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# Angular Orientation of Anti-Aircraft Gun for Interception of a Moving Air Target

Deepak Kumar[1] and R.N. Mishra[2]

[1]*Directorate of Standardisation, Ministry of Defence, New Delhi - 110 011*
[2]*Indian Institute of Technology, Roorkee -247 667*

**ABSTRACT**

The paper considers the problem of angular orientation of anti-aircraft guns towards a moving air target. The factors affecting the collision of the projectile fired from a gun with the moving air target are highlighted. Thereafter, a mathematical model has been developed to estimate the angular orientation of the anti-aircraft gun in terms of bearing and elevation, in the direction of the predicted future position of the moving air target, to enable collision of the projectile with the target.

**Keywords:** Gun laying equations, angular orientation, anti-aircraft guns, future position, moving air target

## NOMENCLATURE

| | |
|---|---|
| $X_t, Y_t, Z_t$ | Component of target's initial position, along X,Y and Z axes |
| $V_{tx}, V_{ty}, V_{tz}$ | Component of target's initial velocity, long X, Y and Z axes |
| $A_{tx}, A_{ty}, A_{tz}$ | Component of target's initial acceleration, along X, Y and Z axes |
| $X_p, Y_p, Z_p$ | Component of projectile's initial position, along X, Y and Z axes |
| $V_{px}, V_{py}, V_{pz}$ | Component of projectile's initial velocity, along X, Y and Z axes |
| $D_{pavx}, D_{pavy}, D_{pavz}$ | Component of projectile's average deceleration, along X, Y and Z axes |
| $D_{pav}$ | Projectile's average deceleration |
| $V_{pav}$ | Projectile's average velocity |
| K | Projectile's initial velocity (1000 m/s) |
| C | Deceleration Constant |
| g | Acceleration due to gravity (9.8 m/s$^2$) |
| $b_p$ | Bearing at which the projectile is fired from the gun |
| $e_p$ | Elevation at which the projectile is fired from the gun |
| t | Time of flight of projectile till collision with target |

## 1. INTRODUCTION

Present day war has become multifaceted with technologically advanced weapon systems being used on land, in air, and at sea. Although ground forces play the most decisive role, the enemy aircrafts and missiles can limit the efficacy of ground forces to an extent that may change the course of a battle. To provide immunity against enemy air attacks, air defence systems comprising anti-aircraft guns and surface- to- air missiles form an integral part of any ground force. Amongst the two, anti-aircraft guns are a more economical option and are invariably used. However, angular alignment of a gun towards a moving air target needs some deliberation.

Consider a target in the three dimensional air space moving with some velocity being fired upon by an anti-aircraft gun located on ground. If the angular orientation of the gun at the time of firing the projectile is aligned in the direction of an imaginary line joining the gun and the position of the target at that instant, then by the time the projectile traverses the distance between the gun and the target, the target itself would have moved some distance away, and thereby, the projectile will miss the target entirely. To ensure collision of the projectile fired by the gun with a moving air target, the angular orientation of the gun has to be aligned in the direction of the future position of the moving air target, taking into account all factors that affect their collision[1].

A variety of configurations are possible for such weapon systems. The air target tracking device and the anti-aircraft guns may be co-located on the same platform or on two different platforms which may themselves be static or mobile.

## 2. FACTORS CONTRIBUTING TO THE IMPACT

A multitude of factors affect the collision between the projectiles fired from the anti-aircraft gun and a moving air target[4,6,8]. All these factors need to be accounted for in the mathematical model to estimate the angular orientation of gun in the direction of the future position of the moving

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air target. These factors are as below:

(a) *Target*:
   (i) Initial position of target along X, Y and Z axes of the stabilised reference frame.
   (ii) Initial velocity of target along X, Y and Z axes of the stabilised reference frame.
   (iii) Initial acceleration of target along X, Y and Z axes of the stabilised reference frame.

(b) *Projectile*:
   (i) Initial position of projectile along X, Y and Z axes of the stabilized reference frame
   (ii) Initial velocity of projectile along X, Y and Z axes of the stabilised reference frame

(c) *Environment*:
   (i) Gravitational acceleration
   (ii) Deceleration due to viscous friction with the atmosphere
   (iii) Wind drift

(d) *System Errors*:
   (i) Computation time
   (ii) Steady state error and settling time of control loops
   (iii) Ballistic drift

## 3. FRAME OF REFERENCE AND ANGULAR REFERENCE

Before evolving the mathematical model, one defines the stabilised reference frame and the angular references for measurement of the variables involved[2-4].

In the three dimensional space, a reference frame is required as datum wrt which the angular measurements of the variables are made, i.e., bearing and elevation of the target as well as the angular orientation of anti aircraft gun. The reference frame will have three mutually perpendicular axes, named arbitrarily as X, Y and Z. Let the XY plane of this frame be always perpendicular to the direction of the gravitational force and parallel to perfectly horizontal surface of the earth. The XZ and the YZ planes are perpendicular to XY plane as well as to each other. However, the reference frame has no particular orientation wrt the Earth's north. For a platform placed on the ground with radar mounted on it, the X, Y and Z axes of the reference frame aligned themselves along the longitudinal, lateral, and vertical axes of the platform, and the centre of gravity of platform is coincident with the origin, as shown in Fig. 1. Such an imaginary reference frame, as described here, is the stabilised reference frame.

The angular references are required for quantifying the angular measurements of the variables, i.e., bearing and elevation, in the stabilised reference frame under consideration. Bearing is measured in XY plane positively from +X towards +Y axis and elevation is measured positively from XY plane towards +Z axis.

## 4. SYSTEM MODEL

To evolve the generalised mathematical model, consider a twin platform system with the target- tracking device,

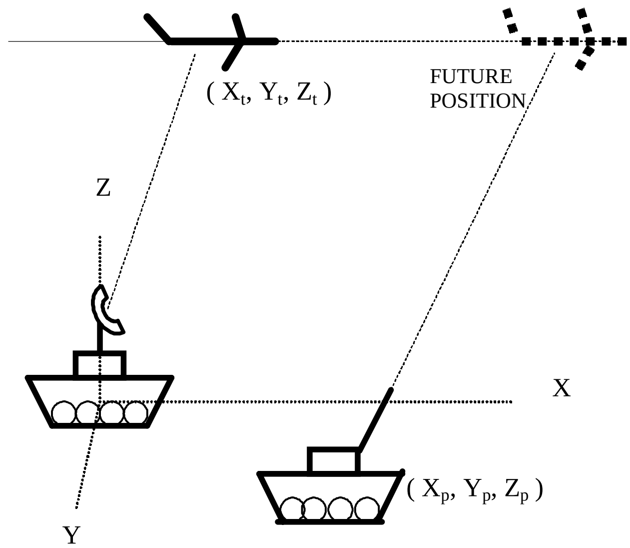


**Figure 1. Two platform system.**

such as the tracking radar on one platform and the anti-aircraft gun on another platform, and also the stabilised reference frame aligned with the platform mounted with the target-tracking device as shown in Fig. 1. The target will be at a certain elevation, bearing, and range wrt the target-tracking platform at different instances of time. Depending on the target's velocity, the target may approach the platform (approacher), fly past the platform (crosser) or recede away from the platform (receder). This three-dimensional scenario may be viewed in the horizontal yaw plane i.e. left / right disregarding the elevation aspect, and also in the vertical pitch plane, i.e., up / down, disregarding the bearing aspect. The target, in either of the two planes, can be depicted as in Fig. 2.

To prevent the target, i.e., enemy aircraft, from causing damage in the area where the anti-aircraft gun is located on the ground, the target has to be hit by the projectile while it is an approacher. Once the target- tracking device acquires the initial target data, i.e., bearing, elevation, and range, the targets initial position, velocity, and acceleration along the three axes are computed and it is assumed that the target continues to fly with the same velocity and acceleration till the time the projectile intercepts it. To enable collision of the projectiles fired from the anti-aircraft gun with the moving air target, the equations of their motions along the X, Y and Z axes may be equated similarly as for the estimation of missile launch angle[4-6] along with two additional terms pertaining to main external influences of gravity and deceleration due to air[8], as below:

$$X_t + V_{tx}.t + ½A_{tx}.t^2 = X_p + V_{px}.t - ½D_{pavx}.t^2 \quad (1)$$

$$Y_t + V_{ty}.t + ½A_{ty}.t^2 = Y_p + V_{py}.t - ½D_{pavy}.t^2 \quad (2)$$

$$Z_t + V_{tz}.t + ½A_{tz}.t^2 = Z_p + V_{pz}.t - ½D_{pavz}.t^2 - ½g.t^2 \quad (3)$$

Substituting values of $V_{px}$, $V_{py}$, $V_{pz}$, $D_{pavx}$, $D_{pavy}$ & $D_{pavz}$ in Eqns (1-3),

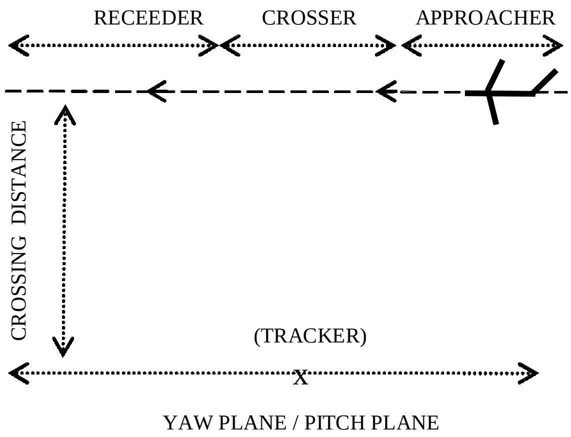


**Figure 2. Approacher/crosser/receder target in yaw/pitch plane.**

$$X_t + V_{tx}.t + ½A_{tx}.t^2 = X_p + KCos\ e_p\ Cos\ b_p.t - ½D_{pav}Cos\ e_p\ Cos\ b_p.t^2 \quad (4)$$

$$Y_t + V_{ty}.t + ½A_{ty}.t^2 = Y_p + KCos\ e_p\ Sin\ b_p.t - ½D_{pav}Cos\ e_p\ Sin\ b_p.t^2 \quad (5)$$

$$Z_t + V_{tz}.t + ½A_{tz}.t^2 = Z_p + K\ Sin\ e_p.t - ½D_{pav}Sin\ e_p.t^2 - ½g.t^2 \quad (6)$$

Rewriting eqn's (4-6),

$$X_t - X_p + V_{tx}.t + ½A_{tx}.t^2 = (K.t - ½D_{pav}.t^2)Cos\ e_p\ Cos\ b_p \quad (7)$$

$$Y_t - Y_p + V_{ty}.t + ½A_{ty}.t^2 = (K.t - ½D_{pav}.t^2)\ Cos\ e_p\ Sin\ b_p \quad (8)$$

$$Z_t - Z_p + V_{tz}.t + ½A_{tz}.t^2 + ½g.t^2 = (K.t - ½D_{pav}.t^2)\ Sin\ e_p \quad (9)$$

Squaring Eqns (7) and (8) and adding them,

$$(X_t - X_p + V_{tx}.t + ½A_{tx}.t^2)^2 + (Y_t - Y_p + V_{ty}.t + ½A_{ty}.t^2)^2 = (K.t - ½D_{pav}.t^2)^2.Cos^2\ e_p \quad (10)$$

Squaring eqn (9),

$$(Z_t - Z_p + V_{tz}.t + ½(A_{tz}+g).t^2)^2 = (K.t - ½D_{pav}.t^2)^2.Sin^2\ e_p \quad (11)$$

Adding eqn (10) and (11),

$$(X_t - X_p + V_{tx}.t + ½A_{tx}.t^2)^2 + (Y_t - Y_p + V_{ty}.t + ½A_{ty}.t^2)^2 + (Z_t - Z_p + V_{tz}.t + ½(A_{tz} + g).t^2)^2 = (K.t - ½D_{pav}.t^2)^2 \quad (12)$$

Expanding Eqn (12) and collecting terms with same power of variable time t,

$$¼\ [A_{tx}^2 + A_{ty}^2 + (A_{tz} + g)^2 - D_{pav}^2]\ t^4 + [A_{tx}.V_{tx} + A_{ty}.V_{ty} + (A_{tz}+g)V_{tz} + K.D_{pav}]t^3 + [(X_t - X_p)A_{tx} + V_{tx}^2 + (Y_t-Y_p)A_{ty} + V_{ty}^2 + (Z_t-Z_p)\ (A_{tz}+g) + V_{tz}^2 - K^2]t^2 + 2[(X_t-X_p)V_{tx} + (Y_t-Y_p)V_{ty} + (Z_t-Z_p)\ V_{tz}]\ t + (X_t-X_p)^2 + (Y_t-Y_p)^2 + (Z_t-Z_p)^2 = 0 \quad (13)$$

The Eqn. (13) above is a fourth-order equation for the variable time $t$ containing terms for $t^4$, $t^3$, $t^2$, $t$ and a constant. The formulae for finding roots of second- third- fourth-order equations are available; above which one must resort to the numerical method. The formula for solving the quadratic is easily applied, but the cubic solution is rather long and for quartic, it is very complicated. It turns out that one usually uses a numerical method for all the equations above the quadratic order[7]. The Lin's method and the Newton's method are two well documented methods[5,7]. Although either of these two methods or any similar method will give the roots, however for the model under consideration, a better way of finding the value of the variable time $t$ would be as under since it may so happen that more than one positive real root may exist for a particular case.

It may be appreciated that the value of the variable time $t$ cannot be imaginary or complex and can have only a positive real value. Further, ones interest lies in finding only that one particular root amongst the possible four that would result in correct estimation of the angular orientation of the gun which will enable collision of the projectile with the target. This value of $t$ when substituted in the left side of the Eqn. (13) will equate with the right side of the Eqn, i.e., zero.

To determine the value of the variable time $t$ which is of interest, consider the target to be stationary at point $A$ in the target / projectile fly plane as shown in Fig. 3. In the three-dimensional stabilised reference frame, the plane containing the straightline trajectory of the target, which also passes through the initial position of the projectile on the ground, is the target / projectile fly plane[4]. For a target initially acquired at point $A$ at a distance $d_1$, the corresponding value of time $t_1$ for flight of the projectile to the target is the most obvious reference in whose vicinity the actual value of the variable time $t$ lies. The initial distance of the target from the initial position of the projectile is[4]:

$$d_1 = [(X_t-X_p)^2 + (Y_t-Y_p)^2 + (Z_t-Z_p)^2]^{1/2} \quad (14)$$

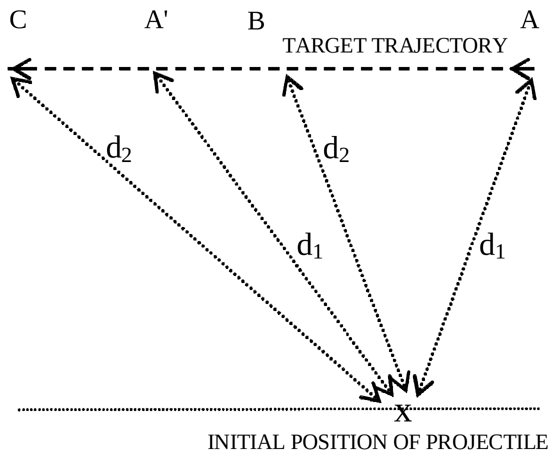


**Figure 3. Target/projectile fly plane.**

If the deceleration of the projectile is disregarded due to viscous friction of the atmosphere as well as the effect of gravity, the time of flight of the projectile will be $t_1 = d_1/K$ as in the case of missile which fly at a constant velocity[4]. In reality, as the projectile moves towards the target it is subjected to viscous friction with the atmosphere due to which its velocity keeps reducing continuously and is accounted for by taking into consideration the average deceleration $D_{pav}$. The deceleration due to gravity may be accounted for by resolving it with cosine of the elevation $e_1$ of the stationary target initially located at point *A*, as shown in Fig. 4. The time of flight $t_1$ may be estimated from its equation of motion as under:

$$d_1 = K.t_1 - \frac{1}{2}D_{pav}.t_1^2 - \frac{1}{2}\,g.\cos(90^o - e_1).t_1^2 \qquad (15)$$

If we neglect the term pertaining to gravity since its value is much smaller than the initial velocity of the projectile *K*, which is approximately 1000 m/s for most anti- aircraft guns today, as well as average deceleration $D_{pav}$, whose value lies between 285 and 140 as will be shown later, the Eqn.(15) reduces to:

$$D_{pav}.t_1^2 - 2K.t_1 + 2d_1 = 0 \qquad (16)$$

The two roots of the quadratic Eqn(16) above are

$$t_1(\text{root 1}) = [K - (K^2 - 2.D_{pav}.d_1)^{1/2}]/D_{pav} \qquad (17)$$

$$t_1(\text{root 2}) = [K + (K^2 - 2.D_{pav}.d_1)^{1/2}]/D_{pav} \qquad (18)$$

To ascertain which one of these roots gives the correct value of the time of flight $t_1$ of the projectile, the values of the two roots tabulated in Table 1 were examined for various values of $d_1$ along with its associated value of average deceleration $D_{pav}$ calculated using Eqn (34).

All the values of the second root corresponding to various distances $d_1$ varying from 500 m to 2500 m fall within 7.1129 s and 8.6601 s, which is obviously not possible, and therefore ruled out. The values of the first root seem more likely to be correct considering the fact that the initial velocity *K* of the projectile is 1000 m/s. Also, even if the values of the second root for various distances were reasonably different, it would be appropriate to consider the first root, being lesser of the two roots, and therefore, representing the time for the shortest flight path of the projectile to the target. Further, numerical evaluation of Eqn (15) for various values of the variables $d_1$ and $e_1$ will reveal that taking into consideration the effect of gravity cause a variation of less than 0.015 s in the value of time of flight $t_1$. During this time of 0.015 s, a target travelling at a maximum velocity of 400 m/s will move only 6 m, which is inconsequential considering the size of the smallest target aircraft which may be 20 m long, 25 m wide, and 3 m tall. Thus the estimated value of $t_1$ using Eqn (17) by disregarding effect of gravity is sufficiently accurate to correctly determine the direction of search for the correct value of time *t* that will satisfy Eqn (13).

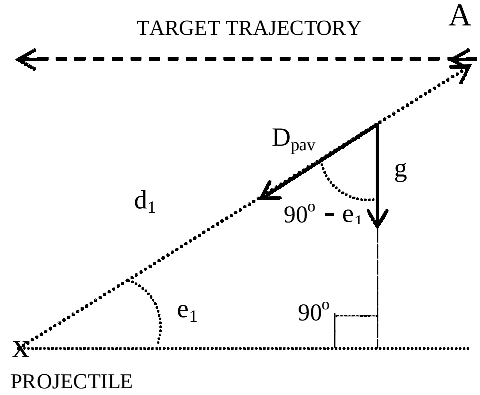


**Figure 4. Accounting gravity in Eqn (15).**

Having calculated the value of time $t_1$ for flight of projectile to the stationary target, let the target now continue to fly. The value that will satisfy Eqn (13) will not be exactly $t_1$ but in its vicinity, either less or more than $t_1$, depending on whether the target is approaching or receding[4]. To determine in which direction the value will lie, consider now the position of the moving target after $t_1$ seconds. The distance will be

$$d_2 = [(X_t + V_{tx}.t_1 + \frac{1}{2}A_{tx}.t_1^2 - X_p)^2 + (Y_t + V_{ty}.t_1 + \frac{1}{2}A_{ty}.t_1^2 - Y_p)^2 + (Z_t + V_{tz}.t_1 + \frac{1}{2}A_{tz}.t_1^2 - Z_p)^2]^{1/2} \qquad (19)$$

If $d_2 > d_1$, the target is a receding one, and consequently the value of variable time *t* that will satisfy Eqn (13) will be greater than $t_1$.. If $d_2 < d_1$, the target is an approaching one and consequently the value of variable time *t* that will satisfy Eqn (13) will be < $t_1$.

To justify the use of variables $t_1$, $d_1$ and $d_2$ in choosing the direction of search let *A*' be a point ahead on the target trajectory which is at the same distance $d_1$ and for which the projectile would take the same time $t_1$ to reach.

Allow the projectile travel from its initial position to the point *A*' and simultaneously the target from point *A*

**Table 1. Evaluating values of two roots of Eqn (16)**

| Variables | Values | | | | |
|---|---|---|---|---|---|
| $d_1$ (meters) | 500 | 1000 | 1500 | 2000 | 2500 |
| $D_{pav} = 285.7 - (0.04857) . d_1$ | 261.415 | 237.100 | 212.845 | 188.560 | 164.275 |
| $t_1$ (root 1) = $[K - (K^2 - 2 . D_{pav} . d_1)^{1/2}] / D_{pav}$ (sec) | 0.5378 | 1.1594 | 1.8736 | 2.6743 | 3.5146 |
| $t_1$ (root 2) = $[K + (K^2 - 2 . D_{pav} . d_1)^{1/2}] / D_{pav}$ (sec) | 7.1129 | 7.2748 | 7.5229 | 7.9324 | 8.6601 |

towards point *A*'. By the time the projectile reaches point *A*', if the target reaches any point *B* which is short of *A*' then the distance $d_2$ will be < $d_1$. It can thus be inferred that if the projectile were to be launched in the correct direction the time taken to intercept the target will be < $t_1$.

Similarly, if the target were to reach some point *C* ahead of point *A*', then the distance $d_2$ will be > $d_1$ and the time taken by the projectile to intercept the target will be > $t_1$.

Having ascertained the direction in which the value of variable time *t* lies, the expression in Eqn (13) is calculated successively starting with value of $t = t_1$ and thereafter, increasing or decreasing it, as determined earlier, in steps of say 0.01 or 0.025 or 0.1, as per the requirement of accuracy. As these calculations are done successively, the value of the expression will approach zero. Once that value of the variable time *t* has been reached for which the value of the expression on the left side of Eqn (13) crosses the zero or is within certain $\pm$ limit of zero, the value of variable time *t* arrived at will be the one that will nearly satisfy Eqn (13). This value of the variable time t so arrived at by time-convergence of Eqn (13) is that pertaining to the final distance $d_f$ of the impact point *D*. The final distance of the target from the initial position of the projectile[4] will be

$$d_f = [(X_t+V_{tx}.t+½A_{tx}.t^2–X_p)^2 + (Y_t+V_{ty}.t+½A_{ty}.t^2–Y_p)^2 + (Z_t+V_{tz}.t+½A_{tz}.t^2–Z_p)^2]^{1/2} \qquad (20)$$

Mathematically, one may be tempted to find the value of variable time t where the slope of Eqn (13) will change by taking the derivative of Eqn (13) and finding the value of the variable time *t* that would have satisfied Eqn (13). However, this may not render the value of variable time *t* for two reasons. Firstly, taking derivative of Eqn (13) will give a third-order equation with three roots and where once again a numerical method will have to be applied. Secondly, the nature of Eqn (13) will depend on its constants and may be such that a change of slope may not exist[4].

However, before implementing the above time-convergence procedure, another aspect regarding the value of the variable average deceleration $D_{pav}$ needs to be considered on which depends the accuracy of the result. More about the variable average deceleration $D_{pav}$ is brought out later including experimental determination as well as approximate estimation for validation of this model by numerical simulation.

While searching for the value of the variable time *t* to satisfy Eqn (13), the most obvious and easily available value $t_1$ was used to start with, pertaining to the initial distance $d_1$ of the target. It would thus be logical to use that initial value of $D_{pav}$ which pertains to the distance $d_1$. However the final distance $d_f$ will invariably be different from $d_1$ for a moving target and the initial assumption of the value of $D_{pav}$ pertaining to the distance $d_1$ will be incorrect. Also, as will be brought out later, greater the distance, lesser will be the value of the variable average deceleration $D_{pav}$. Therefore, if the value of $d_f$ turns out to be greater than $d_1$ then the correct value of $D_{pav}$ would be less than that selected initially. Similarly, if the value of $d_f$ turns out to be less than $d_1$ then the correct value of $D_{pav}$ would be greater than that selected initially. Accordingly the value of $D_{pav}$ is increased or decreased in steps of say 1.0 or 0.5 or 0.1, as per the requirement of accuracy, and the time-convergence procedure is repeated. This procedure of average deceleration convergence, with the time-convergence nested inside, is repeated till the last selected value of $D_{pav}$ in the above convergence procedure is nearly the same as the corresponding value of average deceleration $D_{pavf}$ pertaining to final distance $d_f$.

The method described above may perhaps be the most suitable way of finding the solution of Eqn (13). Having found the desired value of variable time *t*, the same is substituted in Eqn (9) to get the value of variable elevation $e_p$. Further, by substituting the value of the variable time t and elevation $e_p$ in either Eqn (7) or Eqn (8) will give the value of the variable bearing $b_p$.

The Eqn's (7), (8), (9) and (13) are the gun-laying equations for anti-aircraft guns whose problem-specific numerical solution is as elucidated above.

## 5. DETERMINATION OF SYSTEM CONSTANTS

As the projectile travels from the gun towards the target, it experiences deceleration due to the drag force exerted on it by virtue of friction with the atmospheric medium. For small projectiles traveling at low subsonic speeds, where there is laminar flow of air, drag force is directly proportional to the instantaneous velocity as well as coefficient of drag[9] of the projectile. However, for large projectiles traveling at high supersonic velocities, where there is turbulent flow of air, drag force is directly proportional to one-half of coefficient of drag, density of air, cross-sectional area of the projectile perpendicular to flow of air and square of the projectiles instantaneous velocity (or even higher powers of velocity depending on the speed of the projectile)[9]. Although literature mentions this fact but does not specify the velocity at which the relationship changes. Further, the value of coefficient of drag, which is determined experimentally using Doppler radar[10], is not constant and depends on projectiles velocity, viscosity of air, projectile shape as well as roughness of projectiles surface and thus only an average value is used. This drag force when divided by the mass of the projectile gives the deceleration experienced by the projectile[10]. Mathematical models such as Siacci/Mayevski G1 model and the Pejsa model for calculating effects of air resistance are quite complex and the most reliable method of establishing trajectories (Ballistic Tables) is still by empirical measurement[10]. Irrespective of the changing values of coefficient of drag and power of instantaneous velocity to which the deceleration of the projectile is proportional, what can be said with certainty is that the velocity of the projectile keeps reducing continuously, which in turn reduces the deceleration experienced by the projectile. It is thus evident that greater the distance, lesser will be the average velocity and thus the average deceleration as well.

Since the velocity of the projectile is continuously reducing, the best value that can be chosen over a certain distance traveled by the projectile is the average velocity which will render the average deceleration of the projectile over the distance under consideration. Using the most common expression of drag force in which it is proportional to square of instantaneous velocity and representing one-half of coefficient of drag, density of air, cross sectional area of the projectile, and inverse of mass of the projectile by a single constant C which may be called the deceleration constant[9], the average deceleration may be represented mathematically as

$$D_{pav} = C.V_{pav}^{\ 2} \tag{21}$$

To estimate the average deceleration one would instinctively tend to mathematically integrate instantaneous deceleration wrt to time. However, neither deceleration nor the velocity to which deceleration is proportional, can be represented as a function of time since these are interdependent[11]. This aspect, apart from the fact that the velocity at which its relationship changes with deceleration not being known, leads one to infer that experimental determination of average deceleration will yield more realistic results.

The average deceleration of the projectile over a certain distance can be determined by conducting an experiment in the following way. A single projectile is fired from a gun kept at a height $h$ from the ground in a horizontal direction, parallel to the surface of earth as shown in Fig. 5. Thus one can write the equation for its motion along the $Z$ axis as

$$-h = V_{pz}.t - ½g.t^2 - D_{pav}.t^2 \tag{22}$$

If one neglects the viscous friction effect, since the velocity achieved by the projectile along $Z$ axis will not be high for height equal to say up to 10 m or so, and also

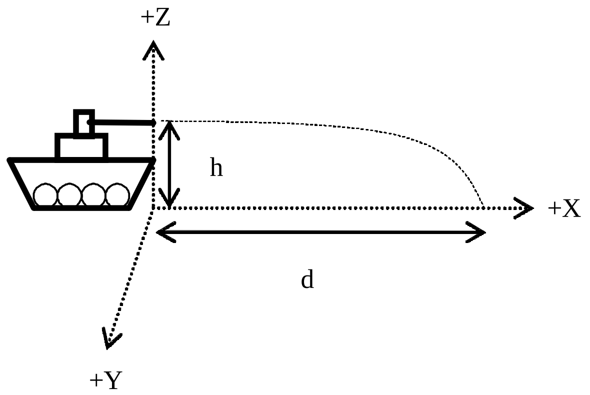


**Figure 5. Experimental determination of deceleration due to viscous friction of the atmosphere.**

since the initial velocity of the projectile along the $Z$ axis is zero, the equation reduces to

$$h = ½g.t^{\ 2} \tag{23}$$

and therefore

$$t = (2h/g)^{1/2} \tag{24}$$

To get a still more accurate value of time $t$, the projectile may simply be dropped from various heights and time $t$ elapsed till the projectile hits the ground be recorded using an accurate time measurement device such as gun chronograph.

Let the projectile travel the distance $d$ before it hits the ground, which can be measured physically. Also, the initial velocity of the projectile along the $X$ axis, i.e. $K$, is known from the gun's muzzle velocity measuring device or from the manufacturer's data sheet. We may thus write the equation for the motion of the projectile along the $X$ axis as

$$d = V_{px}.t - ½D_{pav}.t^2 \tag{25}$$

Substituting value of time $t$ and rearranging,one gets

$$D_{pav} = [K.(2h/g)^{1/2} - d]/(h/g) \tag{26}$$

This experiment is repeated from different heights from which a graph/table can be drawn between the average deceleration of the projectile and the distance.

However, for validation of the model by numerical simulation, one may approximately estimate the value of average deceleration $D_{pav}$ for various distances as described hereafter. Based on experience of Army's Air Defence Regiments, it has been established that attacking enemy aircrafts flying in at a low altitude, ranging between 300 m and 800 m, to avoid radar detection and at speeds no more than 1.2 Mach (Mach 1 is approximately 331 m/s, the speed of sound) to enable precision delivery of conventional explosives. Also, most anti-aircraft guns in use today have maximum muzzle velocity of 1000 m/s and maximum effective range of approximately 3000 m. At this maximum effective range of 3000 m, for the worst case of a receding target at 400 m/s (1.2 Mach), the velocity of the projectile should be at least 400 m/s for it to be able to hit the target. Thus the average velocity of the projectile over this distance of 3000 m would be one-half of summation of the initial and final velocities, i.e., [1000 m/s + 400 m/s] / 2 = 700 m/s and the time of flight of the projectile would be ratio of distance and the average velocity i.e. (3000 m) / (700 m/s) = 4.285 s. From the laws of motion one may write:

$$\text{Distance} = (\text{Initial Velocity x Time}) - ½\,(\text{Average deceleration x Time}^2) \tag{27}$$

Substituting the values,

$$3000 = (1000 \text{ x } 4.285) - ½\,(D_{pav})\,(4.285)^{\,2} \tag{28}$$

and therefore,

$D_{pav}$ = 140. (29)

Thus, a point has been established on the distance Vs average deceleration graph. Substituting the value of $D_{pav}$ as 140 and the value of average velocity as 700 m/s pertaining to the distance 3000 m in Eqn (21), one gets the value of the deceleration constant *C* as

$C = D_{pav} / V_{pav}^2 = 140 / 700^2 = 0.00028571$ (30)

It is also known that the velocity of the projectile at the muzzle of the gun is 1000 m/s when the distance traveled is zero. When substituted in Eqn (21) along with the value of the deceleration constant *C*, as calculated above, the value of $D_{pav}$ is known which pertains to the value of distance as zero.

$D_{pav} = V_{pav}^2 . C = 1000^2 \text{ x } 0.00028571 = 285.71$ (31)

It has thus been established that another point on the distance $V_s$ average deceleration graph. Assuming that the variation in the average velocity of the projectile over various distances is linear, we can now draw a graph by joining a straight line between the two points established above, depicting the relationship between distance and average deceleration as shown in Fig. 6. However, in the real world the variations in average velocity over various distances will not be linear and hence the experimentally determined table/graph of average deceleration over various distances will also not be linear.

From the two points that we have established above and the straight line joining them which depicts the relationship between distance and average deceleration, the value of $D_{pav}$ at any distance *d* can be calculated as under:

$D_{pav} = D_{pav}(\text{initial}) +$

$[(D_{pav}(\text{final}) - D_{pav}(\text{initial}))/(d(\text{final}) - d(\text{initial}))].d$ (32)

Substituting the values, one gets,

$D_{pav} = 285.71 + [(140 - 285.71) / (3000 - 0)] . d$ (33)

and therefore,

$D_{pav} = 285.71 - (0.04857) . d$ (34)

The Eqn (34), which gives a typical approximate estimate of the value of average deceleration $D_{pav}$ for various distances was established only to provide quantitative values to work with for the purpose of validation of the model by numerical simulation in the absence of physical resources for experimental determination of the values. While implementing the model in the real world, only experimentally determined values in the form of look-up table are to be used to achieve accurate and successful results.

Projectile drag/deceleration is directly proportional to air density, which depends on temperature, pressure, and moisture[8]. An increase in pressure or decrease in temperature will result in an increase in density. Although not as obvious, an increase in moisture content will result in a decrease in density. Since the air pressure varies

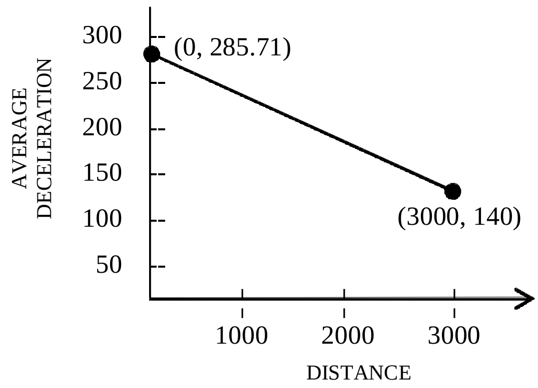


**Figure 6. Estimation of average deceleration at various distances.**

widely at a particular place depending on its altitude and the variations superimposed due to temperature depending on the time of the day, the graph/table between the average deceleration and distance is drawn with a fixed relation between density and altitude for average temperature and moisture levels just like the standard trajectories reflected in the ballistic/firing tables[8]. The value of the average deceleration over a certain distance isthen read off from the graph/table pertaining to the altitude at which the anti-aircraft gun is operating.

## 6. SIMULATION VALIDATION

To gain faith, let an example be considered to and demonstrate the efficacy/validity of Eqn (13). The initial values of the variables have been chosen consistent with the facts based on experience of Army's Air Defence Regiments.

The target is assumed to be initially positioned + 2000 m away along *X* axis, + 2200 m away along *Y* axis, approaching at a constant velocity of –250 m/s along *X* axis, –275 m/s along *Y* axis (approacher target with crossover distance) and with no acceleration. Further, the anti-aircraft gun is assumed to be displaced +50 m along *X* and *Y* axes from the target tracking device which is at the origin of the reference frame. The initial muzzle velocity of the projectile of most anti-aircraft gun's today is approximately 1000 m/s and assumed accordingly. The Eqn (34) have been used to provide values of average deceleration for various distances.

With this data, the Eqn (13) was solved using software made in MATLAB 6.1, as shown in Table 2. The software numerically simulates values of the variables time t and average deceleration $D_{pav}$ which satisfy the Eqn (13) along with the count (n) for the number of times the iterations take place for time convergence and the count (m) for the number of times the average deceleration convergence takes place. Thereafter the analytical values for the final position of the target ($X_{tf}$, $Y_{tf}$, $Z_{tf}$) and the projectile ($X_{pf}$, $Y_{pf}$, $Z_{pf}$) along the three axes, initial and the final distances of the target ($d_1$, $d_f$) and the average deceleration ($D_{pavf}$) corresponding to the final distance is calculated. The result of the simulation after time convergence procedure for the first time as well as after full procedure of average

**Table 2. Software program for time convergence and average deceleration convergence of Eqn (13) to calculate projectile elevation ep and bearing bp.**

```
% anti aircraft gun laying angle calculation program in matlab 6.1 %
Xt = 2000 ; Yt = 2200 ; Zt = 500 ; Vtx = -250 ; Vty = -275 ; Vtz = 0 ; Atx = 0 ; Aty = 0 ; Atz = 0 ;
Xp = 50 ; Yp = 50 ; Zp = 0 ;
K = 1000 ; G = 9.8 ;
d1 = sqrt((Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2) ; % check tgt approaching / receeding
Dpav = 285.7 – 0.04857*d1 ;
t1 = (K-sqrt(K^2-2*d1*Dpav))/Dpav ;
t = t1 ;
d2 = sqrt((Xt+Vtx*t+0.5*Atx*t^2-Xp)^2+(Yt+Vty*t+0.5*Aty*t^2-Yp)^2+(Zt+Vtz*t+0.5*Atz*t^2-Zp)^2) ;
if d1 > d2 % set direction of search for time t
   dir = -1 ;
else
   dir = 1 ;
end
eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
       ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
       (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
       2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
       (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
x = sign(eqn) ; % check initial value of eqn is +ve / -Ve
n = 0 ; % count number of iterations while searching for time t
switch x
   case -1
      while eqn < 0 % limit number of iteration till eqn reaches zero
      n = n + 1 ;
      t = t1 + (dir*n*0.01) ;
      eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
             ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
             (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
             2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
             (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
      end
   case +1
      while eqn > 0 % limit number of iteration till eqn reaches zero
      n = n + 1 ;
      t = t1 + (dir*n*0.01) ;
      eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
             ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
             (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
             2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
             (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
      end
end
% find gun elevation / bearing , position of target/ projectile and distance of impact
e = asin ((Zt-Zp+(Vtz*t)+(0.5*Atz*t^2)+(0.5*G*t^2))/((K*t)-(0.5*Dpav*t^2))) ; % gun elevation
b = atan ((Yt-Yp+(Vty*t)+(0.5*Aty*t^2))/(Xt-Xp+(Vtx*t)+(0.5*Atx*t^2))) ; % gun bearing
Xtf = Xt + Vtx*t + 0.5*Atx*t^2 ; % target's final posn along X axis
Ytf = Yt + Vty*t + 0.5*Aty*t^2 ; % target's final posn along Y axis
Ztf = Zt + Vtz*t + 0.5*Atz*t^2 ; % target's final posn along Z axis
Xpf = Xp + K*t*cos(e)*cos(b) - 0.5*Dpav*cos(e)*cos(b)*t^2 ; % projectile's final posn along X axis
Ypf = Yp + K*t*cos(e)*sin(b) - 0.5*Dpav*cos(e)*sin(b)*t^2 ; % projectile's final posn along Y axis
Zpf = Zp + K*t*sin(e) - 0.5*Dpav*sin(e)*t^2 - 0.5*G*t^2 ; % projectile's final posn along Z axis
df = sqrt((Xtf-Xp)^2+(Ytf-Yp)^2+(Ztf-Zp)^2); % targets's final distance w.r.t projectile
Dpavf = 285.7 – 0.04857*df ; % final value of Dpav corresponding to df
t1, n, t, Xtf, Ytf, Ztf, Xpf, Ypf, Zpf, d1, d2, df, Dpavf, Dpav, e, b %- print results
m = 0 ; % count number of iterations while searching for Dpav
% chk direction of search for correct Dpav
if Dpavf > Dpav
   while Dpavf > Dpav
   m = m + 1 ;
   Dpav = Dpav + 1.0 ;
   t = t1 ;
   n = 0 ;
   eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
          ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
          (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
          2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
          (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
   x = sign(eqn) ;
      switch x
         case -1
            while eqn < 0 % limit number of iteration till eqn reaches zero
            n = n + 1 ;
```

```
          t = t1 + (dir*n*0.01) ;
          eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
               ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
               (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
               2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
               (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
          end
        case +1
          while eqn > 0 % limit number of iteration till eqn reaches zero
          n = n + 1 ;
          t = t1 + (dir*n*0.01) ;
          eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
               ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
               (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
               2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
               (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
          end
      end
    e = asin ((Zt-Zp+(Vtz*t)+(0.5*Atz*t^2)+(0.5*G*t^2))/((K*t)-(0.5*Dpav*t^2))) ; % gun elevation
    b = atan ((Yt-Yp+(Vty*t)+(0.5*Aty*t^2))/(Xt-Xp+(Vtx*t)+(0.5*Atx*t^2))) ; % gun bearing
    Xtf = Xt + Vtx*t + 0.5*Atx*t^2 ; % target's final posn along X axis
    Ytf = Yt + Vty*t + 0.5*Aty*t^2 ; % target's final posn along Y axis
    Ztf = Zt + Vtz*t + 0.5*Atz*t^2 ; % target's final posn along Z axis
    Xpf = Xp + K*t*cos(e)*cos(b) - 0.5*Dpav*cos(e)*cos(b)*t^2 ; % projectile's final posn along X axis
    Ypf = Yp + K*t*cos(e)*sin(b) - 0.5*Dpav*cos(e)*sin(b)*t^2 ; % projectile's final posn along Y axis
    Zpf = Zp + K*t*sin(e) - 0.5*Dpav*sin(e)*t^2 - 0.5*G*t^2 ; % projectile's final posn along Z axis
    df = sqrt((Xtf-Xp)^2+(Ytf-Yp)^2+(Ztf-Zp)^2); % targets's final distance w.r.t projectile
    Dpavf = 285.7 – 0.04857*df ; % final value of Dpav corresponding to df
    end
else
    while Dpavf < Dpav
    m = m + 1 ;
    Dpav = Dpav - 1.0 ;
    t = t1 ;
    n = 0 ;
    eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
         ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
         (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
         2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
         (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
    x = sign(eqn) ;
      switch x
        case -1
          while eqn < 0 %- limit number of iteration till eqn reaches zero
          n = n + 1 ;
          t = t1 + (dir*n*0.01) ;
          eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
               ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
               (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
               2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
               (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
          end
        case +1
          while eqn > 0 %- limit number of iteration till eqn reaches zero
          n = n + 1 ;
          t = t1 + (dir*n*0.01) ;
          eqn = 0.25*(Atx^2+Aty^2+(Atz+G)^2-Dpav^2)*t^4+ ...
               ((Atx*Vtx)+(Aty*Vty)+((Atz+G)*Vtz)+(K*Dpav))*t^3+ ...
               (((Xt-Xp)*Atx)+Vtx^2+((Yt-Yp)*Aty)+Vty^2+((Zt-Zp)*(Atz+G))+Vtz^2-K^2)*t^2+ ...
               2.0*(((Xt-Xp)*Vtx)+((Yt-Yp)*Vty)+((Zt-Zp)*Vtz))*t+ ...
               (Xt-Xp)^2+(Yt-Yp)^2+(Zt-Zp)^2 ;
          end
      end
    e = asin ((Zt-Zp+(Vtz*t)+(0.5*Atz*t^2)+(0.5*G*t^2))/((K*t)-(0.5*Dpav*t^2))) ; % gun elevation
    b = atan ((Yt-Yp+(Vty*t)+(0.5*Aty*t^2))/(Xt-Xp+(Vtx*t)+(0.5*Atx*t^2))) ; % gun bearing
    Xtf = Xt + Vtx*t + 0.5*Atx*t^2 ; % target's final posn along X axis
    Ytf = Yt + Vty*t + 0.5*Aty*t^2 ; % target's final posn along Y axis
    Ztf = Zt + Vtz*t + 0.5*Atz*t^2 ; % target's final posn along Z axis
    Xpf = Xp + K*t*cos(e)*cos(b) - 0.5*Dpav*cos(e)*cos(b)*t^2 ; % projectile's final posn along X axis
    Ypf = Yp + K*t*cos(e)*sin(b) - 0.5*Dpav*cos(e)*sin(b)*t^2 ; % projectile's final posn along Y axis
    Zpf = Zp + K*t*sin(e) - 0.5*Dpav*sin(e)*t^2 - 0.5*G*t^2 ; % projectile's final posn along Z axis
    df = sqrt((Xtf-Xp)^2+(Ytf-Yp)^2+(Ztf-Zp)^2); % targets's final distance w.r.t projectile
    Dpavf = 285.7 – 0.04857*df ; % final value of Dpav corresponding to df
    end
end
m, n, t, Xtf, Ytf, Ztf, Xpf, Ypf, Zpf, d1, d2, df, Dpavf, Dpav, e, b %- print results
%- end of programme
```

deceleration convergence with time convergence nested inside is shown in Table 3.

After the time convergence for the first time in which 172 iterations took place, apparently the values of final position of the target ($X_{tf}$ = 1378.3 m, $Y_{tf}$ = 1516.1 m) and the projectile ($X_{pf}$ = 1376.8 m, $Y_{pf}$ = 1514.5 m) are almost the same, with a difference of 1.5 m and 1.6 m along the two axes, leading us to believe that collision will take place. However, the average deceleration $D_{pavf}$ corresponding to the final distance $d_f$ is 186.5405 and is not the same as the initially chosen value of $D_{pav}$ which is 142.5667. It can thus be inferred that in the real world the projectile will experience an average deceleration other than 142.5667 and hence the values of final position of the projectile along the three axes are incorrect. However, after the average deceleration convergence in which 47 iterations took place and the last iteration nested with another 155 iterations of time convergence, the value of average deceleration $D_{pavf}$ corresponding to the final distance $d_f$ is 189.5145 which is almost same as the value of $D_{pav}$ = 189.5667 arrived at by average deceleration convergence procedure. It can thus be inferred that the projectile will actually experience an average deceleration of 189.5667 and hence the values of final position of the target and the projectile along the three axes as well as the gun elevation and bearing are correct. The corresponding values along $X$ and $Y$ axes of the final position of target ($X_{tf}$ = 1335.8 m, $Y_{tf}$ = 1469.3 m) and final position of the projectile ($X_{pf}$ = 1335.4 m, $Y_{pf}$ = 1468.9 m) are same with a negligible difference of just 0.4 m along both $X$ and $Y$ axes.

The difference of 0.4 m in the values of the final position of target and the projectile along $X$ and $Y$ axes worked out in the example are negligible as compared to size of an aircraft and will vary for other examples with different initial target data. For the choice of 0.01 s as the size of the step in the time convergence procedure, the distance an aircraft would travel in 0.01 s while flying at 1.2 Mach (400 m/s approx) will be only four meters and the difference in the values of the final position of target and the projectile will also be within this range which may be improved by further reducing the size of the step if required. As per practices of anti aircraft gunnery, invariably a burst of eight to ten projectiles are fired and due to recoil of the gun, dispersion takes place. This dispersion of projectiles in 3-D space as well as the use of proximity fuse[1] in the projectile enables the projectile to be effective against the target. Lastly, as regards to the execution time of the software, the results are displayed instantaneously on the computer screen even before the release of the click of the mouse which would be adequate for real time application.

## 7. ERROR COMPENSATION

It will be appropriate to mention that the angular orientation of the gun estimated so far will remain a little inaccurate as long as the error attributable to the computation time of the mathematical model, settling time and steady state error of the closed loop position control system for bearing and elevation channels of the gun as well as the drift due to ballistic effects and wind is not compensated for.

**Table 3. Result of convergence procedures on Eqn (13) for example in Table 2.**

| Results of time convergence for the first time. | Results of average deceleration convergence with time convergence nested inside |
|---|---|
| t1 = 4.2069 | m = 47 |
| n = 172 | n = 155 |
| t = 2.4869 | t = 2.6569 |
| Xtf = 1.3783e+003 | Xtf = 1.3358e+003 |
| Ytf = 1.5161e+003 | Ytf = 1.4693e+003 |
| Ztf = 500 | Ztf = 500 |
| Xpf = 1.3768e+003 | Xpf = 1.3354e+003 |
| Ypf = 1.5145e+003 | Ypf = 1.4689e+003 |
| Zpf = 500.0000 | Zpf = 500.0000 |
| d1 = 2.9453e+003 | d1 = 2.9453e+003 |
| d2 = 1.4294e+003 | d2 = 1.4294e+003 |
| df = 2.0405e+003 | df = 1.9793e+003 |
| Dpavf = 186.5405 | Dpavf = 189.5145 |
| Dpav = 142.5667 | Dpav = 189.5667 |
| e = 0.2622 | e = 0.2723 |
| b = 0.8347 | b = 0.8347 |

In the mathematical model developed so far, it is assumed that the computations are done instantaneously and the gun is aligned immediately without any time lag, which is not so. In the finite time taken to carry out the computations and the settling time taken by the gun to align itself, which will be system specific and known, the target would have moved ahead due to its velocity and acceleration. This distance can be added with the terms $X_t$, $Y_t$ and $Z_t$ in Eqn's (1-3) respectively for compensation. The compensation for angular deviation due to steady state error of the closed loop position control system in bearing and elevation channels can be done by algebraically adding value of the angular deviation to the values of bearing and elevation, estimated in the mathematical model.

Also, to enable the anti-aircraft gun to traverse smoothly without any jerks while changing its orientation from one instance of time to another, a traverse rate may be injected in the control loops of elevation and bearing channel. If $\dot{e}_p$ (n) and $\dot{b}_p$ (n) are the elevation and bearing calculated based on the target data acquired at the t(n) instance of time and similarly $e_p$(n-1) and $b_p$(n-1) for the previous instance of time t(n-1), then the elevation rate $\dot{e}_p$ (n) and bearing rate $\dot{b}_p$ (n) at which the anti-aircraft gun may be made to traverse till the next instance of time t (n+1) will be as under:

$$\dot{e}_p (n) = [e_p(n) - e_p(n-1)]/[t(n) - t(n-1)] \quad (35)$$

$$\dot{b}_p (n) = [b_p(n) - b_p(n-1)]/[t(n) - t(n-1)] \quad (36)$$

When a projectile is fired, it does not go straight but tend to veer off in the direction of its spin. This effect is called drift and is mainly due to Equilibrium Yaw by virtue of projectile's gyroscopic motion[8]. The other factors that contribute to drift, although to a much lesser extent, are the Magnus Effect and Lateral Jump[8]. The drift due to Magnus Effect is very slight and is entirely masked by the drift due to the equilibrium yaw and hence ignored. Lateral jump caused by a slight lateral and rotational movement of the barrel at the instant of firing is also ignored since its effect on bearing is small and varies from round to round. The Coriolis Effect caused by rotation of earth pertain to long range guns firing at static targets located on ground and is not applicable to short range guns firing at mobile air targets. Also, Poisson Effect is no longer accepted since it is not supported by laws of aerodynamics. A practical method of allowing for drift is to assume that it is product of the drift constant and the tangent of the angle of departure of the projectile from the gun[8]. The angle of departure of the projectile from the gun is the vertical acute angle measured from the horizontal plane passing through the weapon to the line tangent to the trajectory at the commencement of free flight[8]. The Drift Constant depends on the muzzle velocity, spin rate, shape of the projectile and other characteristics of the projectile which is determined experimentally and mentioned by the manufacturer of the projectile and gun system[8].

Angular deviation due to drift =
Drift Constant x Tan (angle of departure) (37)

The effect of wind on a projectile in flight is called wind drift. The effect of ballistic wind can be analyzed by reducing it into its two components of range wind, which blows along the trajectory, and cross wind, which blows across the trajectory, by use of wind component tables[8]. The range wind which blows with the projectile (tail wind) offers less resistance while the range wind which blows against the projectile (head wind) offers more resistance. Since the cross-section which projectile offers to range wind is small the effect is negligible as compared to the effect of cross wind to which the cross-section offered by the projectile is much greater. Cross wind tend to carry the projectile with it causing drift and thereby deviation from direction of fire. The amount of drift depends on the cross wind speed acting on the cross-sectional area of the projectile, projectile's time of flight in air and projectile's time of flight in vacuum[12,13]. The projectile's time of flight in vacuum is nothing but ratio of final distance of the target and muzzle velocity of the projectile.

Drift due to crosswind = Crosswind velocity x
(flight time in air – flight time in vacuum) (38)

The effect of drift due to cross wind and ballistic effects can be once again compensated by algebraically adding equivalent angular value of the drift to the value of angular orientation of gun estimated in the mathematical model.

## 8. CONCLUSIONS

In this paper, the problem of angular orientation of the anti-aircraft gun in direction of the predicted future position of the moving air target to enable collision of the projectile fired by the gun with the target has been dicussed. A method and computational algorithm scheme has been developed for determining angular orientation quantitatively in elevation and bearing. The mathematical model developed can be implemented on a digital computer in real time with inputs from the target tracking device and feeding back the outputs of angular orientation to the closed loop position control system in bearing and elevation channels of the anti-aircraft gun system.

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**Contributors**

**Lt Col Deepak Kumar** obtained his BTech from G B Pant University and ME from Delhi College of Engineering and is presently pursuing PhD from the Indian Institute of Technology (IIT), Roorkee. He is serving in the Corps of Electronics and Mechanical Engineers, Indian Army and has been closely associated with providing engineering support to a broad range of Air-Defence equipment in service with the Government of India.

**Dr R.N. Mishra** obtained his BE (Hons) from University of Jabalpur, MTech, from the Indian Institute of Technology, New Delhi, and PhD from University of Leeds, U.K. He is a professor in the Deptt of Electrical Engineering at IIT, Roorkee. He has guided PhD, MPhil and MTech students and published several research papers in international journals of repute. His areas of research are process instrumentation and image processing.