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Ballistic Limit Estimation Approaches for Ballistic Resistance Assessment

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ABSTRACT

The armour technologist conducts ballistic impact testing either for evaluating armour materials and systems or for studying material's defeating mechanism. Most standards make use of the ballistic limit velocity for ballistic assessment. This is the bullet impact velocity that leads to the protection perforation in 50 per cent of the cases. Various models have been emerged to estimate this key metric. The present article summarises the popular models developed for ballistic limit estimation. An attempt is made to point out models' strength and weakness. First, the experimental set-up used for that goal is displayed. Next, a concise overview of ballistic limit estimation methods is presented. Lastly, a discussion is dedicated to model's comparison and analysis. This literature survey reveals that the main drawback of already existing methods is that they are purely statistical. Moreover, existing methods are based on the normality assumption of perforation velocities which tends from -infinity to infinity. The main conclusion of this survey is that the presented methods offer a comparable accuracy in estimating the ballistic limit velocity. However, a given variability is remarked when extreme values estimation is of interest, impact velocities leading to low and high perforation probability. Finally, existing models' performances decay with the reduction of the experimental sample size which represent a constraining requirement in ballistic resistance assessment.

Keyword: Ballistic resistance; Ballistic limit; Normal law; Perforation probability

1. INTRODUCTION

Ballistic experiments are performed to observe the response of a given protection structure subjected to impacts of a specific bullet. Reasonably, the protection wearer is interested in the impact velocity of the potential threat that will never penetrate the protection, the maximum impact velocity with zero perforation probability. Given the complexity and the cost of estimating this critical velocity, assessment standards and technical specification refer to a proofing velocity or ballistic limit velocity for protection resistance evaluation. The proof testing requires the protection resistance to the intended bullet impact velocity for a given number of the total conducted tests while ballistic limit testing estimates the bullet velocity that causes the target perforation in 50 per cent of cases. In the beginning, researchers were interested in the estimation of the V_{50} velocity only. Due to the need to satisfy high precision requirements, researchers manifested the interest in estimating the complete curve of the perforation probability.

Given the risk factor on human life, the end-user of ballistic protections imposes increasing requirements on the accuracy in the evaluation of materials resistance to ballistic threat impacts. Thus, a growing interest is addressed to ballistic resistance assessment approaches. Several methods have been introduced for ballistic limit estimation or the complete characterisation of the perforation probability curve. Different papers[1-4] and books[5-7] have addressed this topic and analysed these methods. However, no recent works have summarised the reached conclusions about accuracy performance of available ballistic limit estimation approaches.

In this paper, a critical and concise overview of existing ballistic limit assessment techniques will be presented. The goal is to compare available methods accuracy in estimating the V_{50} velocity and the perforation probability curve. Moreover, methods advantages and drawbacks are denoted to point out the need for further researches and investigations on this topic. To reach that, first ballistic testing experiments are introduced. Later, a concise summary of existing methods implemented to evaluate the ballistic limit metric of a protection system is given. Lastly, a discussion is dedicated to the analysis of the performance of the various existing approach in this field.

2. TEST CONCEPT AND TERMINOLOGY

2.1 Experimental Test Set-up

Figure 1 (adapted from[8]) illustrates the typical experimental set-up used in the framework of terminal ballistic measurements. It consists of a launcher, the ammunition/bullet, a velocity measurement equipment, the target and possibly a high-speed camera. The shooting range between the launcher muzzle and the target may vary from 5 m (handguns) to 10 m (rifles and shotguns)[9] which is the minimum limit distance essential for the bullet stability to be achieved.

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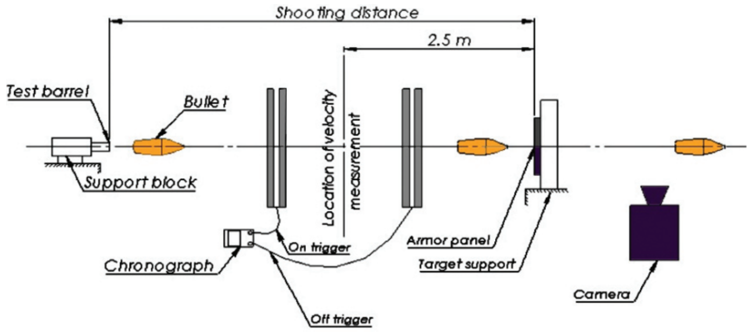

Figure 1. Experimental set-up for ballistic resistance measurements.

- The launcher is the mechanical device adopted to propel the striking projectile at the desired impact velocity. The system is composed of a chamber to contain the acting gases, a barrel to guide the projectile acceleration and a breech to close the system rear face. Powder and gas guns are the two laboratory launchers available to the experimenter to cover the wide range of impact velocity testing situations. The first is a receiver of a complete ammunition while the second simply releases high-pressurised gas.
- The velocity measurement facility is formed of a double optical trigger screens and an electronic time counters installed on the bullet path. The bullet passage through the first light windows activates the electronic chronograph while its exit from the second screen triggers the stop of the time counter. Known the distance separating the light screens and the corresponding projectile travel time, one can calculate the bullet longitudinal velocity (average) at the middle of the double screen base. The barriers midpoint is situated at 2.5 m from the target to measure the bullet impact velocity just before hitting the target. Usually, two chronographs and pairs of screens are used to cross-check the measurement accuracy.
- The target is mounted vertically on a rigid frame to ensure a correct positioning of the target repetitively. The normal impact condition is fulfilled assuming that the projectile trajectory is co-linear with the horizontal firing direction. Finally, the impact locations on the target are distributed according to specific schemes with a minimal prescribed (stipulated within standards) distance between impact locations and plate edges to prevent interaction with previous impacts and edges reflexions.
- The high-speed camera is one of the valuable imaging tools employed in ballistic measurements (in addition to flash radiography and X-ray shadowgraphy techniques). It is placed perpendicular to the bullet path in plane with the target in order to record the entire impact event before, during and after the bullet/target interaction. Firstly, real-time observation serves to the enhancement of the actual knowledge about damage mechanisms. Secondly, the post-treatment of the captured images provides measurements of the displacement of the penetrator relative to the penetration time, the bullet impact/residual velocity, the impact angles (two cameras are required for a 3D description) and the impact process duration. The measurement accuracy depends on the spatial calibration (linking lab space coordinates to the camera image space) and the camera alignment. Finally, a sufficient luminance (flash leds) is recommended to obtain useful images.

2.2 Data Processing

Ballistic limit estimation is the most used technique for ballistic performance assessment. The destructive character of ballistic impact events motivated its adoption from the concept of sensitivity experiments. It is a test-to-failure experiment where the tested equipment is subjected to an increased level of a given stimulus to detect the transition of the equipment performance from no-response to response. Dealing with problem in focus, the projectile strikes the armour system at a given impact velocity (V_i) which constitutes the loading level and upon impact, there is two outcomes that indicate the armour damage, either perforation or no perforation. Accordingly, at each impact velocity V_i, the armour behaviour is coded as a binary outcome U where $U = 0$ if perforation takes place (target failure) and $U = 1$ if not (target resistance). Ideally, there is a threshold impact velocity associated with the target behaviour transition from resistance to failure. However, in reality, there is a zone of mixed results which is limited by two critical impact velocities. To characterize this zone, a sensitivity testing methodology is used to obtain the response function of the tested armour system, as a function of the applied stress induced by the impacting bullet.

Figure 2 displays the binary response (U) of a given tested armour system as a function of the bullet impact velocity (V_i) for a preliminary visual control of the obtained results. It can be experimentally verified that there is a range of impact velocities within which the target response is switching between 0 (failure) and 1 (resistance). On the one hand, the lower limit of the mixed results zone denotes the highest impact velocity at which the bullet repeatedly fails to perforate the armour system. By using a probabilistic approach, this bullet impact velocity, V_0, defines the impact velocity corresponding to zero percent probability of perforation of the impacted target. On the other hand, the upper limit indicates the lowest impact velocity with which the striking bullet constantly perforates the protection. This bullet velocity is recognised as the V_{100} velocity to identify the impact velocity that gives 100 percent probability of target perforation. Intuitively, this

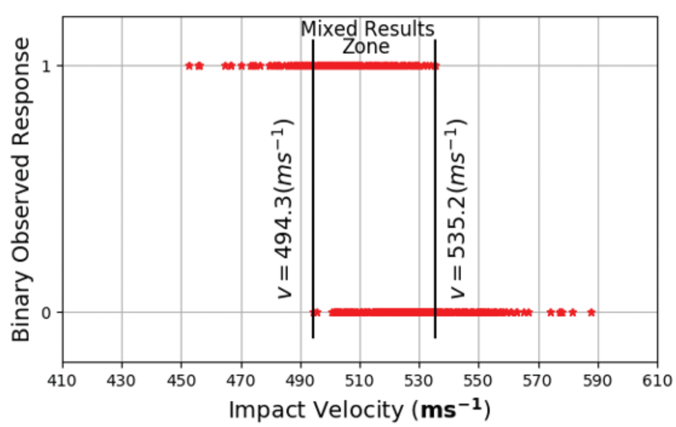

Figure 2 . Armor response versus impact velocity.

testing concept adopts, as a measure of the ballistic resistance for a given structure/bullet combination, the estimation of the perforation probability of the tested system against the bullet impact velocity.

In a series of n impacts at a given impact velocity, the target response follows the binomial distribution $B(n,p)$ corresponding to n Bernouilli experiments where p is the relative perforation probability. In view of the difficulty to control the impact velocity, the experimental results of a total size (N) are grouped into classes (k=1,2,...,s) of impact velocity to form s samples of n_k trials. Thus, the perforation probability (P_k) is estimated, using the relative perforation frequency (F_{n_k}), by dividing the number of perforations (n_{0_k}) observed in the class (k) by the sample class size (n_k). Subsequently, Fig. 2 is modified to draw the perforation probability curve of the tested target versus the striking bullet velocity as is illustrated in Fig. 3. The green points indicate the experimental estimations (V_k , P_k). To generate the perforation probability curve P_{V_i} , one can interpolate between these points. But there are different possible ways to obtain the interpolation curve as shown with the red curves. This points the first difficulty in the analysis of ballistic limit testing results.

It is schematically shown that the probability curve P_{V_i} is bounded by the V_0 and V_{100} velocities where the target perforation probability has undoubtedly (or theoretically) to increase from zero, corresponding to the point (V_2 , P_2) in Fig. 3, to one, corresponding to the point (V_6 , P_6) by rising the bullet velocity. Nevertheless, giving the current challenges in predicting these two measures, the ballistic limit metric (V_{50}) is rather used to characterise the probabilistic character of the ballistic resistance of armour system. The V_{50} is the bullet velocity that causes the target perforation in 50 percent of the cases.

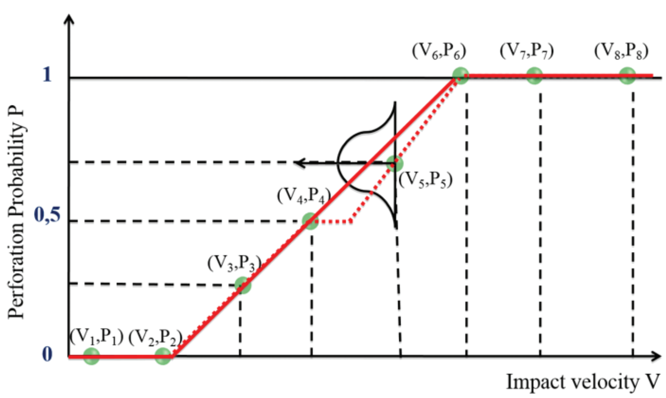

Figure 3. Perforation probability versus impact velocity.

Different methods appeared in literature to estimate the V_{50} velocity. Nevertheless, there is an increasing demand for estimating the entire curve of the perforation probability. This led to models based on assumptions regarding the perforation velocity distribution. The next section aims at introducing these different modelling methods and comparing them.

3. BALLISTIC LIMIT ESTIMATION METHODS

Several approaches have been proposed for the V_{50} estimation over the years with an increasing interest at accurately estimating low and high extreme values of the perforation probability. The present section goal is to explain the theoretical background of the most practiced methods.

3.1 Up-and-Down Method

The Up-and-Down procedure has been fundamentally developed for the examination of explosive sensitivity to shock[1]. In the current research field, the protection structure sample to be tested is subjected to sequential levels of the bullet impact velocity. The experimenter chooses an initial testing impact velocity V_i . If the target is perforated, then the impact velocity is lowered for the next specimen test otherwise a higher speed is selected for the impacting bullet. In that case, the testing process targets the neighbourhood of the mean in an attempt to converge to the V_{50} . The experimental test ends when an equal number of perforations and non-perforations events occurs within a given margin of the bullet speed. A point estimate of V_{50} is computed using the arithmetic mean of the velocities related to the chosen observations (the group of equal perforations and non-perforations). This calculation of the mean uses a limited number of observations without taking into account all there corded results. Furthermore, no extra information is derived, from the supplied experimental database, to model the variability of the armour response and the spread of the data around the mean behaviour V_{50} . This method is broadly employed because it demonstrated an acceptable accuracy when applied to a sample of limited size in addition to its simplicity. However, its precision is conditioned by three hypotheses. The observed variable has to follow a normal distribution. The sample must at least contains 50 items to be representative of the complete population. Finally, it is required to have a prior estimate of the dispersion around V_{50} to optimise the sequential selection of test speeds.

3.2 Langlie Method

Langlie method[2] originally developed for items resistance testing. It is a sequential process too where solicitation velocities have to be selected based on the perceived response of the armour (perforation or not). The obtained database is a collection of N couples (U_j , V_{ij}) where U_j is the binary outcome and V_{ij} is the impact velocity of the j^{th} trial. It has to satisfy three criteria for a valid application of the proposed methodology. First, a mixed zone results must appear during the tests otherwise there will be no variability to model. Second, the lowest impact velocity corresponds to a non-perforation while it is necessary that the highest one gives a perforation in an attempt to cover the entire range of the target response. Next, the data collection procedure requires firing on beforehand mathematically computed stress levels based on the analysis of the previously gathered results. Indeed, the $n^{th}+1$ velocity is the average of V_{in} and V_{ip} where V_{in} is the last recorded impact velocity and V_{ip} is the p^{th} impact velocity which guarantees that the sub-database {n, n-1, ...,p} contains an equal number of perforations and no-perforations. This is one of the main shortcomings of this method, given the challenge of experimentally adjusting these imperative velocities which produces useless shots. Adapted to the current application, Langlie presumes that for a given stress level of impact velocity V_i, the resulting failure (resistance) implies that the tested item critical level V_i^c is lower (higher) than the selected stressing level V_i. Using this equivalence, the perforation probability (target failure) is mathematically computed as follows:

$$\begin{cases} P(U=0|V_i)=P(V_i^c \le V_i) & \to \textit{in case of perforation.} \\ P(U=1|V_i)=P(V_i^c > V_i) & \to \textit{in case of no perforation.} \end{cases} \quad (1)$$

Furthermore, Langlie assumes that the critical impact velocity of the armour system is a normally distributed random variable $V_i^c \sim N(\mu,\sigma^2)$. The mean velocity μ is the critical velocity that leads to the armour response (perforation or failure) in 50 percent of the cases which is nothing else than the V_{50} ballistic limit. However, the standard deviation σ describes the variability of the armour response relative to the ballistic limit. Then, for a given solicitation V_i, the probability that the random variable V_i^c is inferior than the observation level V_i is equal to:

$$P(U=0|V_i)=P(V_i^c \le V_i)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{V_i} exp^{\frac{(V_i^c-\mu)^2}{2\sigma^2}}\, dV_i^c \quad (2)$$

The maximum likelihood inference is implemented for the estimation of the population parameters. The probability $(\hat{\mu},\hat{\sigma})$. to get the observed sample (N couples U_j and V_{ij}. where j designates the j^{th} trial results) is equal to the products of the items outcome probability which is expressed in terms of the first two moment of the assumed normal law (the estimators $\hat{\mu}$ and $\hat{\sigma}$) as :

$$L(\hat{\mu},\hat{\sigma})=\prod_{j=1}^{N}P(U_j|V_{ij}) \quad (3)$$

where

$$P(U_j|V_{ij})=U_j\,(1-P(U_j=0|V_{ij}))+(1-U_j)P(U_j=0|V_{ij}) \quad (4)$$

with $P(U_j|V_{ij})$ is calculated using the equation 2. On the one hand, $\hat{\mu}$ is the estimator of the ballistic limit velocity. On the other hand, the standard deviation $\hat{\sigma}$ characterises the response dispersion. Based on the normality assumption, the estimation of any percentile of interest V_p is given by[10]:

$$V_p = V_{50} - Z_p\hat{\sigma} \quad (5)$$

where V_p is the impact velocity corresponding to p% of perforation probability and Z_p is the equivalent quantile of the standard normal distribution.

3.3 Kneubuehl method (VPAM-KNB)

Kneubuehl[3] applies the theory of descriptive statistics and probability. He considers that the armour response $P(V_i)$ is the cumulative distribution function (CDF) of the bullet impact velocity (the random variable) that causes the target perforation. Although the velocity V_i is theoretically a continuous random variable, Kneubhuel implements the previously explained classes concept for the discretely sampled experimental database as explained in the third paragraph of section 2.2. Hence, the central tendency and dispersion of the perforation velocity distribution may be estimated using the statistical mean (V_m) and standard deviation (σ) as follows, respectively:

$$V_m = \sum_{k=1}^{s} v_k f_{n_k} \quad (6)$$

and,

$$\sigma^2 = \sum_{k=1}^{s} (v_k - V_m)^2 f_{n_k} \quad (7)$$

where v_k is the mid-value of the class k. And, f_{n_k} is the value of the probability density function related to the perforation velocity distribution at the impact velocity v_k. The parameter f_{n_k} is calculated via the difference $F_k - F_{k-1}$ where F_k and F_{k-1} are the relative perforation frequency estimated using class k and class $k-1$ observations. The other important parameter besides the estimator V_m of the ballistic limit V_{50} is the distribution standard deviation σ that models the armour response variability. By making an assumption on the distribution law (and generally a normal distribution is assumed), this method enables the computation of the perforation probability at any impact velocity and inversely throughout equation 5 under the normal law assumption. Unfortunately, the fulfillment of additional requirements, besides the first two ones specified by Langlie, regarding the number and position of empty classes across the recorded velocities constrains the accuracy of the results. Accordingly, V_m and σ estimations depend highly on the classes number or width choice. Instead of the need for the precise control of solicitation velocities values, the inspection of the stress levels concentration over the expected response range is recommended.

3.4 Probit, Logit and C-log-log Methods

Probit, Logit and C-log-log are all linear regression models that belong to the class of generalised linear models. These are the models to use if we seek to analyse the relationship between a non-continuous response variable (binary or multi-level outcomes) and a given set of independent variables. In the present case, the question is how to characterise the dependence of the discrete armour response variable ($U=0$ or $U=1$) on the bullet impact velocity V_i. Since the conditional mean of U given V_i is equal to $\sum_{i=1}^{n_k} U/n_k$ which is the conditional perforation probability $P(U=0|V_i)=P(V_i)$ (the mean of a Bernoulli distribution). This conditional perforation probability P (for a lighter notation) is considered as the continuous independent variable to be modelled. Given that the probability belongs to the interval [0,1], a transformation function that maps [0,1] to R is needed to apply the linear regression. Interested reader is referred to Tang[11], *et al.* book for further details. Different functions can be used for that purpose. The most known are Probit[4], Logit, and complementary-log-log transformation functions (g) that link the conditional probability P to a linear predictor. The corresponding expressions can be written as follows:

$$\begin{cases} \textit{Probit}: & g(P)=\Phi^{-1}(P)=\alpha+\beta V_i \\ \textit{Logit}: & g(P)=\log\left(\dfrac{P}{1-P}\right)=\alpha+\beta V_i \\ \textit{Complementary-log-log}: & g(P)=\log(-\log(1-P))=\alpha+\beta V_i \end{cases} \tag{8}$$

where α and β are the regression parameters to be inferred and (Φ^{-1}) is the inverse of the CDF of the standard normal law N(0,1). Usually, the maximum likelihood inference is developed in order to estimate the models' parameters. Again, the likelihood (L) of a sample N is expressed as follows:

$$L(\alpha,\beta)=\prod_{j=1}^{N}P(U_j=0|V_{ij})^{(1-U_j)}\left(1-P(U_j=0|V_{ij})\right)^{U_j} \tag{9}$$

where the perforation probability $P(V_i)=P(U=0|V_i)$ depends on α and β through the inverse (g^{-1}) of the transformation function (g), displayed in the system 8, evaluated at $\alpha+\beta V_i$ like :

$$\begin{cases} \textit{Probit}: & P(V_i)=g^{-1}(\alpha+\beta V_i)=\Phi(\alpha+\beta V_i) \\ \textit{Logit}: & P(V_i)=g^{-1}(\alpha+\beta V_i)=\dfrac{1}{1+e^{-(\alpha+\beta V_i)}} \\ \textit{Complementary-log-log}: & P(V_i)=g^{-1}(\alpha+\beta V_i)=1-e^{-e^{(\alpha+\beta V_i)}} \end{cases} \tag{10}$$

The derivative of the likelihood function set to zero leads to the determination of the maxima which correspond to the parameter's estimators ($\hat{\alpha}$, $\hat{\beta}$) of the Probit, logistic or complementary-log-log regression models. Further, remembering that the CDF of the normal for the Probit method, logistic for the logit method and Gumbel for the c-log-log method distributions are equal to:

$$\begin{cases} \textit{Normal}, \; V_i \sim N(\mu,\sigma): & P(V_i)=\Phi\left(\dfrac{V_i-\mu}{\sigma}\right): \\ \textit{Logistic}, \; V_i \sim \textit{Logistic}(\mu,\sigma): & P(V_i)=\dfrac{1}{1+e^{-\left(\frac{V_i-\mu}{\sigma}\right)}} \\ \textit{Gumbel}, \; V_i \sim \textit{Gumbel}(\mu,\sigma): & P(V_i)=g^{-1}(\alpha+\beta V_i)=1-e^{-e^{-\left(\frac{V_i-\mu}{\sigma}\right)}} \end{cases} \tag{11}$$

where μ and σ are the characteristic parameters of the related distribution central trend and dispersion. Thus, it is noted by comparing the equations of system 11to the ones in 10that :

$$\begin{cases} \hat{\mu}=\dfrac{-\hat{\alpha}}{\hat{\beta}} \\ \hat{\sigma}=\dfrac{1}{\hat{\beta}} \end{cases} \tag{12}$$

Accordingly, these Probit, logit and complementary-log-log regression models are based on the assumption of a normal, logistic and Gumbel distribution of the velocities leading to a given probability of perforation, respectively. Therefore, the maximum likelihood inference conducted, in this analysis and under these assumptions, serves to characterise the corresponding assumed distributions. In addition, any quantile V_p of interest may be estimated using the quantile function (the inverse of the CDF) of these three distributions, respectively:

$$\begin{cases} \textit{Probit}: & V_p=\hat{\mu}+\hat{\sigma}\Phi^{-1}(P)=\hat{\mu}+\hat{\sigma}Z_p \\ \textit{Logit}: & V_p=\hat{\mu}+\hat{\sigma}\log\left(\dfrac{P}{1-P}\right) \\ \textit{Complementary-log-log}: & V_p=\hat{\mu}+\hat{\sigma} \end{cases} \tag{13}$$

Applying this calculation to the particular value of the V_{50}, which is the 0.5 quantile of interest, gives:

$$\begin{cases} \textit{Probit}: & V_{50}=\hat{\mu} \\ \textit{Logit}: & V_{50}=\hat{\mu} \\ \textit{Complementary-log-log}: & V_{50}=\hat{\mu}+\hat{\sigma}\log(-\log(0.5)) \end{cases} \tag{14}$$

In addition, the armour response variability is characterised by the distribution dispersion $\hat{\sigma}$.

3.5 Brownian Motion-based Approach

Recently, the Brownian motion-based approach[12]is introduced for ballistic resistance evaluation. The approach followed here makes use of the Newton's second law to model the one-dimensional impact phenomena. It's expressed in the form of a set of differential equations which gives the complete motion of the penetrating projectile in the protection structure. To handle the observed stochastic behaviour (the variability of the system response) the stochastic differential equation (SDE) form is used as follows:

$$\begin{cases} dV(t)=a(t,V)dt+\sigma(t,V)dW(t) \\ dX(t)=V(t)dt \\ X(0)=0 \quad \textit{and} \quad V(0)=V_i \end{cases} \tag{15}$$

where $a(t,V)$, $V(t)$ and $X(t)$ are, respectively, the instantaneous bullet deceleration, velocity and position along the through-thickness direction (normal impact). The bullet initial condition is defined by the bullet impact velocity V_i and the bullet initial position X_0 which is equal to zero at the plate frontal face as the bullet just enters in contact with it. Finally, $\sigma(t,V)$is the diffusion coefficient and $W(t)$ is the Wiener (Brownian) process which are intended to reproduce the observed variability of the system response. The Eqn system 15 may be discretised using the Euler-Maruyama scheme as follows:

$$\begin{cases} V(t_j)=V(t_{j-1})+a(t_{j-1},V_{j-1})\Delta t+\sigma(t_{j-1},V_{j-1})\Delta W_{t_{j-1}}^{t_j} \\ X(t_j)=X(t_{j-1})+V(t_{j-1})\Delta t+\dfrac{a(t_{j-1},V_{j-1})}{2}\Delta t^2+\sigma(t_{j-1},V_{j-1})\Delta W_{t_{j-1}}^{t_j}\Delta t \end{cases} \tag{16}$$

Then, a prediction of the bullet trajectory within the target is computed for each Brownian path $W(t)$. In addition, the perforation occurrence of the tested target is identified by a bullet residual velocity different from zero at the exit moment.

The challenging problem of this modelling approach is in the determination of the model parameters ($a(t,V)$, $\sigma(t,V)$) that represent the stochastic nature of the collected experimental database. Chi-square and Kolmogorov-Smirnov inference tools are adopted for model parameters estimation when the system response is coded by a collection of binary outcomes as a function of the bullet impact velocities (U, V_i). The key idea is to use the proposed stochastic model to numerically generate an equivalent sample (simply stated the numerical sample) using the same impact velocity classes of the experimentally observed results. The model reproduces the observed variability if it passes the null hypothesis that simulated data (by the assumed mathematical model) and experimental data come from the same underlying population. Recalling the ballistic limit testing concept and terminology, the conducted tests, of a given threat/armour system, provide a collection of (U, V_i) measurements that may be grouped into s classes of impact velocities. The criterion function of the Chi-square goodness-of-fit tests, of the experimental and numerical sample, can then be expressed as follows:

$$\chi^2_{s-1} = \sum_{k=1}^{s} \frac{\left(n_{0_k} - n_k F_{k,\exp}\right)^2}{n_k F_{k,\exp}} \tag{17}$$

where n_{0_k} is the number of perforations that occur in the k^{th} class over the total number, n_k^{th}, of numerical trials in that class. Finally, $F_{k,exp}$ is the k^{th} experimentally estimated probability of perforation, and χ^2_{s-1} is a Chi-square random variable with $s-1$ degrees of freedom. Analogously, the Kolmogorov-Smirnov test can be brought into play by computing the test statistic based on the difference between the simulated and experimental frequency:

$$F = maximum\left|F_{k,exp} - F_{k,sim}\right| \tag{18}$$

where $F_{k,sim}$ is the k^{th} simulated probability of perforation. The goal is to determine the values of a and σ that minimise the test statistics and F. Accordingly, the perforation probability of the tested bullet/target system can be predicted for each impact velocity V_i.

4. COMPARISON AND DISCUSSION

In view of the various modelling approach, it is required to identify the most reliable model for the ballistic resistance assessment of protection structures. In the literature, an extensive effort has been devoted to the analysis and comparison of existing methods performances.

Under the normality assumption of the perforation velocities, Maldague[13] compared the Langlie, Kneubuehl, Probit and up-and-down methods accuracy and reproducibility using Monte-Carlo simulations. He concluded that these methods have a comparable accuracy in estimating the V_{50}. Moreover, he highlighted the improvement of the estimation repeatability with larger sample size except for the up-and-down which shows a constant trend. Maldague[13]deduced that the up-and-down method is the most accurate and repeatable for estimating the V_{50}, especially with small samples, albeit it does not provide an estimation of σ. The estimation of the distribution standard deviation $\hat{\sigma}$ is gaining much interest from the user's perspective to quantify the armour response range. Regarding this second moment estimation, Maldague[13]concluded that Langlie and Probit methods have a better accuracy than the Kneubuehl one while the latter presents a better reproducibility of the estimation. Later, Maldague[10]inspected the validity of the normality assumption for modelling the distribution of the armour system response and particularly for estimating the extreme values V_1 and V_{99}. To achieve the desired goal, an experimental sample of almost 581 impacts has been collected. Again, he confirmed that the Kneubuehl, Probit, Weibull and classical histogram methods yield comparable predictions of the V_{50}. Whereas, despite the remarkable size of the sample, these methods present a certain variability in predicting the two extreme values V_1 and V_{99}. The Pearson and binomial tests were conducted to examine the adequacy of the normality hypothesis, of the perforation velocities for the tested bullet/plate. The obtained results support the acceptance of this assumption. Nevertheless, a close look to the dispersion of the impact velocities reveals that only 100 impacts out of the 581 total results are covering the extreme range of low and high probabilities of perforation. If more data were located in the distribution tails, the question that needed further investigations would have been whether the normality assumption remains valid or not. Moreover, Maldague[10] showed that the estimation accuracy of V_1 and V_{99} deteriorates considerably as the sample size decreases. Then, the effect of this assumption acceptance (correct/incorrect decision) on the accuracy of the ballistic resistance estimation is an ongoing problem of concern.

In the same context, Mauchant[14]examined the Probit, Logit, and C-log-log regression models to identify the most appropriate model for armour resistance evaluation. He concluded that the Probit and logit models led to a comparable estimation of the V_{50} due to the similarities between the logistic and the normal distribution (symmetrical and asymptotical behaviour between 0 and 1). The little-noticed difference between the two models arises from the rate at which the normal and logistic distribution tends to the asymptotes 0 and 1. The immediate consequence of the distributions tails divergence, heavier for logistic distribution, is that the logit extreme values estimation differs slightly from the Probit ones. Conversely, the complementary log-log regression produces large variation in V_1, V_{99}, and even V_{50} estimation compared to the logit and Probit estimates. This is principally due to the asymmetrical shape of the Gumbel distribution used to catch the ballistic response variability. To sum up, Mauchant[14]concluded that those three regression models provide an equivalent estimation of the V_{50} while discrepancies emerge when estimating extreme values. But, the impact velocities of low and high perforation probability are of increasing concern during the evaluation of a given protection structure performances. For this reason, even though these differences are not significant and usually the three models are indistinguishable, Mauchant[14]recommended to apply the three regression models and find the one that best fit the observed data.

Johnson[15]study focused on analysing Up and Down (UD), Langlie, Delayed Robbins Monroe (DRM), Robbins Monroe Joseph (RMJ), Neyers, three-phase approach (3POD), and K-in-a-Row (KR) models. All of them are sequential designs with specific algorithms for optimising the impact velocities selection. Using Monte-Carlo simulations.

Johnson[15] investigated the sensitivity of V_{50} and V_{10} estimates (accuracy and precision) to choices of the method, the test stopping criteria, the standard deviation σ of the true distribution, the parameters estimator, and the guesstimates of

V_{50} and σ. Regarding the method choice, he showed that the V_{50} accuracy decays using the UD and DRM methods with bad guesses of V_{50} and σ while they yield a better precision of the provided estimate compared with the other methods. However, the Neyer and 3POD methods exhibited the best accuracy in predicting the V_{50}. Furthermore, the 3POD estimation proved to be lesser sensitive to true and prior guess of the distribution parameters. In all cases, Johnson[15]demonstrated that the accuracy and precision of the V_{50}. Estimation deteriorate by increasing the dispersion of the system response (true σ). In addition, he asserted that the tested methods fail to estimate the V_{10} and V_{50} velocities with the same accuracy (minimum bias to the true values). This results from the particular design of each sequential process which locates the impacts velocities either near the V_{10} or V_{50} velocity. Therefore, these techniques operate by converging towards the quantile of interest which make the estimation of all quantiles with the same precision unachievable with these techniques. Lastly, Johnson[15]favored the maximum likelihood estimator than the arithmetic mean one given that it allows the establishment of the whole perforation probability curve.

The Brownian motion-based approach is a newly developed tool that didn't receive reviewer critics yet. Tahenti[16] analysed the model performance in relation to the hypothesis of the model's parameters constancy. It was noted that the model results are in good agreement with the experimental results even under this restrictive hypothesis. In addition, it was pointed out that better characterisation of the model parameters, indeed the impact velocity effect, may improve the quality of agreement between the experimental and model results. However, the inspection of the presently proposed inference method shows that its application imposes the need for a large experimental database. Effectively, a reliable estimation of the probability of perforation at a given impact velocity (or class), as displayed in Eqns 1 and 2, requires a minimal number of observations per class of impact velocities k. Therefore, future work should address the methodology improvements to extend its implementation on a small sample size.

The question that has to be raised at this stage concerns the reasons for this significant interest for precise estimation of the perforation probability curve of a given system. In fact, an armour optimisation based on an erroneous estimation of the armour response leads either to a under-engineered product directly linked to a human life security problem or an over-engineered product directly connected to an extra weight to the soldier's load. Accordingly, this explanation shows the impact of armour performance estimation using the ballistic limit testing method on the final product. Therefore, even if the normality assumption of perforation probability is accepted, it is still of primordial importance to search new methods for perforation probability estimation in the seek of reaching a better accuracy. Then, the ballistic resistance assessment methods, and especially the ballistic limit testing methods, is still an active query of research for the armour designer.

5. CONCLUSION

Previous sections have detailed the different approaches adopted for ballistic limit estimation. Literature review for methods performance comparison showed a comparable accuracy in estimating the ballistic limit velocity. However, low and high perforation velocities estimations exhibit a given variability and uncertainty. In addition to these discrepancies, these statistical methods are fitting methods to the normal law distribution which limits their improvement alternatives. To explain, these modelling approaches omit the consideration of new research knowledge on the physics of the system behaviour. Thus, the accuracy improvement of perforation probability estimation for a given impact velocity imposes the exploration of new routes for this problem modelling. The work presented validates the limitation of ballistic limit estimation approaches in predicting the low chances of perforation and non-perforation especially under sample size limitation. This invites researchers to explore different stochastic modelling tools to identify a new methodology for ballistic resistance estimation based on the tested system observed results.

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