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Thermo Creep Transition in Non-homogeneous Thick-walled Rotating Cylinders

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ABSTRACT

Creep stresses have been derived using transition theory[7]. The results for the combined effects of angular speed and temperature are calculated and depicted graphically. It has been observed that a cylinder made of less compressible material at the internal surface and highly compressible at the outer surface is on the safer side of the design for different values of N, Ω^2 and temperature as compared to highly compressible material at the internal surface and less compressible at the outer surface.

Keywords: Rotating cylinder, creep, thermal stresses, transition theory

1. INTRODUCTION

The problem of a uniformly rotating long, thick-walled circular cylinder arises occasionally in the design of turbine rotors. The creep behavior of circular cylinders rotating about its axis of symmetry has been investigated by many authors[1-5]. Rotating cylinder plays an important role in machine design particularly at elevated temperatures. Rimrott and Luke[5] has obtained the creep stresses of a rotating hollow circular cylinder made of isotropic and homogeneous materials. Wahl[6] has given stress distribution under steady state creep at elevated temperature for long rotating cylinders having axial bores and subjected to external radial tension. Non-homogeneous materials are effectively utilized in aerospace and commercial applications. Some degree of non-homogeneity is present in wide class of materials such as hot rolled metals, aluminum and magnesium allows. Non-homogeneity can also be generated by certain external field, that is thermal field, as the elastic modules of the material vary with the temperature or co-ordinates, etc. The effect of stress distribution caused by external fields is much more pronounced and of larger duration than the effect of thermal stresses themselves.

In this paper, an attempt has been made to obtain the creep stresses for a non-homogeneous thick-walled rotating cylinder by using transition theory[7]. It utilizes the concept of generalized strain measure and asymptotic solution at the turning points or transition points of the governing differential equation defining the deformed field and has been successfully applied to a large number of problems in creep[7-11].

The generalized principal strain measure is defined as[7]

$$e_{ii} = \int_0^{e_{ii}^A} \left[1 - 2e_{ii}^A\right]^{\frac{n}{2}-1} de_{ii}^A = \frac{1}{n}\left[1 - \left(1 - 2e_{ii}^A\right)^{\frac{n}{2}}\right] \tag{1.1}$$

Taking the non-homogeneity as the compressibility of the material in the cylinder as

$$C = C_0 r^{-k} \tag{1.2}$$

where, $a \le r \le b$, a and b are internal and external radii, C_0 and k are constants. Results obtained have been discussed numerically and depicted graphically.

2. GOVERNING EQUATIONS

Consider a thick-walled circular cylinder of internal and external radii a and b respectively, rotating with an angular velocity ω of gradually increasing speed about its axes subjected to temperature $\theta = \theta_0$ at the internal surface. The components of displacement in cylindrical co-ordinates are given by[7]

$$u = r(1-\beta); \qquad v = 0; \qquad w = d\,z \tag{2.1}$$

where β is a function of $r = \sqrt{x^2+y^2}$ only and d is a constant.

The finite components of strain are,

$$\begin{aligned} e_{rr}^A &= \frac{1}{2}\left[1 - \left(r\beta' + \beta\right)^2\right] \\ e_{\theta\theta}^A &= \frac{1}{2}\left[1 - \beta^2\right] \\ e_{zz}^A &= \frac{1}{2}\left[1 - (1-d)^2\right] \\ e_{r\theta}^A &= e_{\theta\theta}^A = e_{zz}^A = 0 \end{aligned} \tag{2.2}$$

where $\beta' = \dfrac{d\beta}{dr}$

Using equation (2.2) in equation (1.1), we get the generalized components of strain as,

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$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right]$$

$$e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right] \tag{2.3}$$

$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right]$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0$$

where n is the measure.

The stress-strain relation for thermo elastic isotropic materials is

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\theta\delta_{ij} \quad (i,\ j = 1,2,3) \tag{2.4}$$

where $I_1 = e_{kk}$; T_{ij} , e_{ij} are stress and strain tensors respectively and

$$\xi = \alpha\left(3\lambda + 2\mu\right)$$

where λ, μ are Lame's constant, δ_{ij} is Kronecker's delta, α being coefficient of thermal expansion and θ is the temperature.

The temperature θ has to satisfy

$$\theta_{ii} = 0 \tag{2.5}$$

Equation (2.4) for this problem become

$$T_{rr} = \lambda I_1 + \frac{2\mu}{n}\left[1-(r\beta'+\beta)^n\right] - \xi\theta$$

$$T_{\theta\theta} = \lambda I_1 + \frac{2\mu}{n}\left[1-\beta^n\right] - \xi\theta \tag{2.6}$$

$$T_{zz} = \lambda I_1 + \frac{2\mu}{n}\left[1-\left(1-d\right)^n\right] - \xi\theta$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = 0$$

where $I_1 = \frac{1}{n}\left[3-\left(r\beta'+\beta\right)^n - \beta^n - \left(1-d\right)^n\right]$; $\xi = \alpha\left(3\lambda+2\mu\right)$

The temperature field satisfying equation (2.6) and

$\theta = \theta_0$ at $r = a$

$\theta = 0$ at $r = b$

where θ_0 is constant, is given by

$$\theta = \left(\theta_0 \log\frac{r}{b}\right)\Big/\left(\log\frac{a}{b}\right) \tag{2.7}$$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(T_{rr}) + \frac{(T_{rr}-T_{\theta\theta})}{r} + \rho\, r\omega^2 = 0 \tag{2.8}$$

where ρ is the density of the rotating cylinder.

Using equation (2.6) in equation (2.8), one gets a non-linear differential equation in β as

$$\begin{aligned} nP\beta(P+1)^{n-1}\frac{dP}{d\beta} = {} & \frac{rC'}{(1-C)}\left[1-\beta^n(P+1)^n\right]\frac{1}{\beta^n} \\ & - nP\left[(1-C)+(P+1)^n\right] \\ & + C\left[1-(P+1)^n\right] + \frac{nC\rho r^2\omega^2}{2\mu\beta^n} \\ & - \frac{Cn\bar{\theta}_0}{2\mu\beta^n}\left[\xi + r\xi'\log\frac{r}{b}\right] \end{aligned} \tag{2.9}$$

where $r\beta' = \beta P$, $C = \frac{2\mu}{\left(\lambda+2\mu\right)}$ and $\bar{\theta}_0 = \frac{\theta_0}{\log\frac{a}{b}}$

The transition point of β in equation (2.9) are $P \to -1$ and $P \to \pm\propto$.

The boundary conditions are

$$T_{rr} = 0 \text{ at } r = a\ ; \quad T_{rr} = 0 \text{ at } r = a \tag{2.10}$$

The resultant forces normal to the plane Z = constant must vanish, that is,

$$\int_a^b rT_{zz}\,dr = 0 \tag{2.11}$$

3. SOLUTION THROUGH THE PRINCIPAL STRESS DIFFERENCE

For finding the creep stresses, the transition function is taken through the principal stress difference[7-11] at the transition point $P \to -1$. The transition function R is defined as

$$R = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}\left[1-\left(P+1\right)^n\right] \tag{3.1}$$

Taking the logarithmic differentiation of equation (3.1) with respect to r and using equation (2.9), one gets

$$\frac{d}{dr}\left(\log R\right) = \frac{nP}{r} + \frac{C'}{C\ (1-C)} - \frac{\left[\begin{array}{l} \frac{r\,C'}{\beta^n(1-C)}\left[1-\beta^n(P+1)^n\right] - n\,p\left[(1-C)+(P+1)^n\right] \\ +C\ \left[1-(P+1)^n\right] - \frac{C\,n\bar{\theta}_0}{2\mu\ \beta^n}\left[\xi + r\xi'\log\frac{r}{b}\right] + \frac{Cn\rho\, r^2\omega^2}{2\mu\ \beta^n} \end{array}\right]}{r\ \left[1-(P+1)^n\right]} \tag{3.2}$$

Taking asymptotic value of equation (3.3) at $P \to -1$ and integrating, one gets

$$R = A\, r^{-2n}\exp f \tag{3.3}$$

where asymptotic value of β as $P \to -1$ is D/r, D being a constant and

$$2\mu = E(2-C)/(3-2C)$$

where E is the Young's modulus, A is constant of integration and

$$f = \int \frac{C'}{(1-C)}\left\{\frac{1}{C} - \frac{r^n}{D^n}\right\} dr$$

$$+(n-1)\int \frac{C}{r}\, dr - \frac{n\,\rho\,\omega^2}{E\,D^n}\int \frac{C\, r^{n+1}(3-2C)}{(2-C)}\, dr$$

$$+ \frac{n\,\alpha\,\bar{\theta}_0}{D^n}\int \left\{(3-2C) + \frac{r\,C'}{(1-C)}\left(\log\frac{r}{b}\right)\right\} dr$$

From equation (3.1) and equation (3.3), it is found

$$T_{rr} - T_{\theta\theta} = A\, r\, F \tag{3.4}$$

where $F = r^{-2n-1}\exp f$

Substituting the value of $T_{rr} - T_{\theta\theta}$ from equation (3.4) into equation (2.8) and integrating, it is found

$$T_{rr} = B - A\int F dr - \frac{\rho\, r^2 \omega^2}{2} \tag{3.5}$$

where B is a constant of integration.

The constants A and B are obtained by using the boundary conditions (2.10) in equation (3.5) as

$$A = \frac{\frac{\rho\omega^2}{2}\left(a^2 - b^2\right)}{\int_a^b F dr} \;;\quad B = \frac{\rho a^2 \omega^2}{2} + A\left[\int F dr\right]_{r=a}$$

Substituting the value of B in equation (3.5), it is found

$$T_{rr} = \frac{\rho\,\omega^2(a^2 - r^2)}{2} - A\int_a^r F dr \tag{3.6}$$

Using equation (3.6) in equation (3.4), it is found

$$T_{\theta\theta} = T_{rr} - ArF \tag{3.7}$$

Equations (2.6) give

$$T_{zz} = \left(\frac{1-C}{2-C}\right)\left(T_{rr} + T_{\theta\theta}\right) + E\, e_{zz} - E\,\alpha\theta \tag{3.8}$$

Substituting equation (3.8) in equation (2.11), one get

$$e_{zz} = \frac{-\rho\omega^2 \int_a^b \frac{r^3(1-C)}{\left(2-C\right)} dr - \int_a^b \frac{r^2 C' T_{rr}}{(2-C)^2} dr + E\,\alpha \int_a^b r\theta\, dr}{\frac{E(b^2 - a^2)}{2}}$$

Equations (3.6) to (3.8) are thermal creep stresses for non-homogeneous thick-walled rotating cylinder.

Non-homogeneity in the cylinder is due to variable compressibility C as given in equation (1.2). Using equation (1.2) in equations (3.6) to (3.8), one gets

$$T_{rr} = \frac{\rho\omega^2(a^2 - r^2)}{2} - A_1\int_a^r F_1 dr \tag{3.9}$$

$$T_{\theta\theta} = T_{rr} - A_1\, r\, F_1 \tag{3.10}$$

$$T_{zz} = \left(\frac{1 - C_0 r^{-k}}{2 - C_0 r^{-k}}\right)\left(T_{rr} + T_{\theta\theta}\right) + E\, e_{zz} - E\,\alpha\,\theta \tag{3.11}$$

where

$$e_{zz} = \frac{-\rho\omega^2 \int_a^b \frac{r^3(1 - C_0 r^{-k})}{\left(2 - C_0 r^{-k}\right)} dr + k\, C_0 \int_a^b \frac{r^{1-k} T_{rr}}{(2 - C_0 r^{-k})^2} dr + \alpha\, E \int_a^b r\theta\, dr}{E\,\frac{\left(b^2 - a^2\right)}{2}} \tag{3.12}$$

$$A_1 = \frac{\rho\,\omega^2\left(a^2 - b^2\right)}{2\int_a^b F_1 dr} \;;\quad F_1 = \frac{r^{-(2n+k+1)}}{(1 - C_0 r^{-k})}\exp f_1 \;;\quad \text{and}$$

$$f_1 = -\left(\frac{n-1}{k}\right) C_0 r^{-k} + \frac{k C_0}{D^n}\int \frac{r^{n-k-1}}{(1 - C_0 r^{-k})} dr$$

$$- \frac{n\rho\,\omega^2 C_0}{E D^n}\int \frac{r^{n-k+1}(3 - 2C_0 r^{-k})}{(2 - C_0 r^{-k})} dr$$

$$+ \frac{\alpha\, n\bar{\theta}_0}{D^n}\int \left\{\left(3 - 2C_0 r^{-k}\right) - \frac{k C_0 r^{-k}\log(r/b)}{(1 - C_0 r^{-k})}\right\} r^{n-1} dr$$

Equations (3.9) to (3.11) give thermal creep stresses for a thick-walled rotating cylinder having variable compressibility.

The non-dimensional components are introduced as

$$R_0 = \frac{a}{b}\;;\; R = \frac{r}{b}\;;\; \sigma_r = \frac{T_{rr}}{E}\;;\; \sigma_\theta = \frac{T_{\theta\theta}}{E}\;;\; \sigma_z = \frac{T_{zz}}{E}\;;\; \Omega^2 = \frac{\rho\omega^2 b^2}{E}$$

The equations (3.9) to (3.12) in non-dimensional form can be written as

$$\sigma_r = \frac{\Omega^2}{2}\left(R_0^2 - R^2\right) - A_2\int_{R_0}^{R} F_2\, dR \tag{3.13}$$

$$\sigma_\theta = \sigma_r - R\, A_2\, F_2 \tag{3.14}$$

$$\sigma_z = \left(\frac{1 - C_0 b^{-k} R^{-k}}{2 - C_0 b^{-k} R^{-k}}\right)\left(\sigma_r + \sigma_\theta\right) + e_{zz} - \alpha\theta \tag{3.15}$$

where

$$e_{zz} = \frac{\left[\begin{array}{c} -\Omega^2 \int_{R_0}^{1} \frac{R^3(1 - C_0 b^{-k} R^{-k})}{(2 - C_0 b^{-k} R^{-k})} dR + k\, C_0 b^{-k}\int_{R_0}^{1} \frac{R^{1-k}\sigma_r}{(2 - C_0 b^{-k} R^{-k})^2} dR \\ + \frac{\alpha\bar{\theta}_0\, b^2}{\log R_0}\int_{R_0}^{1} R\,\left(\log R\right)\, dR \end{array}\right]}{\frac{1}{2}\left(1 - R_0^2\right)} \tag{3.16}$$

$$A_2 = \frac{\Omega^2\left(R_0^2 - 1\right)}{2\int_{R_0}^{1} F_2\, dR} \;;\quad F_2 = \frac{\left(bR\right)^{-(2n+k+1)}}{\left(1 - C_0\, b^{-k} R^{-k}\right)}\exp f_2$$

$$f_2 = -\left(\frac{n-1}{k}\right) C_0\, b^{-k} R^{-k} + \frac{k\, C_0\, b^{n-k}}{D^n}\int \frac{R^{n-k-1}}{(1 - C_0 b^{-k} R^{-k})} dR -$$

$$- \frac{n\,\Omega^2\, C_0\, b^{n-k}}{D^n}\int \frac{R^{n-k-1}(3 - 2C_0 b^{-k} R^{-k})}{(2 - C_0 b^{-k} R^{-k})} dR +$$

$$+ \frac{n\alpha\,\bar{\theta}_0 b^n}{D^n}\int \left\{\left(3 - 2C_0 b^{-k} R^{-k}\right) - \frac{k\, C_0 b^{-k} R^{-k}\left(\log R\right)}{(1 - C_0 b^{-k} R^{-k})}\right\} R^{n-1} dR$$

4. ROTATING CYLINDER WITHOUT THERMAL EFFECTS

For non-homogeneous material with negligible temperature field (θ_0=0), creep stresses (3.13) to (3.15) become

$$\sigma_r = \frac{\Omega^2}{2}\left(R_0^2 - R^2\right) - A_3 \int_{R_0}^{R} F_3 \, dR \qquad (4.1)$$

$$\sigma_\theta = \sigma_r - R\, A_3\, F_3 \qquad (4.2)$$

$$\sigma_z = \left(\frac{1 - C_0 b^{-k} R^{-k}}{2 - C_0 b^{-k} R^{-k}}\right)\left(\sigma_r + \sigma_\theta\right) + e_{zz} \qquad (4.3)$$

where

$$e_{zz} = \frac{\left[-2\,\Omega^2 \int_{R_0}^{1} \frac{R^3(1 - C_0 b^{-k} R^{-k})}{(2 - C_0 b^{-k} R^{-k})} dR + 2kC_0 b^{-k} \int_{R_0}^{1} \frac{R^{1-k}\sigma_r}{(2 - C_0 b^{-k} R^{-k})^2} dR\right]}{\left(1 - R_0^2\right)} \qquad (4.4)$$

$$A_3 = \frac{\Omega^2\left(R_0^2 - 1\right)}{2\int_{R_0}^{1} F_3 \, dR} \; ; \quad F_3 = \frac{\left(bR\right)^{-(2n+k+1)}}{\left(1 - C_0 b^{-k} R^{-k}\right)} \exp f_3$$

$$f_2 = -\left(\frac{n-1}{k}\right) C_0\, b^{-k} R^{-k} + \frac{k\, C_0\, b^{n-k}}{D^n} \int \frac{R^{n-k-1}}{(1 - C_0 b^{-k} R^{-k})} dR - \frac{n\,\Omega^2 C_0\, b^{n-k}}{D^n} \int \frac{R^{n-k-1}(3 - 2C_0 b^{-k} R^{-k})}{(2 - C_0 b^{-k} R^{-k})} dR$$

Equations (4.1) to (4.4) are same as obtained by Gupta *et.al.*[8]

5. STRAIN RATES

When the creep sets in, the strain should be replaced by strain rates. The stress-strain relation (2.4) can be written as

$$\dot{e}_{ij} = \frac{(1+\nu)}{E} T_{ij} - \frac{\nu}{E} \delta_{ij}\, \Theta + \alpha\, \theta \qquad (5.1)$$

where $\dot{e}_{ij}$ is the strain rate tensor with respect to flow parameter t and $\Theta = T_{11} + T_{22} + T_{33}$ and $\nu = \left(1 - C\right)/\left(2 - C\right)$ is the Poisson's ratio.

Differentiating equation (2.3) with respect to t, it is found

$$\dot{e}_{\theta\theta} = -\beta^{n-1}\dot{\beta} \qquad (5.2)$$

For SWAINGER measure (n=1)

$$\dot{\varepsilon}_{\theta\theta} = -\dot{\beta} \qquad (5.3)$$

where $\dot{\varepsilon}_{\theta\theta}$ is SWAINGER strain measure.

The transition value of equation (3.1) at $P \to -1$ gives

$$\beta = \left(\frac{n}{2\,\mu}\right)^{\frac{1}{n}} \left(T_{rr} - T_{\theta\theta}\right)^{\frac{1}{n}} \qquad (5.4)$$

Using equations (5.2), (5.3) and (5.4) in equation (5.1), it is found

$$\dot{\varepsilon}_{\theta\theta} = \left[\frac{n}{2\,\mu}\left(T_{rr} - T_{\theta\theta}\right)\right]^{\frac{1}{n}-1} \left[\frac{\left(1+\nu\right)}{E} T_{ij} - \frac{\nu}{E}\delta_{ij}\Theta + \alpha\,\theta\right]$$

i.e.

$$\dot{\varepsilon}_{rr} = \left[\frac{(3-2C)}{N(2-C)}\right]^{N-1} \left(\sigma_r - \sigma_\theta\right)^{N-1} \left[\sigma_r - \left(\frac{1-C}{2-C}\right)\left(\sigma_z + \sigma_\theta\right) + \alpha\,\theta\right] \qquad (5.5)$$

$$\dot{\varepsilon}_{\theta\theta} = \left[\frac{(3-2C)}{N(2-C)}\right]^{N-1} \left(\sigma_r - \sigma_\theta\right)^{N-1} \left[\sigma_\theta - \left(\frac{1-C}{2-C}\right)\left(\sigma_r + \sigma_z\right) + \alpha\,\theta\right] \qquad (5.6)$$

and

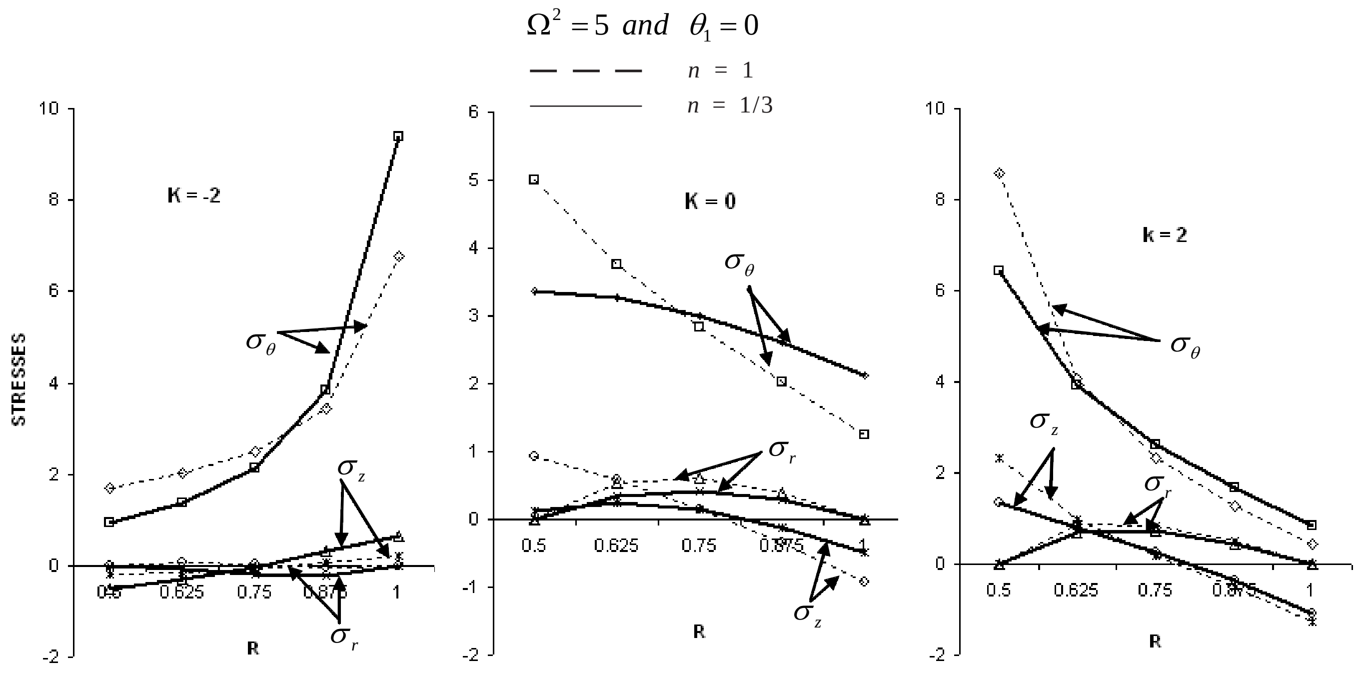

Figure 1. Creep stresses in a non-homogeneous / homogeneous thick-walled rotating cylinder.

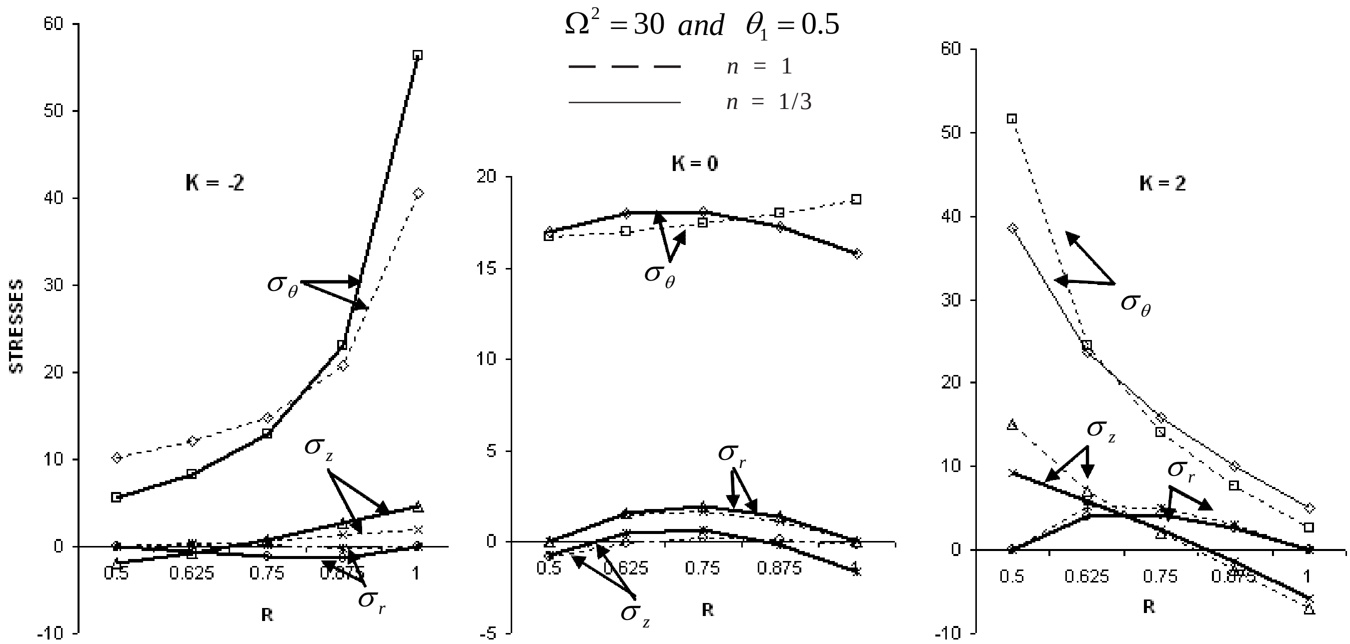

Figure 2. Creep stresses in a non-homogeneous/homogeneous thick-walled rotating cylinder.

Table 1. Values of circumferential stress σ_θ for a non-homogeneous/ homogeneous rotating cylinder along the radius *R* for various values of angular speed Ω^2 and measure *n* with and without thermal effects.

	$\Omega^2 = 5$ and $\theta_1 = 0$					$\Omega^2 = 30$ and $\theta_1 = 0.5$				
R →	**0.5**	**0.625**	**0.75**	**0.875**	**1.0**	**0.5**	**0.625**	**0.75**	**0.875**	**1.0**
n ↓	**K = - 2 (Non-homogeneous material whose compressibility increases radially)**									
1/7	0.78	1.20	2.03	3.92	10.21	4.66	7.21	12.17	23.51	61.24
1/5	0.82	1.25	2.06	3.89	9.96	4.93	7.51	12.38	23.37	59.77
1/3	0.93	1.37	2.14	3.84	9.40	5.59	8.22	12.87	23.04	56.41
1	1.69	2.02	2.48	3.45	6.76	10.14	12.12	14.88	20.69	40.57
n ↓	**K = 0 (Homogeneous incompressible material)**									
1/7	2.98	3.09	3.02	2.79	2.45	16.59	17.91	18.11	17.41	15.91
1/5	3.097	3.15	3.02	2.74	2.35	16.72	17.95	18.10	17.35	15.84
1/3	3.38	3.26	2.99	2.62	2.13	16.95	18.01	18.06	17.28	15.77
1	5	3.75	2.83	2.028	1.25	16.69	17.00	17.42	17.94	18.69
n ↓	**K = 2 (Non-homogeneous material whose compressibility decreases radially)**									
1/7	5.87	3.86	2.71	1.82	1.01	35.20	23.18	16.27	10.94	6.08
1/5	6.04	3.89	2.69	1.78	0.96	36.21	23.33	16.13	10.69	5.78
1/3	6.44	3.94	2.63	1.69	0.85	38.617	23.66	15.79	10.12	5.12
1	8.58	4.09	2.33	1.29	0.45	51.49	24.54	13.99	7.73	2.71

$$\dot{\varepsilon}_{zz} = \left[\frac{(3-2C)}{N(2-C)}\right]^{N-1} \left(\sigma_r - \sigma_\theta\right)^{N-1} \left[\sigma_z - \left(\frac{1-C}{2-C}\right)\left(\sigma_r + \sigma_\theta\right) + \alpha\,\theta\right] = -\dot{\varepsilon}_0 \quad (5.7)$$

These are the constitutive equations for finding the creep stresses. In classical theory, the measure $N = 1/n$. For incompressible material, without thermal effects one obtained the same constitutive equations as given by Odquist[3].

6. NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating the stresses and strain rate distributions based on the above analysis, the definite integral in equation (3.13) to equation (3.16) have been evaluated by using Simpson's rule (with the use of Mathematica) by taking $D = 1$. Curves have been made for stresses and strain rates for measuring $n = 1$, 1/3 with angular speed Ω^2=5, 30 at temperature $\theta_1 = \alpha\theta_0$= 0.5 [thermal expansion coefficient α = 5.0 x 10^{-5} °F^{-1} (for Methyl Methacrylate) and θ_0= 10000° F] with respect to radii ratio *R* shown in Fig. 1 and Fig. 2 for *k* = –2, 0, 2, respectively. In classical theory, the measure *N* is equal to .

(i) For *k* = –2, the compressibility of the material varies as $C = C_0 r^2$, that is, from the lowest value at the internal surface to the highest value at the external surface. It has been observed from Fig. 1 that circumferential stress for non-homogeneous rotating cylinder with an angular speed Ω^2=5, is maximum at the external surface for the measure *n* = 1, 1/3. It has also been observed from Table 1 that circumferential stress goes on increasing with the increase in measure. From Fig. 2, it has been observed that circumferential stress goes on increasing

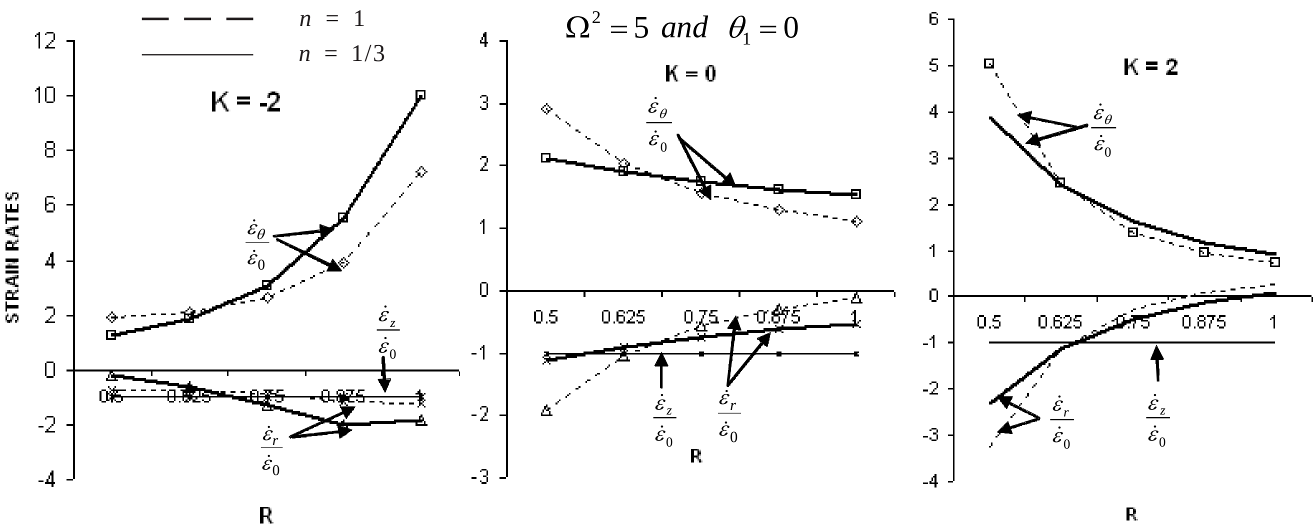

Figure 3. Strain rates in a non-homogeneous/homogeneous thick-walled rotating cylinder.

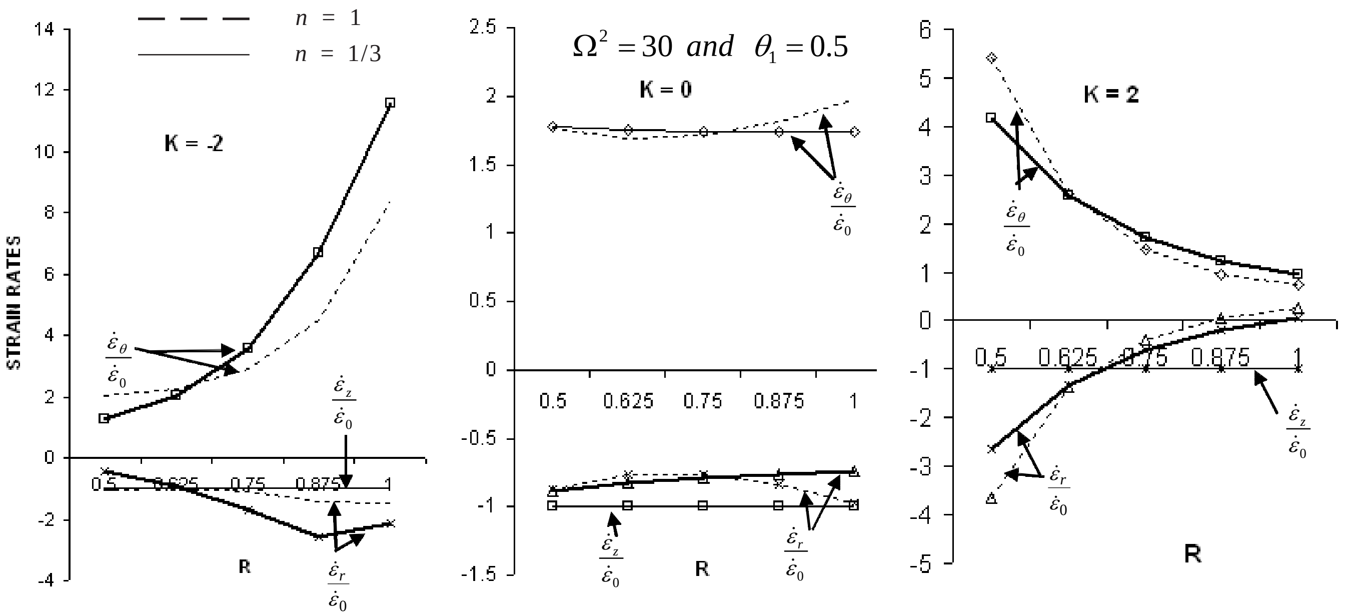

Figure 4. Strain rates in a non-homogeneous/homogeneous thick-walled rotating cylinder.

with the increase in angular speed and inclusion of thermal effects.

(ii) For k = 0, for homogeneous incompressible rotating cylinder, it has been observed from Fig. 1 that with angular speed Ω^2=5, circumferential stress is maximum at the internal surface. It has been observed from Table 1 that with the increase in measure, the circumferential stress goes on decreasing at the internal surface. From Fig. 2, it has been observed that circumferential stress goes on increasing with the increase in angular speed and inclusion of thermal effects.

(iii) For k = 0, the compressibility of the material varies as $C=C_0/r^2$, that is, from the higher value at the internal surface to the lower value at the external surface. It has been observed from Fig. 1 that circumferential stress is maximum at the internal surface and goes on decreasing with the increase in measure N (as seen from Table 1). From Fig. 2, it has been seen that the circumferential stress goes on increasing at the internal surface with the increase in angular speed and inclusion of thermal effects.

It can be seen from Fig. 3 that there is a contraction in the radial direction at its internal surface for a non-homogeneous rotating cylinder whose compressibility increases radially (k=–2) for n = 1, 1/3 respectively and it increases for homogeneous incompressible rotating cylinder (k=0) and non homogeneous rotating cylinder whose compressibility decreases radially (k=2).With the increase in angular speed and inclusion of thermal effects, there is more contraction in radial direction for k=–2 and k=2 while less contraction for incompressible material k=0 (as seen from Fig. 4). The circumferential strain rate is maximum at the external surface for k=–2 and at internal surface for k=0, k=2.

7. CONCLUSION

From the above observations, it can be concluded that a cylinder made of less compressible material at the internal surface and highly compressible at the outer surface is on the safer side of the design for different values of N, Ω^2 and temperature as compared to highly compressible at the internal surface and less compressible at the outer surface.

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