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Conversion Method of Impact Dispersion in Substitute Equivalent Tests Based on Error Propagation

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ABSTRACT

Since there are discrepancies of test environment, target and range between substitute equivalent tests and overall operational tests, this study explores a method for converting the impact dispersion of missiles with homing radar from the substitute equivalent test to that in the overall operational test. According to the law of error synthesis, differences between impact dispersion in different tests are decomposed and propagated to the measurement elements of the homing radar by analysing factors which influence the impact dispersion. Furthermore, the measurement errors in different tests were drawn from their related parameters of radar measurement elements in different tests by analysing the error sources. Then, the different measurement errors were propagated and synthesised into the impact dispersion in different environments. Precision of the conversion process and reliability of the conversion results have also been analysed. Feasibility and effectiveness of this method have been proved in simulation tests.

Keywords: Aircraft testing, error analysis, estimation, measurement errors, radar navigation, impact dispersion, substitute tests, operational tests, dispersion conversion, strategic weapons

1. INTRODUCTION

Since guided weapons can only be assessed through a few operational flight tests due to high cost[1], a lot of substitute tests are designed to help their assessment. However, because of the differences of environment and target between substitute tests and overall operational tests, the measurement can not reflect missile performance directly and precisely. Thus the problem arises of dispersion conversion from the substitute test to the overall operational test.

Researchers have been making efforts to analyse and convert impact dispersion. In overall tests, test trajectory was used for assessing strategic weapons instead of overall trajectory. Fire dispersion indices were systematically decomposed to meet the demand of fire dispersion assessment, and decomposed errors were converted into the overall trajectory[2]. However, current researches mainly focus on dispersion conversion between different ranges, none touching upon the conversion method between different environments, targets, and testing states.

With regard to substitute equivalent calculation, performances in substitute equivalent tests are different from that in the overall operational tests because of discrepancies of test environments, targets, and ranges. Therefore, many results in substitute tests can not be directly applied to the assessment process. As a result, how to analyse the relationship and differences between substitute tests and overall operational tests and how to convert the results in the substitute test into the overall operational test become the main problems requiring consideration.

Along with the advancement of the range testing technique, some methods for converting the results of substitute tests into the equipment operational tests have been put forward based on the synthetic evaluation theory, which comprises mathematical simulation test, equipment operational test, and substitute equivalent calculation. The radar operating distance measured in substitute equivalent tests can be converted into equipment operational test through deduction of the radar equation[3]. Some researchers now apply this method in radar netting[4]. Moreover, researchers have proposed an equivalent calculation method for operating distance in infrared searching systems based on infrared equations[5]. However, none of the studies has investigated conversion between different environments, targets and test types for missile testing and assessment.

This study seeks to solve the problem of impact dispersion analysis and conversion between different tests for cluster warhead missiles with homing radar. Main factors influencing impact dispersion in different tests were analysed, decomposed, and propagated to error sources of radar measurement. Then, a model was constructed. Based on the model and radar measurements of the substitute test, precision of the substitute test was converted into that in the overall operational test. Finally, errors of the influence factors were integrated and synthesised into impact dispersion of the overall operational test according to the theory of error synthesis.

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2. CONVERSION THEORY OF THE SUBSTITUTE TEST

The basic idea of the conversion is as follows:

- Firstly, to select the representative factors which influence main index to be assessed in the substitute test; then, to construct a correlation model of the index and these factors; and ultimately, to calculate the expected index through putting the influence factors of expected state into the correlation model.
- Mark the index to be assessed as T, the variable influence factors of the index as $\alpha_1, \alpha_2, \ldots, \alpha_m$, and the invariant ones as $\beta_1, \beta_2, \ldots, \beta_n$. Therefore, the correlation model between the index and its influence factors can be expressed by the response function $f(.)$ as:

$$T = f\left(\alpha_1, \alpha_2, \cdots, \alpha_m; \beta_1, \beta_2, \cdots, \beta_n\right) + \varepsilon \tag{1}$$

Meanwhile, mark the index which has been measured in the substitute test as $\tilde{T}$. The correlation between the measurement index and its influence factors can also be expressed by the response function $f(.)$:

$$\tilde{T} = f\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_m; \beta_1, \beta_2, \cdots, \beta_n\right) + e \tag{2}$$

where $\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_m$ are the variable influence factors in the substitute test.

The process of index conversion is to calculate the index in the expected state from the measurement in the substitute test by the response function and the variable parameters. The key is the design of the response function $f(.)$ and the selection of the variable parameters in different states.

As influence factors of the impact dispersion are complex, it's hard to design an accurate and comprehensive response function. So the authors tried to decompose and propagate the impact dispersion, and further to propagate and synthesise the conversion value between different states from the error sources of radar measurement. The specific idea is shown in Fig. 1.

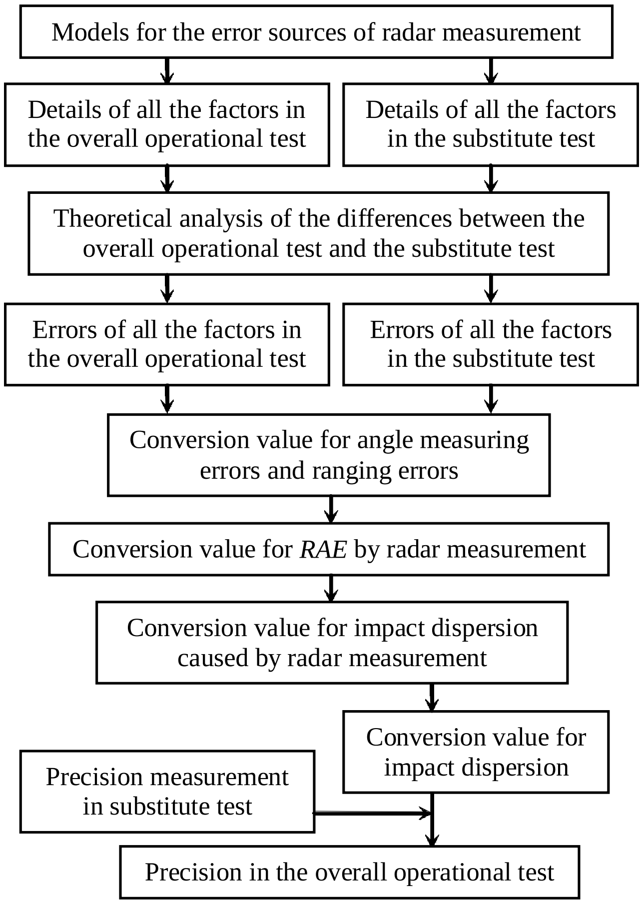

Figure 1. Conversion scheme of the impact dispersion from the substitute test to the operational test. (Conversion value in this paper refers to the difference between assessed indexes in different states).

3. DECOMPOSITION AND PROPAGATION OF CONVERSION VALUE

The impact dispersion ΔL of cluster warhead missiles with active homing radar is mainly composed of the measurement error of radar navigation system ΔL_{Rad}, the instrumental error of inertial navigation system (INS) ΔL_{INS}, the method error of terminal guidance ΔL_{M}, the error of distribution ΔL_{D} and the random error ΔL_{Ran}. According to the law of error synthesis[6], ΔL can be decomposed as follows:

$$\Delta L^2 = \Delta L_{\mathrm{Rad}}^2 + \Delta L_{\mathrm{INS}}^2 + \Delta L_{\mathrm{M}}^2 + \Delta L_{\mathrm{D}}^2 + \Delta L_{\mathrm{Ran}}^2 \tag{3}$$

The instrumental error of INS refers to the position error accumulated by the velocity error of INS after the radar seeker shuts down; the method error of terminal guidance refers to the impact dispersion caused by the trajectory command tracker error of the guidance and control system and the external interference factors such as aerodynamic disturbance and wind disturbance.

The impact dispersion is decomposed in different tests as follows:

$$\begin{cases} \Delta L^2 = \Delta L_{\mathrm{Rad}}^2 + \Delta L_{\mathrm{INS}}^2 + \Delta L_{\mathrm{M}}^2 + \Delta L_{\mathrm{D}}^2 + \Delta L_{\mathrm{Ran}}^2 \\ \Delta \tilde{L}^2 = \Delta \tilde{L}_{\mathrm{Rad}}^2 + \Delta \tilde{L}_{\mathrm{INS}}^2 + \Delta \tilde{L}_{\mathrm{M}}^2 + \Delta \tilde{L}_{\mathrm{D}}^2 + \Delta \tilde{L}_{\mathrm{Ran}}^2 \end{cases} \tag{4}$$

As the missile-borne devices used in the substitute test were the same as those in the overall operational test, the impact dispersion caused by the instrumental error of INS and the distribution error of the warhead distribution system were the same in the two tests. The method error of terminal guidance can also be viewed as the same in the substitute test and the operational test. Therefore, one only needs to consider the impact dispersion caused by homing radar when converting the measurement in the substitute test to the results of the overall operational test. The impact dispersion difference of the two tests is as follows:

$$\Delta L^2 - \Delta \tilde{L}^2 = \Delta L_{\mathrm{Rad}}^2 - \Delta \tilde{L}_{\mathrm{Rad}}^2 \tag{5}$$

where ΔL is the impact dispersion in the overall operational test, $\Delta \tilde{L}$ the measured impact dispersion in the substitute test, ΔL_{Rad} the assessed impact dispersion caused by the error of radar measurement in the overall operational test, and $\Delta\tilde{L}_{\mathrm{Rad}}$ the measured impact dispersion caused by the error of radar measurement in the substitute test.

Now one needs to get the difference between ΔL_{Rad} and $\Delta\tilde{L}_{\mathrm{Rad}}$. For cluster warhead missiles with active homing radar, the measurement error of the warhead-target relative positions *x*, *y*, *z* equals to its impact dispersion caused by the error of radar measurement. Thus one has:

$$\begin{cases}\Delta L^2_{\mathrm{Rad}X}-\Delta\tilde{L}^2_{\mathrm{Rad}X}=\Delta_x^2-\tilde{\Delta}_x^2\\ \Delta L^2_{\mathrm{Rad}Y}-\Delta\tilde{L}^2_{\mathrm{Rad}Y}=\Delta_y^2-\tilde{\Delta}_y^2\\ \Delta L^2_{\mathrm{RadZ}}-\Delta\tilde{L}^2_{\mathrm{RadZ}}=\Delta_z^2-\tilde{\Delta}_z^2\end{cases}\tag{6}$$

As a result, the problem of calculating the difference between the impact dispersion in different tests in Eqn (5) turns into the problem of calculating the difference between errors of the warhead-target relative positions *x*, *y*, *z*. But the measurement elements in radar are *R*, *A*, *E*, so one has to consider how to propagate the difference between errors of *x*, *y*, *z* to the difference between errors of *R*, *A*, *E*. The relationship between *x*, *y*, *z* and *R*, *A*, *E* is shown in Fig. 2.

$$\begin{cases}x=R\cos E\cos A\\ y=R\sin E\\ z=R\cos E\sin A\end{cases}\tag{7}$$

According to the Gauss's law of error propagation[7], the relationship between errors of *x*, *y*, *z* and that of *R*, *A*, *E* can be described as:

$$\begin{cases}\Delta_x=\dfrac{\partial x}{\partial R}\Delta_R+\dfrac{\partial x}{\partial E}\Delta_E+\dfrac{\partial x}{\partial A}\Delta_A\\ \Delta_y=\dfrac{\partial y}{\partial R}\Delta_R+\dfrac{\partial y}{\partial E}\Delta_E+\dfrac{\partial y}{\partial A}\Delta_A\\ \Delta_z=\dfrac{\partial z}{\partial R}\Delta_R+\dfrac{\partial z}{\partial E}\Delta_E+\dfrac{\partial z}{\partial A}\Delta_A\end{cases}\tag{8}$$

where Δ_x, Δ_y, Δ_z are errors of warhead-target relative positions, and Δ_R, Δ_A, Δ_E errors of radar measurement *R*, *A*, *E*.

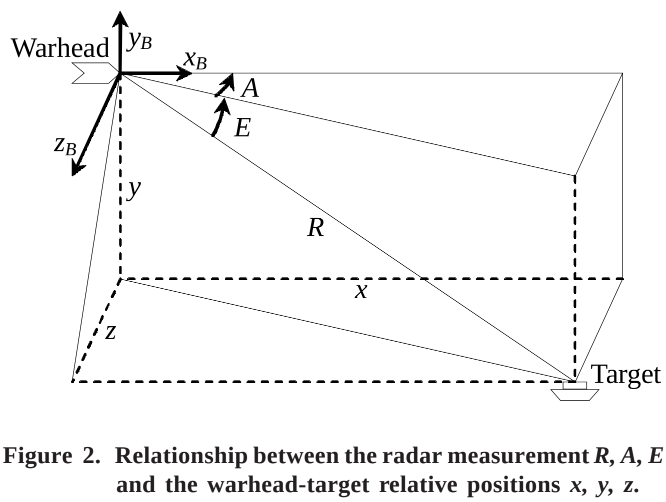

Figure 2. Relationship between the radar measurement *R*, *A*, *E* and the warhead-target relative positions *x*, *y*, *z*.

Their relationship in the overall operational test can be obtained by combining Eqns (7) and (8).

$$\begin{cases}\Delta_x=\cos E\cos A\cdot\Delta_R-R\sin E\cos A\cdot\Delta_E-R\cos E\sin A\cdot\Delta_A\\ \Delta_y=\sin E\cdot\Delta_R+R\cos E\cdot\Delta_E\\ \Delta_z=\cos E\sin A\cdot\Delta_R-R\sin E\sin A\cdot\Delta_E+R\cos E\cos A\cdot\Delta_A\end{cases}\tag{9}$$

where Δ_x, Δ_y, Δ_z are the measurement errors of warhead-target relative positions *x*, *y*, *z* in the overall operational test, and Δ_R, Δ_A, Δ_E the errors of *R*, *A*, *E* measured by radar in the overall operational test. Similarly, their relationship in the substitute test can be expressed as:

$$\begin{cases}\tilde{\Delta}_x=\cos\tilde{E}\cos\tilde{A}\cdot\tilde{\Delta}_R-\tilde{R}\sin\tilde{E}\cos\tilde{A}\cdot\tilde{\Delta}_E-\tilde{R}\cos\tilde{E}\sin\tilde{A}\cdot\tilde{\Delta}_A\\ \tilde{\Delta}_y=\sin\tilde{E}\cdot\tilde{\Delta}_R+\tilde{R}\cos\tilde{E}\cdot\tilde{\Delta}_E\\ \tilde{\Delta}_z=\cos\tilde{E}\sin\tilde{A}\cdot\tilde{\Delta}_R-\tilde{R}\sin\tilde{E}\sin\tilde{A}\cdot\tilde{\Delta}_E+\tilde{R}\cos\tilde{E}\cos\tilde{A}\cdot\tilde{\Delta}_A\end{cases}\tag{10}$$

where $\tilde{\Delta}_x$, $\tilde{\Delta}_y$, $\tilde{\Delta}_z$ are the measurement errors of warhead-target relative positions *x*, *y*, *z* in the substitute test, and $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ the errors of *R*, *A*, *E* measured by radar in the substitute test.

Now the problem of converting the impact dispersion in Eqn (5) turns into how to get the radar measurement errors $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ in the substitute test and Δ_R, Δ_A, Δ_E in the overall operational test, respectively.

4. CALCULATION AND SYNTHESIS OF CONVERSION VALUE

The method for calculating $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ in the substitute test can be introduced. First the telemetric data $\tilde{R}_{\mathrm{Tel}}$, $\tilde{A}_{\mathrm{Tel}}$, $\tilde{E}_{\mathrm{Tel}}$ measured by missile-borne radar is obtained, and the observed tracking data $\tilde{X}_{\mathrm{Tra}}$, $\tilde{Y}_{\mathrm{Tra}}$, $\tilde{Z}_{\mathrm{Tra}}$ in the target coordinate system by measuring equipments on the ground. As the target in the substitute test is motionless, the tracking data equals to the warhead-target relative positions *x*, *y*, *z*. So, $\tilde{X}_{\mathrm{Tra}}$, $\tilde{Y}_{\mathrm{Tra}}$, $\tilde{Z}_{\mathrm{Tra}}$ can be propagated to the radar measurement *R*, *A*, *E* through the spatial relation in Fig. 2. Then one gets:

$$\begin{cases}\tilde{R}_{\mathrm{Tra}}=\sqrt{\tilde{X}^2_{\mathrm{Tra}}+\tilde{Y}^2_{\mathrm{Tra}}+\tilde{Z}^2_{\mathrm{Tra}}}\\ \tilde{A}_{\mathrm{Tra}}=\cot\left(\tilde{Z}_{\mathrm{Tra}}\big/\tilde{X}_{\mathrm{Tra}}\right)\\ \tilde{E}_{\mathrm{Tra}}=\cot\left(\tilde{Y}_{\mathrm{Tra}}\Big/\sqrt{\tilde{X}^2_{\mathrm{Tra}}+\tilde{Z}^2_{\mathrm{Tra}}}\right)\end{cases}\tag{11}$$

As the telemetric data $\tilde{R}_{\mathrm{Tel}}$, $\tilde{A}_{\mathrm{Tel}}$, $\tilde{E}_{\mathrm{Tel}}$ are measured by missile-borne radar and the tracking data $\tilde{X}_{\mathrm{Tra}}$, $\tilde{Y}_{\mathrm{Tra}}$, $\tilde{Z}_{\mathrm{Tra}}$ by measuring equipments on the ground, errors of measuring *R*, *A*, *E* are included in $\tilde{R}_{\mathrm{Tel}}$, $\tilde{A}_{\mathrm{Tel}}$, $\tilde{E}_{\mathrm{Tel}}$ but not in $\tilde{X}_{\mathrm{Tra}}$, $\tilde{Y}_{\mathrm{Tra}}$, $\tilde{Z}_{\mathrm{Tra}}$. Their differences are as in Eqn (12):

$$\begin{cases}\tilde{\Delta}_R=\tilde{R}_{\mathrm{Tel}}-\tilde{R}_{\mathrm{Tra}}\\ \tilde{\Delta}_A=\tilde{A}_{\mathrm{Tel}}-\tilde{A}_{\mathrm{Tra}}\\ \tilde{\Delta}_E=\tilde{E}_{\mathrm{Tel}}-\tilde{E}_{\mathrm{Tra}}\end{cases}\tag{12}$$

where $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ are errors of *R*, *A*, *E* measured by missile-borne radar. Errors of warhead-target relative positions *x*,

y, z in the substitute test, i.e., $\tilde{\Delta}_x$, $\tilde{\Delta}_y$, $\tilde{\Delta}_z$, can then be calculated by Eqn (10). Now, the values of Δ_R, Δ_A, Δ_E are required to calculate the conversion value of the impact dispersion.

The respective conversion values of radar measurement errors in the substitute test and the overall operational test are described as follows:

$$\begin{cases} \delta_R^2 = \left|\Delta_R^2 - \tilde{\Delta}_R^2\right| \\ \delta_A^2 = \left|\Delta_A^2 - \tilde{\Delta}_A^2\right| \\ \delta_E^2 = \left|\Delta_E^2 - \tilde{\Delta}_E^2\right| \end{cases} \tag{13}$$

Now one needs to analyse error sources of R, A, E measured by missile-borne radar. Errors of radar tracking measurement can be classified into angle measuring error, ranging error, and velocity measuring error, but only the former two are used for measuring R, A, E. The monopulse radar system was taken as an example to specifically analyse main influence factors of the angle measuring error and the ranging error[8,9].

Table 1. Factors influencing the measurement precision of homing radar

	Angle Measuring Error	Ranging Error
Environment-relevant factors	Clutter and interference	Clutter and interference
Target-relevant factors	Angular glint dynamic lag	Range glint dynamic lag
Radar-relevant factors	Thermal noise phase imbalance	Thermal noise

There are still many other factors influencing the measurement precision, but some of them are intrinsic factors in radar, which are independent of external circumstances, and some are extremely particle. So Table 1 only lists major error sources. (for models measuring this kind of error sources, see refs. 8,9)

Based on analysis of the error sources of radar measurement R, A, E, the ultimate error of angle measuring can be described by the law of error synthesis as follows:

$$\begin{aligned} U^2 &= \Delta^2_{\text{Radar-relevant}} + \Delta^2_{\text{Target-relevant}} + \Delta^2_{\text{Environment-relevant}} + \cdots \\ &= \Delta^2_{\text{TN}} + \Delta^2_{\text{PU}} + \Delta^2_{\text{AG}} + \Delta^2_{\text{DL}} + \Delta^2_{\text{CI}} + \cdots \end{aligned} \tag{14}$$

where Δ_{TN}, Δ_{PU}, Δ_{AG}, Δ_{DL}, Δ_{CI} are the errors caused by thermal noise, phase imbalance, angular glint, dynamic lag, clutter and interference respectively.

Through Eqns (13) and (14), it is seen that:

$$\begin{aligned} \delta_A^2 &= \left|\Delta_A^2 - \tilde{\Delta}_A^2\right| = \left|U^2 - \tilde{U}^2\right| \\ &= \begin{vmatrix} \left(\Delta^2_{\text{TN}} + \Delta^2_{\text{AG}} + \Delta^2_{\text{DL}} + \Delta^2_{\text{CI}} + \cdots\right) \\ -\left(\tilde{\Delta}^2_{\text{TN}} + \tilde{\Delta}^2_{\text{AG}} + \tilde{\Delta}^2_{\text{DL}} + \tilde{\Delta}^2_{\text{CI}} + \cdots\right) \end{vmatrix} \end{aligned} \tag{15}$$

where the abbreviated terms represent the same errors in the two tests, so one can simplify Eqn (15) as:

$$\delta_A^2 = \begin{vmatrix} \left(\Delta^2_{\text{TN}} - \tilde{\Delta}^2_{\text{TN}}\right) + \left(\Delta^2_{\text{AG}} - \tilde{\Delta}^2_{\text{AG}}\right) \\ + \left(\Delta^2_{\text{DL}} - \tilde{\Delta}^2_{\text{DL}}\right) + \left(\Delta^2_{\text{CI}} - \tilde{\Delta}^2_{\text{CI}}\right) \end{vmatrix} \tag{16}$$

The conversion value of radar measurement angle A, i.e., A, can be calculated by Eqn (16) together with the models of error sources[8,9] and the corresponding parameters in both the tests. The magnitude of Δ_A^2 and $\tilde{\Delta}_A^2$ need to be estimated while calculating to determine their relationship with δ_A. Δ_A can be denoted by Eqn (13) as:

$$\begin{cases} \Delta_A^2 = \tilde{\Delta}_A^2 + \delta_A^2, & U^2 - \tilde{U}^2 > 0; \\ \Delta_A^2 = \tilde{\Delta}_A^2, & U^2 - \tilde{U}^2 = 0; \\ \Delta_A^2 = \tilde{\Delta}_A^2 - \delta_A^2, & U^2 - \tilde{U}^2 < 0. \end{cases} \tag{17}$$

Similarly, one can get the conversion value of radar measurement R and E, i.e., δ_R and δ_E, and the errors of radar measurement Δ_R and Δ_E. Values of Δ_R, Δ_E and Δ_A are used to calculate Δ_x, Δ_y, Δ_z by Eqn (9).

Combining Δ_x, Δ_y, Δ_z with $\tilde{\Delta}_x$, $\tilde{\Delta}_y$, $\tilde{\Delta}_z$, which are calculated previously by Eqn (6), one has:

$$\begin{cases} \Delta L_X^2 - \Delta\tilde{L}_X^2 = \Delta L^2_{\text{Rad}X} - \Delta\tilde{L}^2_{\text{Rad}X} = \Delta_x^2 - \tilde{\Delta}_x^2 \\ \Delta L_Y^2 - \Delta\tilde{L}_Y^2 = \Delta L^2_{\text{Rad}Y} - \Delta\tilde{L}^2_{\text{Rad}Y} = \Delta_y^2 - \tilde{\Delta}_y^2 \\ \Delta L_Z^2 - \Delta\tilde{L}_Z^2 = \Delta L^2_{\text{RadZ}} - \Delta\tilde{L}^2_{\text{RadZ}} = \Delta_z^2 - \tilde{\Delta}_z^2 \end{cases} \tag{18}$$

Then one gets:

$$\begin{cases} \Delta L_X^2 = \Delta\tilde{L}_X^2 + \Delta_x^2 - \tilde{\Delta}_x^2 \\ \Delta L_Y^2 = \Delta\tilde{L}_Y^2 + \Delta_y^2 - \tilde{\Delta}_y^2 \\ \Delta L_Z^2 = \Delta\tilde{L}_Z^2 + \Delta_z^2 - \tilde{\Delta}_z^2 \end{cases} \tag{19}$$

where $\Delta\tilde{L}_X$, $\Delta\tilde{L}_Y$, $\Delta\tilde{L}_Z$ are the impact dispersions measured in the substitute test.

Hereto, one gets the conversion results of impact dispersion in the overall operational test.

5. PRECISION ANALYSIS OF CONVERSION PROCESS

In the conversion process of the impact dispersion, one also needs to consider precision of the conversion process and reliability of the conversion results. Based on the law of error propagation[7], the relationship of variances for response function $b=f\,(a_1,a_2,a_3,\ldots)$ is:

$$\sigma_b^2 = \left(\frac{\partial f}{\partial a_1}\right)^2 \sigma_{a_1}^2 + \left(\frac{\partial f}{\partial a_2}\right)^2 \sigma_{a_2}^2 + \left(\frac{\partial f}{\partial a_3}\right)^2 \sigma_{a_3}^2 + \cdots \tag{20}$$

The error of tracking data $\tilde{R}_{\text{Tra}}$, $\tilde{A}_{\text{Tra}}$, $\tilde{E}_{\text{Tra}}$ in the substitute test is propagated to R, A, E in the operational test. Combining Eqn (11) for transformation from position parameters to measurement parameters, based on the law of error propagation, the relationship between the variances of tracking data $\tilde{R}_{\text{Tra}}$, $\tilde{A}_{\text{Tra}}$, $\tilde{E}_{\text{Tra}}$ and the variances of tracking data $\tilde{X}_{\text{Tra}}$, $\tilde{Y}_{\text{Tra}}$, $\tilde{Z}_{\text{Tra}}$ is:

$$
\begin{aligned}
\tilde{\sigma}_{R\mathrm{Tra}}^2 &= \frac{\tilde{X}_{\mathrm{Tra}}^2\tilde{\sigma}_{X\mathrm{Tra}}^2+\tilde{Y}_{\mathrm{Tra}}^2\tilde{\sigma}_{Y\mathrm{Tra}}^2+\tilde{Z}_{\mathrm{Tra}}^2\tilde{\sigma}_{Z\mathrm{Tra}}^2}{\tilde{X}_{\mathrm{Tra}}^2+\tilde{Y}_{\mathrm{Tra}}^2+\tilde{Z}_{\mathrm{Tra}}^2} \\
\tilde{\sigma}_{A\mathrm{Tra}}^2 &= \frac{\tilde{Z}_{\mathrm{Tra}}^2\tilde{\sigma}_{X\mathrm{Tra}}^2+\tilde{X}_{\mathrm{Tra}}^2\tilde{\sigma}_{Z\mathrm{Tra}}^2}{\sin^4\left(\tilde{Z}_{\mathrm{Tra}}/\tilde{X}_{\mathrm{Tra}}\right)\cdot\tilde{X}_{\mathrm{Tra}}^4} \\
\tilde{\sigma}_{E\mathrm{Tra}}^2 &= \frac{\tilde{X}_{\mathrm{Tra}}^2\tilde{Y}_{\mathrm{Tra}}^2\tilde{\sigma}_{X\mathrm{Tra}}^2+\left(\tilde{X}_{\mathrm{Tra}}^2+\tilde{Z}_{\mathrm{Tra}}^2\right)^2\tilde{\sigma}_{Y\mathrm{Tra}}^2+\tilde{Y}_{\mathrm{Tra}}^2\tilde{Z}_{\mathrm{Tra}}^2\tilde{\sigma}_{Z\mathrm{Tra}}^2}{\sin^4\left(\tilde{Y}_{\mathrm{Tra}}\Big/\sqrt{\tilde{X}_{\mathrm{Tra}}^2+\tilde{Z}_{\mathrm{Tra}}^2}\right)\left(\tilde{X}_{\mathrm{Tra}}^2+\tilde{Z}_{\mathrm{Tra}}^2\right)^3}
\end{aligned}
\tag{21}
$$

where $\tilde{\sigma}_{R\mathrm{Tra}}^2$, $\tilde{\sigma}_{A\mathrm{Tra}}^2$, $\tilde{\sigma}_{E\mathrm{Tra}}^2$ are the variances of tracking data $\tilde{R}_{\mathrm{Tra}}$, $\tilde{A}_{\mathrm{Tra}}$, $\tilde{E}_{\mathrm{Tra}}$, and $\tilde{\sigma}_{x\mathrm{Tra}}^2$, $\tilde{\sigma}_{y\mathrm{Tra}}^2$, $\tilde{\sigma}_{z\mathrm{Tra}}^2$ the variances of tracking data $\tilde{X}_{\mathrm{Tra}}$, $\tilde{Y}_{\mathrm{Tra}}$, $\tilde{Z}_{\mathrm{Tra}}$. In the process of calculating $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ in the substitute test by Eqn (12), the measurement precision of telemetric data and tracking data are propagated into $\tilde{\sigma}_R^2$, $\tilde{\sigma}_A^2$, $\tilde{\sigma}_E^2$ as:

$$
\begin{cases}
\tilde{\sigma}_R^2 = \tilde{\sigma}_{R\mathrm{Tel}}^2 + \tilde{\sigma}_{R\mathrm{Tra}}^2 \\
\tilde{\sigma}_A^2 = \tilde{\sigma}_{A\mathrm{Tel}}^2 + \tilde{\sigma}_{A\mathrm{Tra}}^2 \\
\tilde{\sigma}_E^2 = \tilde{\sigma}_{E\mathrm{Tel}}^2 + \tilde{\sigma}_{E\mathrm{Tra}}^2
\end{cases}
\tag{22}
$$

where $\tilde{\sigma}_{R\mathrm{Tel}}^2$, $\tilde{\sigma}_{A\mathrm{Tel}}^2$, $\tilde{\sigma}_{E\mathrm{Tel}}^2$ are the measurement variances of telemetering data $\tilde{\sigma}_R^2$, $\tilde{\sigma}_A^2$, $\tilde{\sigma}_E^2$ the variances of radar measurement *R, A, E*.

In the process of propagating $\tilde{\Delta}_R$, $\tilde{\Delta}_A$, $\tilde{\Delta}_E$ to $\tilde{\Delta}_x$, $\tilde{\Delta}_y$, $\tilde{\Delta}_z$ by Eqn (10), the relationship of their variances is:

$$
\begin{cases}
\tilde{\sigma}_x^2 = \left(\cos\tilde{E}\cos\tilde{A}\right)^2\tilde{\sigma}_R^2 + \left(-\tilde{R}\sin\tilde{E}\cos\tilde{A}\right)^2\tilde{\sigma}_E^2 + \left(-\tilde{R}\cos\tilde{E}\sin\tilde{A}\right)^2\tilde{\sigma}_A^2 \\
\tilde{\sigma}_y^2 = \left(\sin\tilde{E}\right)^2\tilde{\sigma}_R^2 + \left(\tilde{R}\cos\tilde{E}\right)^2\tilde{\sigma}_E^2 \\
\tilde{\sigma}_z^2 = \left(\cos\tilde{E}\sin\tilde{A}\right)^2\tilde{\sigma}_R^2 + \left(-\tilde{R}\sin\tilde{E}\sin\tilde{A}\right)^2\tilde{\sigma}_E^2 + \left(\tilde{R}\cos\tilde{E}\cos\tilde{A}\right)^2\tilde{\sigma}_A^2
\end{cases}
\tag{23}
$$

where $\tilde{\sigma}_x^2$, $\tilde{\sigma}_y^2$, $\tilde{\sigma}_z^2$ are the measurement variances of warhead-target relative positions *x, y, z* in the substitute test.

Similarly, in the overall operational test, their relationship is:

$$
\begin{cases}
\sigma_x^2 = (\cos E\cos A)^2\sigma_R^2 + (-R\sin E\cos A)^2\sigma_E^2 + (-R\cos E\sin A)^2\sigma_A^2 \\
\sigma_y^2 = (\sin E)^2\sigma_R^2 + (R\cos E)^2\sigma_E^2 \\
\sigma_z^2 = (\cos E\sin A)^2\sigma_R^2 + (-R\sin E\sin A)^2\sigma_E^2 + (R\cos E\cos A)^2\sigma_A^2
\end{cases}
\tag{24}
$$

where σ_x^2, σ_y^2, σ_z^2 are the measurement variances of warhead-target relative positions *x, y, z* in the overall operational test and σ_R^2, σ_A^2, σ_E^2 the variances of radar measurement *R, A, E* in overall operational test.

Because the variances of radar measurement *R, A, E* are equivalent in different tests, one has:

$$\sigma_R^2 = \tilde{\sigma}_R^2,\ \ \sigma_A^2 = \tilde{\sigma}_A^2,\ \ \sigma_E^2 = \tilde{\sigma}_E^2$$

Based on the law of error propagation and Eqn (23),(24) one can describe the relationship of the variances as:

$$
\begin{cases}
\sigma_{LX}^2 = \tilde{\sigma}_{LX}^2 + \sigma_x^2 + \tilde{\sigma}_x^2 \\
\sigma_{LY}^2 = \tilde{\sigma}_{LY}^2 + \sigma_y^2 + \tilde{\sigma}_y^2 \\
\sigma_{LZ}^2 = \tilde{\sigma}_{LZ}^2 + \sigma_z^2 + \tilde{\sigma}_z^2
\end{cases}
\tag{25}
$$

where $\tilde{\sigma}_{LX}^2$, $\tilde{\sigma}_{LY}^2$, $\tilde{\sigma}_{LZ}^2$ are the variances of the impact dispersion in the substitute test.

Hereto, we get σ_{LX}^2, σ_{LY}^2, σ_{LZ}^2 the variances of impact dispersion in the overall operational test.

6. ANALYSIS OF SIMULATION AND RESULTS

With the above method, results of impact dispersion in the substitute test can be converted to that in the overall operational test. The following simulation was designed to demonstrate feasibility of the conversion process and reliability of the conversion results.

6.1 Design of the Demonstration Test

In the substitute test, different targets and environments were set in the range to conduct the operational test. Measurement data, such as parameters of environment and target in the response function of radar measurement precision, were then collected and put into the conversion model to convert the impact dispersion between different tests. The demonstration process was finished after comparing the conversion results with measurements in the substitute test.

The demonstration method is as follows:

- Set two different targets in the same environment for two substitute tests, convert the measurement in one test into the other, and then compare the conversion results with the measurement in the previous substitute test;
- Set one target in two different environments for two substitute tests. The demonstration process is similar to the above one;
- Set two different targets in two different environments for two substitute tests, convert mutually the measurement in one test to the other, and then compare the conversion results with the measurement in each substitute test.

6.2 Simulation of the Conversion Process

To demonstrate the proposed method for converting impact dispersion, the following conversion process was designed (see Fig. 3):

To meet the simulation requirement, relevant parameters of the active homing radar were set[8,9] as follows:

- Half power width of antenna beam: $\theta_B = 1.5^{\circ}$;
- Error slope of monopulse antenna pattern: $k_m = 1.57$;
- Signal bandwidth $B\times$ pulse width τ. $B\tau = 1$;
- Repetition rate of radar: $f_r = 10\text{KHz}$;
- Servo bandwidth: $\beta_n = 7.958\text{Hz}$;
- Independent hits of signal-to-clutter ratio in invariableness tracking time: $n_c = 100$;
- Mean square root bandwidth of radar signal: $\beta = 1\text{MHz}$;
- Velocity error constant: $K_v = \alpha$, acceleration error constant: $K_\alpha = 486$.

Based on the impact dispersion of some representative ground-to-ground tactical missiles like the US ATACMS-2A , Russia's SS-21 and Tender, the impact dispersion in the substitute test was set:

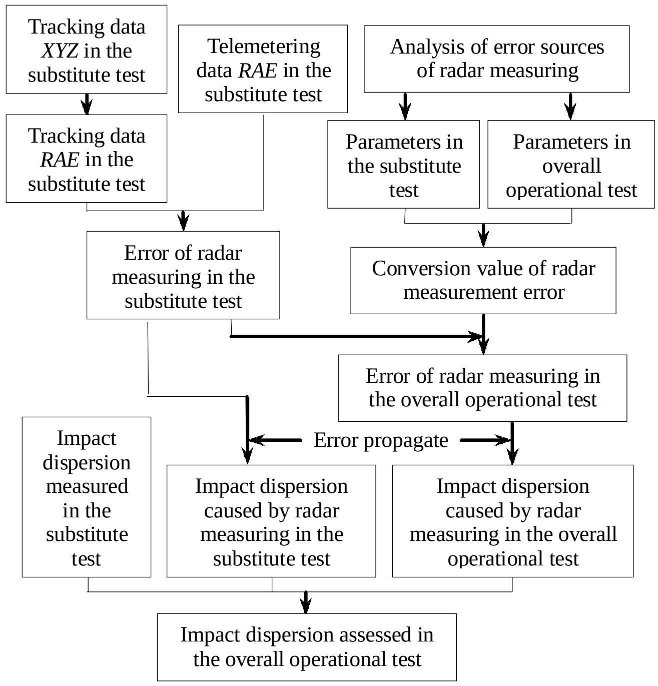

Figure 3. Flow chart of impact dispersion conversion.

$\Delta\tilde{L}_X = 50$ m , $\Delta\tilde{L}_Z = 70$ m

The measurement precision of telemetering data:

$\tilde{\sigma}_{R\mathrm{Tel}} = 2$ m , $\tilde{\sigma}_{A\mathrm{Tel}} = 0.0005$ rad , $\tilde{\sigma}_{E\mathrm{Tel}} = 0.0005$ rad

The measurement precision of tracking data:

$\tilde{\sigma}_{x\mathrm{Tra}} = 3$ m , $\tilde{\sigma}_{y\mathrm{Tra}} = 3$ m , $\tilde{\sigma}_{z\mathrm{Tra}} = 3$ m

Simulation times: 100.

6.3 Simulation Result and Analysis

Through different selections of such parameters as target, environment, and radar in the substitute test and the overall operational test, one can get the following simulation results:

ΔL_X 51.9826 m, ΔL_Z 71.9136 m

σ_{LX} 2.4561 m, σ_{LZ} 3.4441 m

Because the measurement errors are modelled as zero-mean normal distribution, the impact dispersions in the overall operational test must be modelled as normal distribution too. Then, one can get the confidence interval of the simulation result when the confidence is 0.95:

Direction X: [51.5012 m, 52.4640 m]

Direction Z: [71.2386 m, 72.5886 m]

Moreover, if the parameters of target and environment as well as other factors influencing the impact dispersion are kept invariable, by reconverting the results to the substitute test, one can get:

$\Delta\tilde{L}'_X = 50.0049 \text{ m} \approx \Delta\tilde{L}_X$, $\Delta\tilde{L}'_Z = 70.0026 \text{ m} \approx \Delta\tilde{L}_Z$

σ_{LX} 2.8399 m, σ_{LZ} 3.8371 m

Similarly, the confidence interval of the results when the confidence is 0.95 is:

Direction X: [49.4483 m, 50.5616 m].

Direction Z: [69.2506 m, 70.7547 m].

The following conclusions can thus be drawn from the above theoretical analysis and simulation:

(1) Because there are differences of targets, environments and ranges in the substitute test and the overall operational test, the radar measurement precisions in the two tests present differences;

(2) The conversion value of impact dispersion can be propagated to the conversion value of radar measurement precision by decomposing and propagating the impact dispersion of missiles;

(3) Each of the radar measurement error sources imposes influence on the conversion results. The larger a conversion value of the radar measurement precision is, the greater is the difference of the conversion values of the impact dispersion between the two tests; and

(4) Simulation results show that the impact dispersion in the two tests can be converted between each other, which fully demonstrate feasibility of the proposed method in this paper.

7. CONCLUSIONS

Based on the laws of error synthesis and error propagation, a conversion method of impact dispersion for missiles with homing radar in substitute equivalent tests is proposed, solving the problem of impact dispersion conversion between different tests for cluster warhead missiles with homing radar. Main factors influencing differences between impact dispersion in different tests were analysed, and impact dispersion was decomposed and propagated to error sources of radar measurement. Based on the model of the error sources and radar measurements in the substitute tests, the conversion values of error sources were synthesised and propagated into the impact dispersion assessed in the overall operational test.

The conversion method is significant for converting the impact dispersion of missiles between different tests. With the feasibility and effectiveness being proved, this method has been applied to dispersion assessment of operational test for missiles with homing radar. While calculating the conversion values between different tests, it's critical to select accurately the various involved parameters of radar such as the half power width of antenna beam, the error slope of monopulse antenna pattern, the signal bandwidth, the pulse width, the repetition rate of radar, the servo bandwidth, the acceleration error constant, etc. In view of this, one needs to collect more data and improve the method in future tests. In addition, to further demonstrate feasibility and effectiveness of this method, we should design more substitute tests in which more parameters in the conversion of the impact dispersion between different tests can be measured.

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