among the images, are required in our approach.

## 3. ESTIMATION OF THE UNKNOWN VARIABLES

In this section, the $(4 n+3 m)$ unknown variables are iteratively estimated by the following two steps: the initial values of the unknown variables are given, then, the correction for each initial value is computed, and eventually the correction is added to the initial value to produce more precise value of the unknown variables.

### 3.1 Determination of the Initial Values

The initial values of the camera pose angles are near to zero, i.e. $\varphi_{i 0}=0, \omega_{i 0}=0$, and $\kappa_{i 0}=0$, the initial values of the camera focal length can be determined by $f_{i 0}=\frac{1}{n} \sum_{i=1}^{n} \tilde{f}_{i}$, and the initial values of the coordinates of $\mathrm{P}_{j}$ can be respectively calculated by $X_{j 0}=\frac{1}{n} \sum_{i=1}^{n} X_{\mathrm{s}_{i}}, \quad Y_{j 0}=\frac{1}{n} \sum_{i=1}^{n} Y_{\mathrm{s}_{i}}$, $Z_{j 0}=\frac{1}{n} \sum_{i=1}^{n}\left(Z_{s_{i}}-\tilde{H}_{i}\right)$, where $\tilde{f}_{i}$ and $\tilde{H}_{i}$ are the low accuracy
values measured on-line by onboard instruments.

### 3.2 Computation of the Corrections

Linearizing the nonlinear function on the right hand side of Eqn (1), gives ${ }^{14}$

$$
\begin{align*}
x_{i j}= & \hat{x}_{i j}+\frac{\partial x_{i j}}{\partial X_{j}} d X_{j}+\frac{\partial x_{i j}}{\partial Y_{j}} \mathrm{~d} Y_{j}+\frac{\partial x_{i j}}{\partial Z_{j}} \mathrm{~d} Z_{j}+\frac{\partial x_{i j}}{\partial \varphi_{i}} \mathrm{~d} \varphi_{i} \\
& +\frac{\partial x_{i j}}{\partial \omega_{i}} \mathrm{~d} \omega_{i}+\frac{\partial x_{i j}}{\partial \kappa_{i}} \mathrm{~d} \kappa_{i}+\frac{\partial x_{i j}}{\partial f_{i}} \mathrm{~d} f_{i} \\
y_{i j}= & \hat{y}_{i j}+\frac{\partial y_{i j}}{\partial X_{j}} d X_{j}+\frac{\partial y_{i j}}{\partial Y_{j}} \mathrm{~d} Y_{j}+\frac{\partial y_{i j}}{\partial Z_{j}} \mathrm{~d} Z_{j}+\frac{\partial y_{i j}}{\partial \varphi_{i}} \mathrm{~d} \varphi_{i}  \tag{3}\\
& +\frac{\partial y_{i j}}{\partial \omega_{i}} \mathrm{~d} \omega_{i}+\frac{\partial y_{i j}}{\partial \kappa_{i}} \mathrm{~d} \kappa_{i}+\frac{\partial y_{i j}}{\partial f_{i}} \mathrm{~d} f_{i}
\end{align*}
$$

where $\hat{x}_{i j}$ and $\hat{y}_{i j}$ are the estimation of the coordinates of common point $\mathrm{p}_{i j}, \mathrm{~d} f_{i}, \mathrm{~d} \varphi_{i}, \mathrm{~d} \omega_{i}, \mathrm{~d} \kappa_{i}, \mathrm{~d} X_{j}, \mathrm{~d} Y_{j}$, and $\mathrm{d} Z_{j}$ are the corrections to be estimated whose coefficients can be calculated by

$$
\begin{align*}
& \frac{\partial x_{i j}}{\partial X_{j}}=-\frac{a_{i 1} f_{i}+a_{i 3} x_{i j}}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{4}\\
& \frac{\partial x_{i j}}{\partial Y_{j}}=-\frac{b_{i 1} f_{i}+b_{i 3} x_{i j}}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{5}\\
& \frac{\partial x_{i j}}{\partial Z_{j}}=-\frac{c_{i 1} f_{i}+c_{i 3} x_{i j}}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{6}\\
& \frac{\partial y_{i j}}{\partial X_{j}}=-\frac{a_{i 2} f_{i}+a_{i 3} y_{i j}}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{7}\\
& \frac{b_{i 2} f_{i}+b_{i 3} y_{i j}}{\partial Y_{j}}=-\frac{c_{i 2} f_{i}+c_{i 3} y_{i j}}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{8}\\
& \frac{\partial y_{i j}}{\partial Z_{j}}=-\frac{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}{a_{i j}}  \tag{9}\\
& \frac{\partial x_{i j}}{\partial \varphi_{i}}=y_{i j} \mathrm{~s}_{\omega_{i j}}-\left[\frac{x_{i j}}{f_{i}}\left(x_{i j} \mathrm{c}_{\kappa_{i}}-y_{i j} \mathrm{~s}_{\kappa_{i}}\right)+f_{i} \mathrm{c}_{\kappa_{i}}\right] \mathrm{c}_{a_{i}}  \tag{10}\\
& \frac{\partial x_{i j}}{\partial \omega_{i}}=-f_{i} \mathrm{~s}_{\kappa_{i}}-\frac{x_{i j}}{f_{i}}\left(\mathrm{~s}_{\kappa_{i}}+y_{i j} \mathrm{c}_{\kappa_{i}}\right)  \tag{11}\\
& \frac{\partial x_{i j}}{\partial \kappa_{i}}=y_{i j}  \tag{12}\\
& \frac{\partial x_{i j}}{\partial f_{i}}=-\frac{a_{i 1}\left(X_{j}-X_{s_{i}}\right)+b_{i 1}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 1}\left(Z_{j}-Z_{s_{i}}\right)}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)}  \tag{13}\\
& \frac{\partial y_{i j}}{\partial \varphi_{i}}=-x_{i j} \mathrm{~s}_{a_{i j}}-\left[\frac{x_{i j}}{f_{i}}\left(x_{i j} \mathrm{c}_{\kappa_{i}}-y_{i j} \mathrm{~s}_{\kappa_{i}}\right)-f_{i} \mathrm{~s}_{\kappa_{i}}\right] \mathrm{c}_{a_{i}}  \tag{14}\\
& \frac{\partial y_{i j}}{\partial \omega_{i}}=-f_{i} \mathrm{c}_{\kappa_{i}}-\frac{y_{i j}}{f_{i}}\left(x_{i j} \mathrm{~s}_{\kappa_{i}}+y_{i j} \mathrm{c}_{\kappa_{i}}\right) \tag{15}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial y_{i j}}{\partial \kappa_{i}}=-x_{i j}  \tag{16}\\
& \frac{\partial x_{i j}}{\partial f_{i}}=-\frac{a_{i 2}\left(X_{j}-X_{s_{i}}\right)+b_{i 2}\left(Y_{j}-Y_{s_{i}}\right)+c_{i 2}\left(Z_{j}-Z_{s_{i}}\right)}{a_{i 3}\left(X_{j}-X_{s_{i}}\right)+b_{i 3}\left(Y_{j}-Y_{S_{i}}\right)+c_{i 3}\left(Z_{j}-Z_{s_{i}}\right)} \tag{17}
\end{align*}
$$

For convenience of expression, Eq. (3) is rewritten in a brief matrix form as

$$
\begin{equation*}
\mathbf{A}_{i j} \mathbf{U}_{j}+\mathbf{B}_{i j} \mathbf{V}_{i}=\mathbf{C}_{i j} \tag{18}
\end{equation*}
$$

where matrix $\mathbf{A}_{i j}, \mathbf{U}_{j}, \mathbf{B}_{i j}, \mathbf{V}_{i}$ and $\mathbf{C}_{i j}$ are defined as

$$
\begin{align*}
& \mathbf{A}_{i j}=\left[\begin{array}{lll}
\frac{\partial x_{i j}}{\partial X_{j}} & \frac{\partial x_{i j}}{\partial Y_{j}} & \frac{\partial x_{i j}}{\partial Z_{j}} \\
\frac{\partial y_{i j}}{\partial X_{j}} & \frac{\partial y_{i j}}{\partial Y_{j}} & \frac{\partial y_{i j}}{\partial Z_{j}}
\end{array}\right]  \tag{19}\\
& \mathbf{B}_{i j}=\left[\begin{array}{lll}
\frac{\partial x_{i j}}{\partial \varphi_{i}} & \frac{\partial x_{i j}}{\partial \omega_{i}} & \frac{\partial x_{i j}}{\partial \kappa_{i}} \\
\frac{\partial x_{i j}}{\partial y_{i j}} \\
\frac{\partial y_{i j}}{\partial \varphi_{i}} & \frac{\partial y_{i j}}{\partial \omega_{i}} & \frac{\partial y_{i j}}{\partial \kappa_{i}}
\end{array}\right]  \tag{20}\\
& \mathbf{U}_{j}=\left[\begin{array}{lll}
\mathrm{d} X_{j} & \mathrm{~d} Y_{j} & \mathrm{~d} Z_{j}
\end{array}\right]^{\mathrm{T}}  \tag{21}\\
& \mathbf{V}_{i}=\left[\begin{array}{llll}
\mathrm{d} \varphi_{i} & \mathrm{~d} \omega_{i} & \mathrm{~d} \kappa_{i} & \mathrm{~d} f_{i}
\end{array}\right]^{\mathrm{T}}  \tag{22}\\
& \mathbf{C}_{i j}=\left[\begin{array}{lll}
x_{i j}-\hat{x}_{i j} & y_{i j}-\hat{y}_{i j}
\end{array}\right]^{\mathrm{T}} \tag{23}
\end{align*}
$$

Equation (18) shows that each point $p_{i j}$ on the image I can produce two constraints on $\mathrm{d} f_{i}, \mathrm{~d} \varphi_{i}, \mathrm{~d} \omega_{i}, \mathrm{~d} \kappa_{i}, \mathrm{~d} X_{j}, \mathrm{~d} Y_{j}$, and $\mathrm{d} Z_{i}$. Therefore, $m$ points on $I$ can generate $2 m$ constraints, which can be written in a brief matrix form as follows

$$
\begin{equation*}
\mathbf{A}_{i} \mathbf{U}+\mathbf{B}_{i} \mathbf{V}_{i}-\mathbf{C}_{i}=\mathbf{0} \tag{24}
\end{equation*}
$$

Here, the matrices $\mathbf{A}_{i}, \mathbf{B}_{i}, \mathbf{V}_{i}, \mathbf{C}_{i}$ and are defined as

$$
\begin{align*}
& \mathbf{A}_{i}=\operatorname{diag}\left\{\mathbf{A}_{i 1}, \mathbf{A}_{i 2}, \cdots, \mathbf{A}_{i m}\right\}  \tag{25}\\
& \mathbf{U}=\left[\begin{array}{llll}
\mathbf{U}_{1}^{\mathrm{T}} & \mathbf{U}_{2}^{\mathrm{T}} & \cdots & \mathbf{U}_{m}^{\mathrm{T}}
\end{array}\right]^{\mathrm{T}}  \tag{26}\\
& \mathbf{B}_{i}=\left[\begin{array}{llll}
\mathbf{B}_{i 1}^{\mathrm{T}} & \mathbf{B}_{i 2}^{\mathrm{T}} & \cdots & \mathbf{B}_{i m}^{\mathrm{T}}
\end{array}\right]^{\mathrm{T}}  \tag{27}\\
& \mathbf{C}_{i}=\left[\begin{array}{llll}
\mathbf{C}_{i 1}^{\mathrm{T}} & \mathbf{C}_{i 2}^{\mathrm{T}} & \cdots & \mathbf{C}_{i n}^{\mathrm{T}}
\end{array}\right]^{\mathrm{T}} \tag{28}
\end{align*}
$$

where $\operatorname{diag}\left\{\mathbf{A}_{i 1}, \mathbf{A}_{i 2}, \cdots, \mathbf{A}_{i m}\right\}$ stands for the diagonal matrix whose diagonal elements are $\mathbf{A}_{i 1}, \mathbf{A}_{i 2}, \ldots$ and $\mathbf{A}_{i m}$.

If $n$ images from nonlinear viewpoints, along with $m$ identifiable common points among the images, are used for geo-location, there are the $(4 n+3 m)$ unknown corrections that need to be esti-mated by calculating 2 nm constraint equations. These equations are written as

$$
\begin{equation*}
\mathbf{A U}+\mathbf{B V}=\mathbf{C} \tag{29}
\end{equation*}
$$

where the matrices $\mathbf{A}, \mathbf{B}, \mathbf{V}$ and $\mathbf{C}$ are constructed by

$$
\mathbf{A}=\left[\begin{array}{llll}
\mathbf{A}_{1}^{\mathrm{T}} & \mathbf{A}_{2}^{\mathrm{T}} & \cdots & \mathbf{A}_{n}^{\mathrm{T}} \tag{30}
\end{array}\right]^{\mathrm{T}}
$$

$$
\begin{align*}
& \mathbf{B}=\operatorname{diag}\left\{\mathbf{B}_{1}, \mathbf{B}_{2}, \cdots, \mathbf{B}_{n}\right\}  \tag{31}\\
& \mathbf{V}=\left[\begin{array}{llll}
\mathbf{V}_{1}^{\mathrm{T}} & \mathbf{V}_{2}^{\mathrm{T}} & \cdots & \mathbf{V}_{n}^{\mathrm{T}}
\end{array}\right]^{\mathrm{T}}  \tag{32}\\
& \mathbf{C}=\left[\begin{array}{llll}
\mathbf{C}_{1}^{\mathrm{T}} & \mathbf{C}_{2}^{\mathrm{T}} & \cdots & \mathbf{C}_{n}^{\mathrm{T}}
\end{array}\right]^{\mathrm{T}} \tag{33}
\end{align*}
$$

If $2 m n \geq 3(n+m)$, all the $(4 n+3 m)$ corrections can be estimated through Eqn (29).

### 3.3 Algorithms for Estimation of the Unknown Variables

Using the mathematic models mentioned above, the algorithms for computing the unknown variables $f_{i}, \varphi_{i}, \omega_{i}$, $\kappa_{i}, X_{j}, Y_{j}$, and $Z_{j}$ with $i=1,2, . . n$ and $j=1,2, \ldots m$ can be listed as follows:
Step 1. Determine the initial values $f_{i 0}, \varphi_{i 0}, \omega_{i 0}, \kappa_{i 0}$ and $\left(X_{j 0}, Y_{j 0}, Z_{j 0}\right)$.
Step 2 Initialize the variables $f_{i}, \varphi_{i}, \omega_{i}, \kappa_{i},\left(X_{j}, Y_{j}, Z_{j}\right)$ with $f_{i 0}, \varphi_{i 0}, \omega_{i 0}, \kappa_{i 0},\left(X_{j 0}, Y_{j 0}, Z_{j 0}\right)$ respectively.
Step 3 Extract the coordinates $\left(x_{i j}, y_{i j}\right)$ of $p_{i j}$ from the image I.
Step 4 Calculate the camera coordinates ( $X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}$ ) using the UAV navigation data.
Step 5 Substitute $\varphi_{i}, \omega_{i}$, and $\kappa_{i}$ into Eqn (2) to get $\mathbf{R}_{i}$.
Step 6 Substitute $\left(x_{i j}, y_{i j}\right),\left(X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}\right),\left(X_{j}, Y_{i}, Z_{j}\right), f_{i}$, and the elements of $\mathbf{R}_{i}$ into Eqns (4)-(17) to get the matrices $\mathbf{A}_{i j}$ and $\mathbf{B}_{i j}$.
Step 7 Substitute $\left(X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}\right),\left(X_{j}, Y_{j}, Z_{j}\right), f_{i}$, and the elements of $\mathbf{R}_{i}$ into Eqn (1) to get ( $\hat{x}_{i j}, \hat{y}_{i j}$ ).
Step 8 Substitute $\left(x_{i j}, y_{i j}\right)$ and ( $\hat{x}_{i j}, \hat{y}_{i j}$ ) into Eqn (33) to get $\mathbf{C}_{i j}$.
Step 9 Constructs A, B, C and with $\mathbf{A}_{i j}, \mathbf{B}_{i j}$ and $\mathbf{C}_{i j}$.
Step 10 Using Eqn (29)to compute all the corrections $\mathrm{d} f_{i}$, $\mathrm{d} \varphi_{i}, \mathrm{~d} \omega_{i}, \mathrm{~d} \kappa_{i}, \mathrm{~d} X_{i}, \mathrm{~d} Y_{j}$, and $\mathrm{d} Z_{j}$.
Step 11 Update the unknown variables by

$$
\begin{align*}
& f_{i}=f_{i}+\mathrm{d} f_{i} \\
& \varphi_{i}=\varphi_{i}+\mathrm{d} \varphi_{i} \\
& \omega_{i}=\omega_{i}+\mathrm{d} \omega_{i} \\
& \kappa_{i}=\kappa_{i}+\mathrm{d} \kappa_{i} \\
& X_{j}=X_{j}+\mathrm{d} X_{j}  \tag{34}\\
& Y_{j}=Y_{j}+\mathrm{d} Y_{j} \\
& Z_{j}=Z_{j}+\mathrm{d} Z_{j}
\end{align*}
$$

Step 12 If the estimated corrections are accurate enough for geo-location, Section 4 is referred for triangulation measurement. Otherwise, refer to Step 5 to compute more accu-rate corrections.

## 4. FORWARD INTERSECTION FOR GEOLOCATION

If target itself is one of the common points, its coordinates can be calculated by Eqn (29) directly. Otherwise, the coordinates of the target are computed by forward intersection ${ }^{12,13}$.

Suppose that the two images used in forward intersection are taken from viewpoints ( $X_{s_{i}}, Y_{s_{i}}, Z_{s_{i}}$ ) and ( $X_{s_{j}}, Y_{s_{j}}, Z_{s_{j}}$ ), respectively. Then, the coordinates of the target, denoted by ( $X_{m}, Y_{m}, Z_{m}$ ), can be estimated using the following models

$$
\left.\begin{array}{l}
X_{m}=\left[\left(X_{\mathrm{s}_{i}}+N_{i} u_{i m}\right)+\left(X_{\mathrm{s}_{j}}+N_{j} u_{j m}\right)\right] / 2 \\
Y_{m}=\left[\left(Y_{\mathrm{s}_{i}}+N_{i} v_{i m}\right)+\left(Y_{\mathrm{s}_{j}}+N_{j} v_{j m}\right)\right] / 2  \tag{35}\\
Z_{m}=\left[\left(Z_{\mathrm{s}_{i}}+N_{i} w_{i n}\right)+\left(Z_{\mathrm{s}_{j}}+N_{j} w_{j m}\right)\right] / 2
\end{array}\right\}
$$

Here, $\left(u_{i m}, v_{i m}, w_{i m}\right)$ and $\left(u_{j m}, v_{j m}, w_{j m}\right)$ are defined by

$$
\left[\begin{array}{c}
u_{i m}  \tag{36}\\
v_{i m} \\
w_{i m}
\end{array}\right]=\mathbf{R}_{i}\left[\begin{array}{c}
x_{i m} \\
y_{i m} \\
-f_{i}
\end{array}\right],\left[\begin{array}{l}
u_{j m} \\
v_{j m} \\
w_{j m}
\end{array}\right]=\mathbf{R}_{j}\left[\begin{array}{c}
x_{j m} \\
y_{j m} \\
-f_{j}
\end{array}\right]
$$

where $\left(x_{i m}, y_{i m}\right)$ and $\left(x_{j m}, y_{j m}\right)$ are respectively the coordinates of the target on the image $\mathrm{I}_{i}$ and $\mathrm{I}_{j}, \mathbf{R}_{i}, \mathbf{R}_{j}, f_{1}$ and $f_{2}$ can be estimated using the mathematical models given in Section 3. $N_{i}$ and $N_{j}$ are defined as

$$
\left.\begin{array}{l}
N_{i}=\frac{\left(X_{S_{j}}-X_{s_{i}}\right) w_{j m}-\left(Z_{s_{j}}-Z_{s_{i}}\right) u_{j m}}{u_{i m} w_{j m}-u_{j m} w_{i m}} \\
N_{j}=\frac{\left(X_{s_{j}}-X_{s_{i}}\right) w_{i m}-\left(Z_{s_{j}}-Z_{s_{i}}\right) u_{i m}}{u_{i m} w_{j m}-u_{j m} w_{i m}} \tag{37}
\end{array}\right\}
$$

## 5. GEO-LOCATION ACCURACY

In this section, both a Monte Carlo simulation and a real aerial image experiment are used to assess the accuracy of the proposed geo-location method.

### 5.1 Monte Carlo Simulation

The Monte Carlo simulation data are produced as follows and more details are shown in ${ }^{15}$.
a) The simulated UAV altitude is 1500 m and it flies along a 1000 m radius circle shown in Fig.1. Three ground points spanned in the field of vision are used as common points. Three images taken from different viewpoints which are equally spaced on a half-circle are used for simulation.
b) The initial value of each camera pointing angle is assumed to have $5^{\circ}$-deviation from its corresponding true values. The deviation of the initial focal length from the true value is assumed to be 5 mm .
c) The standard deviation of error in marking the common points is assumed to be 0.5 pixels. The camera locations corresponding to the viewpoints are all assumed to have a 1.5 m standard deviation. These two kinds of input errors are all assumed to be Gaussian distribution with zero mean values.
Figure 2 shows the cumulative histogram of horizontal and vertical errors of 2000 Monte Carlo trials, where the resultant median (50\%) horizontal and vertical errors in target location are less than 4 m , and the maximum errors are less than 14 m .


Figure 1. Configuration of simulation.

### 5.2 Experiments with Real Aerial Images

In this subsection, the three real aerial images, along with the four common points denoted by white cross are used for investigating the geo-location method further. These three images, shown in Fig. 3, are taken from three viewpoints whose 3D coordinates are given by DGPS. The four common points which are denoted by white cross in Fig. 3 (a) are selected based on the methods given in ${ }^{16}$ and matched between multiple images using the algorithms in ${ }^{17}$ and. The final geo-location accuracy is 9.3 m CEP.

## 6. CONCLUSIONS AND FUTURE WORK

The approach proposed in this paper requires only several images and associated camera locations to estimate the geodetic coordinates. Neither DEM nor measured pose angles of the camera is used. Camera positioning error is the only main factor that degrades the geo-location accuracy.


Figure 2. Geo-location errors generated in Monte Carlo simulation.


Figure 3. (a) The first frame of the images, (b) The second frame of the images, and (c) The third frame of the images. Figure is the real aerial image used for the accuracy assessment (the white cross on the first frame denotes the position of the common points used for geo-location, and the white triangle denotes the target to be located).

Both simulation and real image experiments show that the new method can give an accuracy up to 14 m CEP when the camera is located by DGPS with an accuracy of 5 m CEP. The method has potential applications for precise targeting against emerging targets, area surveillance for pre- and post-battle assessment, area mapping, and local measurements of size and separations for military operations
in urban terrain.
Future work will address the difficult, high-priority problem of automated common-point placement under various operational conditions.

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