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Estimation of Compressional Wave Speed in Marine Sediments using Biot-Stoll Model and Buckingham's Grain-shearing Model

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ABSTRACT

Acoustic properties of seafloor sediments can be estimated using theoretical models by giving geophysical properties of sediments as inputs to the respective models. Empirical relations connecting the geophysical and geoacoustic properties are available in literature. In this study an experimental assessment of two such theoretical models viz., Biot-Stoll model (BSM), a poro-elastic model and the Buckingham's grain shearing (GS) model, a visco-elastic model is done by estimating the compressional wave speed. Compressional wave speed is measured using in-house developed sediment velocimeter and is compared with the speed estimated using both the models and a regression analysis was done. It was observed that the Coefficient of determination R^2 for BSM and GS model are 0.769 and 0.729, respectively. It shows that once the constants used in GS model are evaluated for the Indian waters, then it can be used to estimate the acoustic properties of sediments.

Keywords: Compressional speed; Geoacoustic modelling; Marine sediments

1. INTRODUCTION

Geoacoustic properties of sediments i.e., compressional speed, compressional attenuation, shear speed and shear attenuation (as functions of frequency, geophysical properties and depth) are crucial inputs to Geoacoustic model which aids in predicting the effects of sound propagation in seafloor. The geoacoustic properties depend on the geophysical properties such as mean grain size, density and porosity, which can be estimated in the laboratory by following standard procedures[1].

Considering the complexities associated with *in-situ* measurement of geoacoustic parameters, sediment models which can simulate the propagation of acoustic wave using physical properties of sediments is required. Empirical models also have been used to estimate the geoacoustic properties of sediments from laboratory measured geophysical properties[2]. But the accuracy of the empirical models is greatly influenced by the identification of the parameters (physical properties of sediments, constants) upon which the acoustic properties depend and also in defining the inter relationship between the parameters.

Biot has developed an approach based on the mathematical and physical principles of the acoustic waves' propagation in saturated porous media and on a comprehensive theory of porous media which relates geoacoustic properties to geophysical properties. Later, Stoll refined the theory to suit marine sediments[3]. The disadvantage of the Biot-Stoll Model (BSM) is in the high number of parameters and the theory is formulated based on the existence of a frame or hard skeleton whose existence is doubtful[4].

Buckingham's theory assumes that marine sediments can be regarded as a granular material with internal friction (which is more important than fluid viscosity) saturated by fluid. He has developed four dispersion relations representing compressional speed, compressional attenuation, shear speed and shear attenuation as function of frequency, porosity, density, grain size and overburden pressure which enables one to estimate the compressional wave speed as a function of frequency and depth[5].

In this study, an attempt is made to assess the theoretical models by comparing the compressional wave speed predicted using BSM and grain shearing (GS) model with the data estimated using in-house developed Sediment Velocimeter.

2. METHODS

2.1 Biot-Stoll Theory

Biot-Stoll Model, a poro-elastic model is applicable to marine sediments whose pores may be completely saturated with gas or fluid or it can be unconsolidated. The dispersion relations developed by Biot-Stoll are dependent on frequency and the attenuation of compressional and shear waves happens due to intergranular friction and viscous friction[6].

Inputs to the BSM are geophysical properties of the sediments. The outputs are the compressional speed I, compressional speed II, compressional attenuation, shear speed and shear attenuation. The inputs to the BSM with specific references are given in Table 1.

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Table 1. Parameters in Biot-Stoll model

Material parameter	Value
Pore fluid density[7](ρ_w)	Variable
Porosity[8] (n)	Variable
Bulk density(ρ_{sat})	Weight-volume method using Pycnometer.
Grain density(ρ_s)	
Viscosity of pore fluid[9,10](η)	Variable
Permeability[3,11](k)	$k=\left(\frac{1}{k'S_o^2}\frac{n^3}{(1-n)^2}\right)$ S_o - the specific surface of the sediment particles (cm^{-1}). $S_o=6/d$, where d is the d iameter in cm k' - empirical constant ~5
Pore-size parameter[12](a)	$a=\sqrt{2[K'(k/n)]}$
Fluid bulk modulus[13] (K_f)	$K_f=C^2(t)\rho_w$ $C^2(t)$- Sound speed in pore fluid t - temperature ρ_w - Pore fluid density
Grain Bulk modulus(K_r) $(\times 10^{11} dyn/cm^2)$	4.0 - sand sediments[14] 3.5 - silty clay[15] 3.6 - soft sediments[15]
Imaginary part of the bulk modulus[3]	Zero (anelasticity of the sediment grains is negligible).
Structure factor[16]	$a'=1-r_o(1-1/n)$ $r_o=0.5$
Frame bulk modulus[17](K_b) $(\times 10^9 dyn/cm^2)$	For natural sands: $\log K_b=2.70932-4.25391n$ For natural silty clays: $\log K_b=2.73580-4.25075n$
Imaginary part of the frame bulk modulus - logarithmic decrement of longitudinal wave δ_e .	0.1 - granular sediments 0.01 - silty clay
Shear modulus[3] (μ)	$[3K_b(1-2\sigma)]/[2(1+\sigma)]$ σ - frame Poisson's ratio 0.2 - granular sediments 0.3 - silty clay 0.5 - soft sediments
Imaginary part of the frame shear modulus[18]	$\delta_s\,\mu/\pi$ $\delta_s=0.1$

2.2 Grain Shearing Model

The GS model is derived from the linearised Navier-Stokes equation and is based on the inter granular interactions during the passage of compressional and shear waves. The dispersion relations are developed based on the Kramers-Kronig relationships[4]. The advantages of using Buckingham's GS model is that the speed and attenuation of compressional and shear waves can be computed using simple algebraic expressions which are functions of the physical properties of the sediments. According to Buckingham, the dispersion relation to estimate the compressional speed and the inputs to the GS model are as follows[5].

$$c_p=\frac{c_o}{\mathrm{Re}\left[1+\frac{\gamma_p+(4/3)\gamma_s}{\rho_o c_o^2}(j\omega T)^n\right]^{-1/2}} \tag{1}$$

$$\rho_o=N\rho_w+(1-N)\rho_g \tag{2}$$

$$\frac{1}{k_o}=N\frac{1}{k_w}+(1-N)\frac{1}{k_g} \tag{3}$$

$$c_o=\sqrt{\frac{k_o}{\rho_o}} \tag{4}$$

$$\gamma_p=\gamma_{po}\left[\frac{(1-N)u_g d}{(1-N_o)u_{go}d_o}\right]^{1/3} \tag{5}$$

$$\gamma_s=\gamma_{so}\left[\frac{(1-N)u_g d}{(1-N_o)u_{go}d_o}\right]^{2/3} \tag{6}$$

where c_p - compressional wave speed, c_o - sound speed without inter granular interactions, γ_p - Compressional moduli, γ_s - shear moduli. ρ_o - Bulk density of the medium k_o - bulk modulus of sediment. $j=\sqrt{-1}$, ω - Angular frequency and T - arbitrary time (1 s). The other parameters used in the grain shearing model are given in Table 2.

Table 2. Parameters in Grain shearing model

Material parameter	Symbol	Value
Pore fluid density (*kg/m*³)	ρ_w	1005
Grain density (*kg/m*³)	ρ_g	2730
Bulk modulus of pore fluid (*Pa*)	k_w	2.3743 x 10^9
Grains Bulk modulus (*Pa*)	k_g	3.6 x 10^{10}
Compressional coefficient (*Pa*)	γ_{po}	3.888 x 10^8
Shear coefficient (*Pa*)	γ_{so}	4.588 x 10^7
Strain-hardening index	n	0.0851
Porosity	N	Variable
Reference porosity	N_o	Variable
Mean grain diameter (μ*m*)	u_g	Variable
Reference grain diameter (μ*m*)	u_{go}	Variable
Depth in sediment (*m*)	d	Variable
Reference depth in sediment (*m*)	d_o	0.3

2.3 Measurement of Compressional Wave Speed

2.3.1 Sediment Sample Collection and Estimation of Physical Properties

The sediment samples used in this study were collected in the continental shelf, west coast of India stretching from Gujarat to Cochin using gravity corer. The sediment samples were analysed in the laboratory by following standard laboratory methods[19]. Mean grain size of the sediments, M_ϕ ($-\log_2$ of grain diameter in mm) were estimated. Dry density, Bulk density and Porosity were estimated using the phase relationships. The physical properties of sediments estimated in laboratory are shown in Figs. 1 and 2.

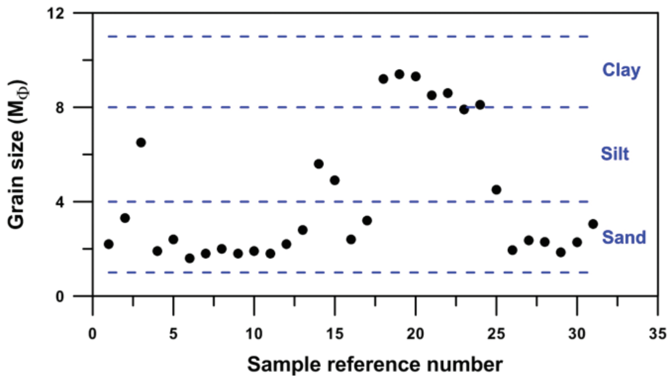

Figure 1. Grain size of the marine sediments (31 samples) estimated in the laboratory. It is expressed in M_ϕ ($-\log_2$ of grain diameter in mm). X axis represents the serial number of the samples.

2.3.2 *Laboratory Measurement of Compressional Speed*

Compressional wave speed is measured using the sediment velocimeter whose block diagram is shown in Fig. 3. An oscillator and a gating circuit is used to generate the signal. A frequency of 500 kHz was selected for the measurement of Compressional speed considering high accuracy and relatively low attenuation.

A crystal oscillator is used to generate a highly stable 1 MHz signal and it was transformed to a 500 kHz square wave using a frequency divider. To modulate the oscillator output, a gating signal was generated. Pulse width and pulse repetition rate was kept in the range of 5-50 μs and 0.1 s to 1 s, respectively.

Signal from the gating circuit is fed to power amplifier, which drives the transmitter. On either side of the sediment sample, transmitter and receiver are fixed. It was made sure that there is no air film in the space between sample and transducer since it causes impedance mismatch. The time delay was recorded using the oscilloscope.

Calibration using Distilled Water: Sound speed through distilled water is measured at 26.1 °C. The travel time through the distilled water was noted down for different path lengths. The spacing between the transducers is measured using vernier calipers. The speed of sound (function of temperature andpressure) is derived using an empirical relation[20]. Average of the three observations is 1483 m/s (~15 m/s i.e. 1% variation). To make corrections for moulding material ('time-delay' between the transmitted and received signals), a value of 8.4 μs was reduced from the observed time delay.

Marine Sediments: Sound speed of the seawater and sediments are V_1 and V_1, respectively. Sound speed of the seawater is known and the sound speed of the sediments is calculated by measuring the transit times ie. the samples are placed at a known distance between the transducers which are kept in the seawater. Compressional speed of the sediments is measured by comparing the transit times in the seawater (t_1) and sediment (t_2). Compressional speed of the sediments (V_2) is calculated using the relation $V_2 = (V_1 t_1 / t_2)$[21].

Measurements are carried out at a temperature of 27.5 °C and using a seawater with 35 PSU salinity. Corrections for temperature and pressure changes were not done since its effect on sound speed are insignificant[8].

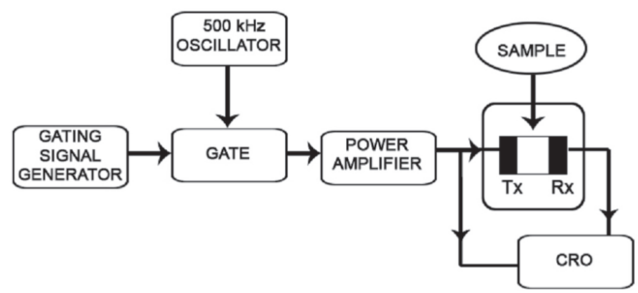

Figure 3. Block diagram of in house developed sediment velocimeter.

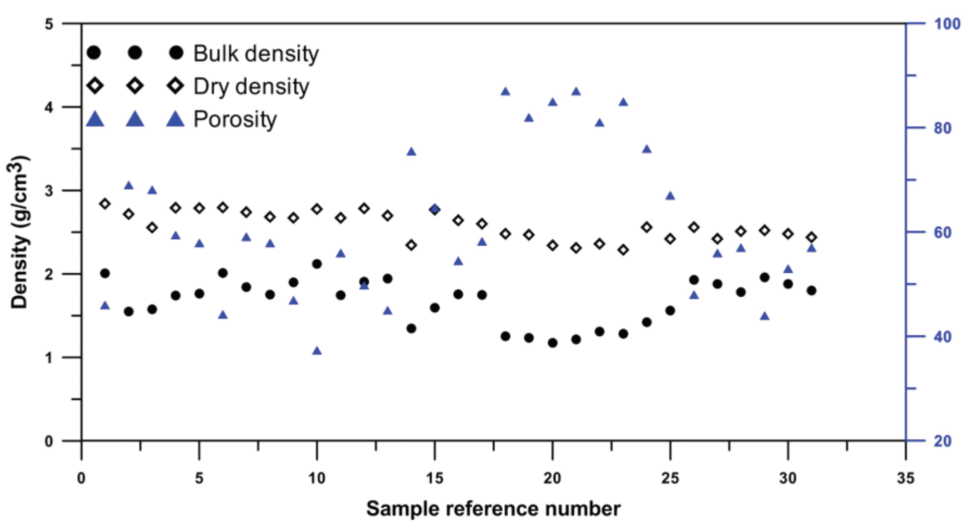

Figure 2. Bulk density, dry density and porosity of the marine sediments estimated in the laboratory. X axis represents the serial number of the samples.

3. RESULTS AND DISCUSSION

The compressional wave speed measured using velocimeter and predicted using BSM and GS model are given in Table 3. The compressional wave speeds of 31 sediment samples were measured at 500 kHz using sediment velocimeter. The measured sound speed was corrected to *in-situ* conditions. Compressional wave sound speeds in sediments were predicted using BSM and GS model at 500 kHz and the comparison is shown in Fig. 4. To find the correlation between the measured compressional speed with that of predicted values a regression analysis was done and R^2 was found to be 0.769 and 0.729 for BSM and GS model respectively as shown in Fig. 4.

The compressional wave speed was measured using sediment velocimeter and predicted using BSM and Buckingham's GS model. The values of the reference porosity (N_o) and Reference grain diameter (u_{go}) for each type of sediment such as sand, silt and clay were taken as the average of the measured porosity and the grain size of the respective type of sediments. The results were plotted as % difference between measured and predicted values and are shown in Fig. 5.

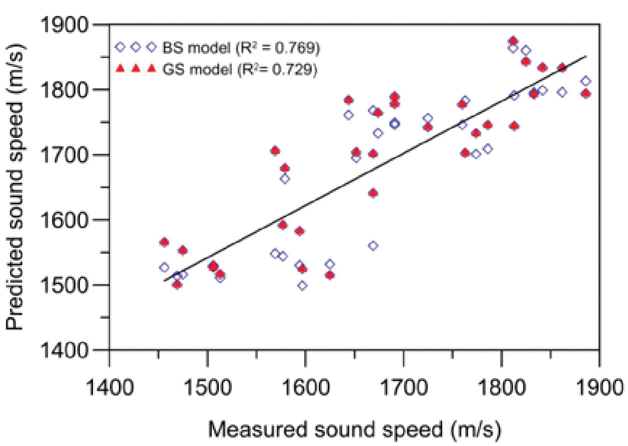

Figure 4. Variation of sound speed predicted using Biot-Stoll and GS model *vs* sound speed measured using velocimeter.

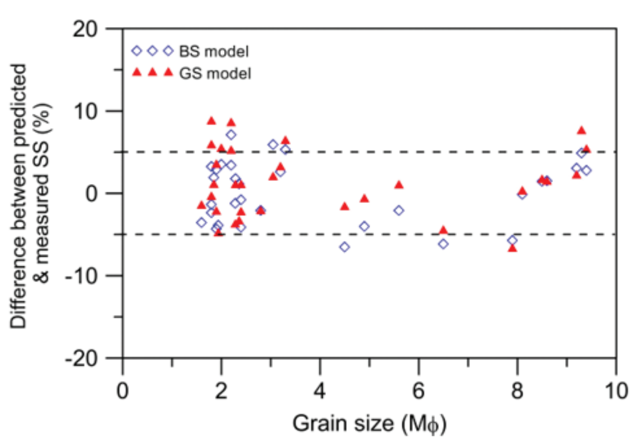

Figure 5. % deviation from the measured sound speed plotted vs grain size. The dotted lines represent ± 5% deviation.

The variation of the compressional wave speed with frequency is studied for the three types of sediments viz., sand, silt and clay. For this purpose, a representative sample is taken from the measured value and the compressional speed is predicted using Buckingham model for the frequency range of 1 - 50 kHz. The estimated variation in sound speed is shown in Fig. 6.

It was observed that in some sandy sediments ($< 4\ M_\phi$), the predicted sound speeds are less than the measured values. This may be attributed to the Calcium Carbonate ($CaCO_3$) content in these samples. As the $CaCO_3$ content in the sediment increases, the sound speed increases and it affects the predictability of sound speeds in shallow-water (< 1500 m)[22,23]. In the sediment samples collected for the study, shells of marine organisms

Table 3. Measured and predicted sound speed in sediments

Water depth (m)	Sediment type	Location	Compressional wave speed (m/s)		
			Measured	Predicted	
			Velocimeter	Biot-Stoll	GS
90	Sand	Off Gujarat	1644	1761	1784
70	Silty sand	Off Gujarat	1579	1663	1679
58	Clayey silt	Off Gujarat	1597	1499	1524
83	Sand	Off Bombay	1786	1709	1746
73	Sand	Off Bombay	1774	1701	1733
77	Sand	Off Bombay	1862	1796	1834
80	Sand	Off Bombay	1569	1548	1706
78	Sand	Off Bombay	1674	1733	1764
75	Sand	Off Bombay	1842	1799	1834
65	Sand	Off Bombay	1812	1864	1875
70	Sand	Off Bombay	1691	1746	1790
113	Sand	Off Panaji	1691	1749	1778
35	Silty sand	Off Panaji	1833	1795	1793
320	Sandy silt	Off Karwar	1577	1544	1592
100	Sandy silt	Off Bhatkal	1594	1530	1583
42	Sand	Off Mangalore	1760	1746	1777
270	Silty sand	Off Kasargode	1652	1695	1704
13	Clay	Off Cochin	1469	1514	1501
12	Clay	Off Cochin	1475	1516	1553
11	Clay	Off Cochin	1456	1527	1566
10	Clay	Off Cochin	1506	1528	1530
10	Clay	Off Cochin	1506	1529	1527
10	Silty clay	Off Cochin	1625	1532	1515
9	Silty clay	Off Cochin	1513	1511	1516
10	Silty sand	Off Cochin	1669	1560	1641
51	Sand	Off Cochin	1886	1813	1794
40	Sand	Off Cochin	1763	1783	1703
46	Sand	Off Cochin	1725	1756	1742
67	Sand	Off Cochin	1825	1860	1843
80	Sand	Off Cochin	1813	1791	1744
50	Sand	Off Cochin	1669	1768	1702

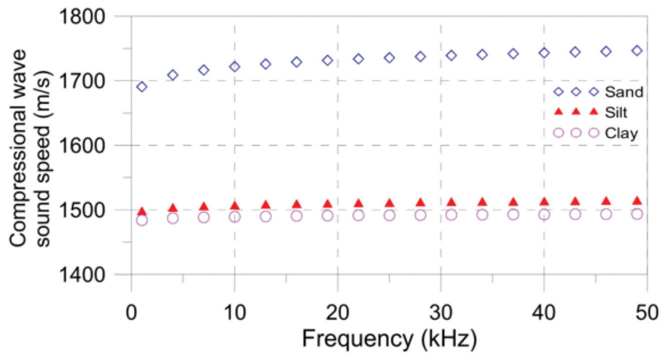

Figure 6. Variation of compressional speed predicted using GS model *vs* frequency.

were observed in sandy sediments and in some fine sediments. $CaCo_3$ content was not estimated in this study.

Buckingham has stated that the geoacoustic properties depends on porosity, grain size and depth. The dispersion relations also involve parameters such as n, γ_p and γ_s which represent the microscopic processes occurring at the intergranular contacts. Although acoustic properties can be calculated using GS model, there are difficulties in evaluating the constants used in the dispersion relations developed by Buckingham. The estimates of the three GS parameters viz., Strain-hardening index (n), Compressional moduli (γ_p) and Shear moduli (γ_s) are essential for the estimation of the wave speed and the attenuation and the accuracy can be improved by calculating the same using known values of compressional speed, compressional attenuation and shear speed. The estimate of n is a critical step and relies on accurate measurement of porosity, compressional sound speed and attenuation[24].

4. CONCLUSIONS

The compressional wave speed was measured using sediment velocimeter and predicted using BSM and GS. The results show that the sound speed predicted by BSM using the thirteen input parameters is well within ± 5% of the measured sound speed with some outliers. On the other hand the sound speed predicted by the GS model using geophysical data such as Porosity, density and mean grain size is also well within the ± 5% range of the measured data with some outliers.

From the regression analysis, it was observed that the R^2 value for BSM and GS model were 0.769 and 0.729 respectively. Effect of the grain size in the compressional speed was studied by estimating compressional speed as a function of frequency for a representative sample in sand, silt and clay. The compressional speed was found to be increasing with increase in frequency and it was higher for sediments whose mean grain size is coarser which matches with the earlier results of Stoll.

The major concern in using the BSM to estimate the acoustic properties of marine sediments is the complexity of the model with approximately ten physical parameters affecting the dispersion relation. The inputs to the model can be obtained from direct measurements, physical/empirical relationship or values obtained from the literature[6] which constrains the use of BSM. Hence GS model can be used to estimate the acoustic properties of the marine sediments if the GS constants calculated accurately with known values of phase speed and attenuation of both types of wave are present. In this case the constants (N_o, u_{go} and d_o) were taken as average of the measured data and the results seem to agree with the measured data at 500 kHz. Once the GS constants are established for Indian waters using the measured datasets, the geoacoustic parameters can be calculated easily with only three physical properties of sediments such as porosity, density and grain size using GS model rather than the BSM.

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Mr C.P. Uthaman obtained his MSc (Geology) from Kannur University. Presently working as Technical Officer 'B' at DRDO-Naval Physical and Oceanographic Laboratory, Kochi. He is working in the field of marine geology and geophysics, he is involved in data collection, processing and analysing the same. His areas of interest include sea bottom characterisation using various models.
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In the current study, he has carried out the estimation of marine geophysical parameters using sediment velocimeter and prediction using Biot-Stoll model and reviewing the manuscript.