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Approximate Solution of Riccati Differential Equation via Modified Green's Decomposition Method

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ABSTRACT

Riccati differential equations (RDEs) plays important role in the various fields of defence, physics, engineering, medical science, and mathematics. A new approach to find the numerical solution of a class of RDEs with quadratic nonlinearity is presented in this paper. In the process of solving the pre-mentioned class of RDEs, we used an ordered combination of Green's function, Adomian's polynomials, and Pade` approximation. This technique is named as green decomposition method with Pade` approximation (GDMP). Since, the most contemporary definition of Adomian polynomials has been used in GDMP. Therefore, a specific class of Adomian polynomials is used to advance GDMP to modified green decomposition method with Pade` approximation (MGDMP). Further, MGDMP is applied to solve some special RDEs, belonging to the considered class of RDEs, absolute error of the obtained solution is compared with Adomian decomposition method (ADM) and Laplace decomposition method with Pade` approximation (LADM-Pade`). As well, the impedance of the method emphasised with the comparative error tables of the exact solution and the associated solutions with respect to ADM, LADM-Pade`, and MGDMP. The observation from this comparative study exhibits that MGDMP provides an improved numerical solution in the given interval. In spite of this, generally, some of the particular RDEs (with variable coefficients) cannot be easily solved by some of the existing methods, such as LADM-Pade` or Homotopy perturbation methods. However, under some limitations, MGDMP can be successfully applied to solve such type of RDEs.

Keywords: Green's function; Adomian polynomials; Taylor's series; Pade` approximation; Riccati differential equation; Adomian decomposition method

1. INTRODUCTION

The solution of first order linear/nonlinear differential equations exhibit an important role in various models of defence, physical sciences, biological sciences, engineering, statistics and economics. Every linear and nonlinear differential equation cannot be solved analytically, then various numerical method are used to solve such problems.

The RDEs are one of the vital case of the first order nonlinear differential equations and are affined with optimal control systems, network synthesis, financial mathematics, one-dimensional Schrödinger equations, solitary wave solutions, DNA repair models, Integrated missile guidance and control[1-5].

The form of RDE solved in this article, is written as

$$y'(t) - P(t)y(t) = f(t) + Q(t)y^2(t),\ y(a) = y_a \tag{1}$$

where $P(t), f(t), Q(t)$ are some known functions in interval $[a,b]$.

If a particular solution of Eqn. (1) is given or known, then the associated analytical solution can be determined. As well, every RDE of the form Eqn. (1) can be converted into a self adjoint differential equation[6], and the respective general solutions can be obtained in some of the particular cases. Unfortunately, there is no any general method to obtain the analytical (or closed form) solution. In any case, some numerical methods like ADM, Homotopy perturbation method, iterated He's Homotopy perturbation method, He's variation iteration method, LADM-Pade`[7-12] are presented to find an approximate solution of Eqn. (1). Although, these methods give quite good approximate solutions. However, either the computation of undetermined coefficients or implementation of multiple operations is required to solve Eqn. (1). Moreover, these methods are also very restrictive with variable coefficients and they have their own advantage and limitations. Therefore GDMP[13], is proposed to solve Eqn. (1). This method is an ordered amalgamation of green's function, Adomian decomposition method, and Pade` approximation. The method is easy to use under some limitations but in some of the examples, obtained solution coincides with the solution given by LADM-Pade`. Since to increase the accuracy of GDMP, a modified green decomposition method (MGDMP) is described in this article. In order to solve aforementioned RDEs with MGDMP, a specific class of adomian polynomials[14] is used, in spite of the most contemporary class. This exhibits a hike in the degree of absolute error, results more accuracy than previously existing methods.

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1.1 An observation of ADM

Adomian decomposition method (ADM)[15-21] has been used to solve multiple problems by a number of authors. We consider a dynamical system represented by a deterministic operator equation of the form

$$F(y) = g(t) \tag{2}$$

where F is a nonlinear ordinary differential operator with linear and nonlinear terms and g is a known function called source term and y is unknown function. Decompose the operator F as $L+R+N$, where L is the linear operator with highest order derivative. In this case L must be invertible. R is the remaining part of the linear operator and N is the nonlinear operator.

Now, Eqn. (2) can be written as,

$$Ly + Ry + Ny = g$$

where L is a first order differential operator, whose inverse L^{-1}, is an integral operator defined as,

$$L[.] = \frac{d}{dt}[.], \text{ and } L^{-1}[.] = \int_a^t [.]ds$$

Applying L^{-1} both side, we have

$$y = \phi + L^{-1}g - L^{-1}(\mathrm{Ry}) - L^{-1}(\mathrm{Ny}) \tag{3}$$

where the function ϕ represents the terms which has been arisen by using the given initial or boundary conditions.

Now decompose the solution $y(\mathrm{t})$, and the nonlinear term $N(\mathrm{y})$ in converging infinite series, such as

$$y = \sum_{k=0}^{\infty} y_k \tag{4}$$

$$N(\mathrm{y}) = \sum_{k=0}^{\infty} A_k \tag{5}$$

where $A_k\text{'}s$ are Adomian's polynomials, which can be use to solve various types of nonlinear functions, given by the formula[7]:

$$A_k = \sum_{\nu=1}^{k} c(\nu,\mathrm{k}) f^{\nu}(\mathrm{y}_0), \quad \mathrm{k} \geq 0$$

Now using Eqns. (4) and (5) in Eqn. (3), and comparing both sides of the obtained equation

$$\begin{cases} y_0 = \phi + L^{-1}g, \\ y_{k+1} = -L^{-1}R(\mathrm{y}_k) - \mathrm{L}^{-1}\mathrm{A}_k \end{cases} \tag{6}$$

Therefore iterative scheme Eqn. (6), is recognised as ADM.

1.2 Pade` approximation

Pade` approximation[22,23] is used to approximate truncated power series of a function. This tool rationalises the truncated power series and gives a large radius of convergence in the comparison of the original truncated power series. Moreover, Pade` approximation gives the asymptotic behaviour of those models, which cannot be solved easily. To understand Pade` approximation further, suppose a function $f(\mathrm{t})$, which can be approximated in a power series about point $t=0$ such that,

$$f(\mathrm{t}) = \sum_{k=0}^{\infty} c_k t^k$$

$$f(t) \approx c_0 + c_1 t + c_2 t^2 + \cdots + c_{2n} t^{2n}$$

Now to increase the radius of convergence of the truncated power series of $f(\mathrm{t})$, $[m/n]$ Pade` approximation takes place such as,

$$\begin{aligned}[m/n] &= \frac{a_0 + a_1 t + a_2 t^2 + \cdots + a_m t^m}{1 + b_1 t + b_2 t^2 + \cdots + b_n t^n} \\ &= c_0 + c_1 t + c_2 t^2 + \cdots + c_{m+n} t^{m+n}\end{aligned} \tag{7}$$

By solving the system of equations obtained by simplification of Eqn. (7), the unknown coefficients occurred in numerator and denominator $(a_i\text{'s and } b_i\text{'s})$ can be determined easily.

The motive of this article is to give a better approximation with the help of Green's function[24,25] and a specific class of Adomian polynomials[14]. The discussed method is easy to use and increases the radius of convergence of the solution of RDEs. Some numerical examples are solved to show the validation of the method.

2. ANALYSIS OF THE MODIFIED GREEN'S DECOMPOSITION METHOD WITH PADE` APPROXIMATION FOR RDE

The Green's function for the homogeneous differential equation corresponding to Eqn. (1)

$y'(t) - P(t)y(t) = 0, \quad y(a) = 0,$ is given as

$$G(t,\xi) = \begin{cases} 0 & \text{if } a \leq t < \xi \leq b \\ e^{\int_\xi^t P(s)ds} & \text{if } a \leq \xi < t \leq b \end{cases}$$

Therefore, the solution of the RDE Eqn. (1) is

$$y(t) = y_a e^{\int_a^t P(s)ds} + \int_a^t G(t,\xi) f(\xi) d\xi + \int_a^t G(t,\xi) Q(\xi) y^2(\xi) d\xi \tag{7}$$

The solution of Eqn. (7) is not easy to calculate analytically, therefore the series solution of RDE Eqn. (1) is considered as

$$y(t) = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \sum_{i=0}^{k} y_i(t) \tag{8}$$

and the nonlinear term in operator form can be decompose as

$$N[y] = y^2(t) = \sum_{i=0}^{\infty} A_i^{**}(y_0(t), y_1(t), y_2(t), \cdots) \tag{9}$$

where $A_i^{**}\text{'s}$, $i = 0,1,2,\cdots$ are specially defined polynomials. These polynomials are known as Adomian polynomials[14], which are estimated by the components $y_j,\ j = 0,1,2,\cdots,$.

On combining Eqns. (8), (9), and (7), an iterative scheme, namely, Green decomposition method (GDM)[13], is proposed as

$$\begin{aligned} y_0(t) &= y_a e^{\int_a^t P(s)ds} + \int_a^t G(t,\xi) f(\xi) d\xi, \ and\ y_i(t) \\ &= \int_a^t G(t,\xi) Q(\xi) A_{i-1}^{**}(\xi) d\xi \end{aligned} \tag{10}$$

where $G(t,\xi) f(\xi)$ is R-Integrable in $[a,b]$. Since a specific class of Adomian polynomials can be used to improve the approximate solution of the given problem. Here class A_n^{**} is introduced. Now first four polynomials of class A_n^{**} shall be,

$$\begin{cases} A_0^{**} = N(y_0), \\ A_1^{**} = y_1 N^{(1)}(y_0) + \frac{1}{2!} y_1^2 N^{(2)}(y_0), \\ A_2^{**} = y_2 N^{(1)}(y_0) + \frac{1}{2!}(2y_1y_2 + y_2^2) N^{(2)}(y_0) + \frac{1}{3!} y_1^3 N^{(3)}(y_0), \\ A_3^{**} = y_3 N^{(1)}(y_0) + \frac{1}{2!}(2y_1y_3 + 2y_2y_3 + y_3^2) N^{(2)}(y_0) \\ \quad + \frac{1}{3!}(3y_1^2 y_2 + 3y_1y_2^2 + y_2^3) N^{(3)}(y_0) \\ \quad \frac{1}{4!} y_1^4 y_0 N^{(4)}(y_0) \end{cases}$$

Therefore, to apply the Pade` approximation, S_k can be written in a power series form with the help of Taylor's series expansion about point t=0, such that

$$\begin{aligned} S_k &\approx S_T(\mathrm{t}), \\ &= \sum_{n=0}^{N} \frac{S_k^{(\mathrm{n})}(0)}{n!} t^n, \\ &= a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n \end{aligned} \tag{11}$$

For Eqn. (11), $\left[\frac{P}{Q}\right]$ Pade` approximation can be defined as

$$\left[\frac{P}{Q}\right] = \frac{c_0 + c_1 t + c_2 t^2 + \cdots + c_P t^P}{1 + d_1 t + d_2 t^2 + \cdots + d_Q t^Q} \tag{12}$$

where $P + Q \le N$.

If $P = Q$ then Eqn. (12) is called diagonal Pade` approximation.

3. EXAMPLES

In this section, we demonstrate some numerical examples based on the discussed approach.

Example 1. Consider the following nonlinear RDE

$$y'(t) - 2y(t) = 1 - y^2(t), \quad y(0) = 0, \; 0 \le t \le 1 \tag{13}$$

The exact solution of Eqn (13) is

$$y(t) = 1 + \sqrt{2}\tanh\left[\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]$$

Now, using iterative scheme Eqn. (10), approximated series solution S_4 will be,

$$\begin{aligned} S_4(t) &= \sum_{i=0}^{4} y_i(t), \\ &= -\frac{116\cdots399}{214\cdots648} + \frac{565\cdots073}{104\cdots000} e^{2t} - \cdots - \frac{x^8 e^{16t}}{2064384} - \frac{1}{4} e^{2t}\sinh(2t) \end{aligned}$$

Here $S_4(\mathrm{t})$ is an approximated solution of Eqn. (13). Now, to perform the MGDMP, the Taylor's series expansion of $S_4(\mathrm{t})$ about point $t = 0$ is given by

$$S_4(\mathrm{t}) \approx \mathrm{t} + \mathrm{t}^2 + \frac{t^3}{3} - \frac{t^4}{3} - \frac{7t^5}{15} - \frac{7t^6}{45} + \frac{53t^7}{315} + \cdots + \frac{1522069t^{13}}{24324300} + O(\mathrm{t}^{14}) \tag{14}$$

Applying $\left[\frac{7}{6}\right]$ Pade` approximation for Eqn. (14),

$$\left[\frac{7}{6}\right] = \frac{189\cdots760t - 614\cdots880t^2 + \cdots + 115\cdots748t^7}{7(271\cdots680 - 114\cdots520t + \cdots + 698\cdots440t^6)}$$

For the further study of the proposed MGDMP, a comparison is made with ADM and LADM-Pade` in Table 1.

Table 1 shows that k term approximation for $k = 4$, of the MGDMP gives better results than ADM and LADM-Pade` methods.

Table 1. A comparison table of the absolute error (AE) function with ADM, LADM-Pade`, and MGDMP

t	Exact solution	AE using ADM[26]	AE using LADM-Pade`[12]	AE using MGDMP
0.0	0	0	0	0
0.2	0.241977	5.31195×10^{-7}	4.45809×10^{-10}	7.84803×10^{-11}
0.4	0.567812	1.14515×10^{-4}	7.50816×10^{-7}	1.93950×10^{-7}
0.6	0.953566	1.01464×10^{-3}	2.75132×10^{-5}	9.32366×10^{-6}
0.8	1.346360	8.56290×10^{-4}	2.76256×10^{-4}	1.03482×10^{-4}
1.0	1.689500	1.36606×10^{-2}	1.41356×10^{-3}	5.58310×10^{-4}

Example 2. Consider the nonlinear RDE

$$y'(t) - y(t) = 1 + y^2(t), \quad y(0) = 0, \quad 0 \le t \le 1 \tag{15}$$

The exact solution of Eqn. (15) is

$$y(t) = -\frac{1}{2} + \frac{\sqrt{3}}{2}\tan\left(\frac{\sqrt{3}}{2}\left(t + \frac{\pi}{3\sqrt{3}}\right)\right)$$

Now, using iterative scheme Eqn. (10), approximated series solution S_4 will be,

$$\begin{aligned} S_4(t) &= \sum_{i=0}^{4} y_i(t), \\ &= -673 - \frac{224\cdots703}{485\cdots000} e^t + \frac{132\cdots009}{128\cdots000} e^{2t} - \cdots - 4e^t\cosh t + 4e^t\sinh t \end{aligned}$$

Here $S_4(t)$ is an approximated solution of Eqn. (15) using MGDM. Now to perform the MGDMP, the Taylor's series expansion of $S_4(\mathrm{t})$ about point $t = 0$ is given by

$$S_4(t) \approx t + \frac{t^2}{2} + \cdots + \frac{841\cdots533}{162\cdots000} t^{21} + \frac{442\cdots589}{124\cdots000} t^{22} + O(t^{23}) \tag{16}$$

Applying $\left[\frac{11}{11}\right]$ Pade` approximation for Eqn. (16),

$$\left[\frac{11}{11}\right] = \frac{2(-579\cdots200\,\mathrm{t} - 227\cdots800\,\mathrm{t}^2 + \cdots + 142057t^{11})}{(-115\cdots400 - 396\cdots400t + \cdots + 153\cdots263t^{11})}$$

For the further study of the proposed MGDMP, a comparison is made with ADM and LADM-Pade` methods in the Table 2.

As shown in Table 2 that five term approximation for the MGDMP gives better results than ADM and LADM-Pade` methods.

Table 2. A comparison table of the absolute error (AE) function with ADM, LADM-Pade`, and MGDMP.

t	Exact solution	AE using ADM	AE using LADM-Pade`	AE using MGDMP
0.0	0	0	0	0
0.2	0.224724	6.71967×10^{-7}	3.14604×10^{-10}	5.47622×10^{-11}
0.4	0.526540	1.49837×10^{-4}	1.17971×10^{-6}	2.17779×10^{-7}
0.6	0.986295	4.85752×10^{-3}	2.01829×10^{-4}	4.28426×10^{-5}
0.8	1.840630	7.53604×10^{-2}	1.07390×10^{-2}	2.96453×10^{-3}
1.0	4.227710	9.49491×10^{-1}	3.74040×10^{-1}	1.63617×10^{-1}

Example 3. Consider the nonlinear RDE

$$y'(t)+2y(t)=-1-y^2(t),\quad y(0)=1,\quad 0\le t\le 1 \qquad (17)$$

The exact solution of Eqn. (17) is

$$y(t)=\frac{1-2t}{1+2t}$$

Now, using iterative scheme Eqn. (10), approximated series solution S_4 will be,

$$S_4(t)=-\frac{281}{1024}+\frac{729e^{-12t}}{2048}-\cdots+\frac{3}{4}e^{-4t}t^4+\frac{1}{12}e^{-2t}t^4+\frac{1}{80}e^{-2t}t^5$$

To perform the Pade` Approximation, the Tayor's series expansion of S_4 about $t=0$ is given by

$$S_4\approx 1-4t+8t^2-16t^3+\cdots-\frac{54295821446t^{15}}{30405375}+\frac{137247812311t^{16}}{91216125} \qquad (18)$$

Using $\left[\frac{8}{8}\right]$ Pade` approximation for Eqn. (18),

$$\left[\frac{8}{8}\right]=\frac{9497\ldots.9375+2937\ldots.3000t+\ldots-9547..0643t^8}{9497\ldots.9375+6736\ldots.0500t+\ldots-9993...1593t^8}$$

For the future study of the proposed MGDMP, a comparison is made with ADM and LADM- Pade` in the Table 3, it can be seen that MGDMP for solving RDE Eqn. (17) working better than ADM and LADM- Pade` methods.

Table 3. A Comparison tables of the AE function for ADM, LADM- Pade` and MGDMP.

t	Exact solution	AE using ADM	AE using LADM-Pade`	AE using MGDMP
0.0	1	0	0	0
0.2	0.428571	6.53871×10^{-3}	4.32515×10^{-5}	4.11424×10^{-6}
0.4	0.111111	1.26703×10^{-1}	2.05989×10^{-4}	1.34154×10^{-5}
0.6	−0.0909091	6.06866×10^{-1}	2.51433×10^{-4}	9.11772×10^{-6}
0.8	−0.230769	1.65096	1.85803×10^{-4}	1.73317×10^{-6}
1.0	−0.333333	3.29559	1.22405×10^{-4}	4.38205×10^{-6}

Example 4. Consider the nonlinear RDE

$$ty'(t)=t^2+t^2y^2(t),\quad y(0)=0,\quad 0\le t\le 1 \qquad (19)$$

If $t=0$, then $y(t)=0$ is the only solution of Eqn. (19). For $0<t\le 1$, Eqn. (19) can be written as

$$y'(t)-\frac{1}{t}y(t)=t+ty^2,\quad y(0)=0$$

Using the iterative scheme Eqn. (10), approximated series solution $S_4(\mathrm{t})$ will be,

$$S_4(t)=\sum_{i=0}^{4}y_i(\mathrm{t})=t^2+\frac{t^6}{5}+\frac{2t^{10}}{45}+\frac{29t^{14}}{2925}+\frac{22t^{18}}{9945}+\frac{19594t^{22}}{46990125} \qquad (20)$$

In Eqn. (20) variable t is in a power series form. So Pade` approximation $[11/11]$ is used to increase the efficiency of the method.

$$\left[\frac{11}{11}\right]=\frac{9945t^2-351t^6+t^{10}}{9\left(1105-260t^4+3t^8\right)}$$

The residual error function for the RDE Eqn. (19) can be given as

$$Residual\ Error=\left|\left[\frac{11}{11}\right]'-\frac{1}{t}\left[\frac{11}{11}\right]-t-t\left[\frac{11}{11}\right]^2\right|$$

For the further study of the proposed MGDMP, a comparison table is made with ADM.

Table 4. A comparison table of the residual error (RE) function for ADM and MGDMP

t	AE using ADM	AE using MGDMP
0.0	0	0
0.2	4.24729×10^{-2}	2.77556×10^{-17}
0.4	6.47210×10^{-2}	5.08038×10^{-13}
0.6	6.02195×10^{-2}	5.25542×10^{-10}
0.8	4.55738×10^{-2}	8.02026×10^{-8}
1.0	0.174587	4.91640×10^{-6}

In Example 4, the numerical solution is not easy to find via LADM- Pade` methods. Hence, MGDMP can be easily applied in this example for the numerical solution. As it can be observed that exact solution of RDE Eqn. (19) is not known. Therefore to investigate the efficiency of MGDMP, the residual error is being used here. With the help of the residual error, the comparative study of MGDMP can be investigated with ADM.

4. CONCLUSIONS

In this paper, a method to solve a particular class of RDEs, namely MGDMP, is proposed. Some specific numerical problems are solved to confirm the robustness of the proposed approach. Examples 1, 2, and 3 validate the applied method. In Example 4, a particular class of RDE is discussed, which cannot be generally solved using LADM-Pade` technique. In the virtue of the solved examples, it can be concluded that MGDMP is an efficient method and provide a better solution in the given interval, in comparison to some of the classical methods or techniques like ADM and LADM-Pade`. Although, the applied class of Adomian polynomials and Pade` approximation have much computational work. However, using MGDMP, the numerical solutions of the specified class of RDEs are more accurate and effective in the given interval.

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