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System Reliability Estimation of Divert Attitude Control System of a Launch Vehicle using Bayesian Networks

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ABSTRACT

Divert attitude and control system (DACS) is a one-shot system and provides attitude correction and translation of the Launch vehicle. DACS consists of many flight critical sub systems which are arranged in a series configuration. The traditional Reliability block diagram and Fault tree diagram methods are unsuitable for reliability modelling, when considering uncertainty among the components and system. Bayesian network is the natural choice to model dependencies among the components and system. DACS being one shot system, it is very expensive and time consuming to test more number of systems during the design and development. Hence the data is drawn from component level, subsystem level and expert opinion is used for reliability estimation. In this paper, Bayesian network modelling of DAC system was carried out for estimating the reliability using multi-level data. An algorithm is developed for computation of Conditional probabilities in Bayesian network. Posterior probability distribution of components is calculated using Markov Chain Monte Carlo (MCMC) simulations and results are compared with Junction tree based exact inference algorithm. MATLAB code is developed to estimate the reliability of DAC system.

Keywords: Propulsion system; Reliability; Bayesian networks; MCMC; Weighted sum algorithm; Conditional probability

NOMENCLATURE

AB	Air bottle
PV	Pyro valve
PR	Pressure regulator
FC	Fuel chamber
BDV	Bust diaphragm valve
LT	Large thrusters
MT	Medium thrusters
ST	Small thrusters

1. INTRODUCTION

Divert attitude control system is a quick reaction system which provides attitude control and precise positioning of pay load of military and space launch vehicles. It is a liquid propellant based system, where in liquid bi-propellants thrusters are used to generate the thrust. Thrusters provide roll, pitch and yaw control of the flight vehicle and act in different combinations using short, forceful pulses. It is a high precision, a light weight system and consists of many flight critical sub systems, which are arranged in series configuration. All sub systems must function correctly in flight; otherwise there is a risk to the mission and its payload. To ensure the failure free operation of liquid propulsion system much attention needs to be paid for reliability in the early stage of design and development. Initially reliability predicted is based on historical data, expert knowledge and simulation results. Later system Reliability is estimated quantitatively using field data and system test data and this value is compared with predicted reliability**.** Conducting full system tests is difficult due to cost and time constraints. This situation calls for a method to develop reliability models for complex systems and to integrate all available information for predicting system reliability. Wilson[1], *et al.* presented approaches for combining of multiple sources of information to predict the reliability of a component. Further they developed approaches for system reliability calculation by combining multiple levels of data. Wilson[2], *et al.* developed reliability models using Bayesian network for combining multiple level data addresses the concern of complex system reliability estimation when limited full test data is available. Wilson[3], *et al.* carried out a detailed review for assessing the reliability of military sub systems using Bayesian approach when limited test data is available. Methodology demonstrated with many types of data like, historical data, information from similar systems and computer simulated test data.

In design and development phase, system developers have limited knowledge about the system and are not aware of its behaviour with operating environment. Therefore, interaction between the system and its components is to be studied to learn about the failure modes. Same to be modelled with suitable reliability models. The existing reliability modelling tools like Reliability Block diagrams and Fault Trees are used for reliability modelling of simple systems and inadequate in analysing more complex systems. In Fault tree analysis, events

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are assumed to be statistically independent, modelled using AND or OR logic gates and dependencies among events are not considered. Hamada[4], *et al.* has modelled a special case of fault tree using multilevel binary data. Bayesian networks (BN) is widely used to model dependency relationship between the components and system and used to solve the many real-world problems. This feature will be helpful especially during the early stage of product's design process when the limited knowledge about system is available. Another important advantage of BN over the traditional approach is its ability of combining information from different sources. Sources of data are Acceptance test and Qualification test data, maintenance data of similar system can also be used. In addition to above, available expert knowledge, simulation results at component, sub system and system level can be considered. Guo[5], *et al.* had modelled a methodology to calculate the reliability of the system with different types of information Real challenge is to combine these data available at different levels to learn about the reliability of major system. Zhai[6], *et al*. described the advantages of Bayesian networks in reliability analysis like, flexibility in modelling framework, strict mathematical formula derivation, a precise reasoning. Liu[7], *et al.* estimated prior probabilities by synthesising the expert opinion. This data can be used to determine system level reliability estimation, which signifies the importance of multi-level information integration. Das[8], proposed an algorithm to find the conditional probabilities using compatible parental configurations.

Inference in BN model is carried out by exact inference and approximate inference methods. Markov Chain Monte-Carlo simulations (MCMC) are used for performing approximate inference in Bayesian networks. MCMC algorithm draw samples from the targeted joint posterior distribution of model parameters. To implement MCMC, a Win-BUGS Statistical software[9], is used. Exact inference in Bayesian Networks is carried out by Junction Tree Algorithm (JTA)[10]. In exact inference, nodes are connected and the form of junction-tree representation. Inference in JTA is carried out in moralisation, triangulation and message passing steps.

The objective of present work is to develop a suitable methodology for reliability estimation of divert attitude control system (DACS) considering the dependencies between the components and the system with multi-level data. Dependency between components to be elicited based on expert opinion and sub system conditional probabilities are not directly available. Computation of conditional probabilities for complex system like a DACS is a complicated process by expert opinion alone. Weighted sum algorithm is used for finding the conditional probabilities of all the possible combinations. Bayesian Inference in DACS BN model is carried out using Junction Tree Algorithm and reliability is estimated.

2. BAYESIAN NETWORKS FOR SYSTEM RELIABILITY

In recent years Bayesian networks have been increasingly employed in a wide range of applications including bio informatics, computer science, etc. The basic fundamental of Bayesian network and Bayesian interface is based on Bayes theorem which updates probabilities based on new information.

Bayesian network is a direct acyclic graph (DAG) which represents the relationship between the set of random variables and their conditional dependencies. In the DAG, every node represents a random variable, arcs between these nodes represents the dependencies among variables. In BN, nodes with arrows directed towards them are called child nodes, and the nodes without any arrows are called parent node. The parent node is characterised with marginal probabilities and the child nodes have a conditional probability distribution associated with it. Bayesian network which consists of four nodes A_1, A_2, A_3, and A_4 as shown in Fig. 1. The joint probability distribution of Bayesian network is as given by the Eqn. (1).

$$\Pr(A_1, A_2, A_3, A_4) = \Pr(A_1)\Pr(A_2) \Pr(A_3 / A_2, A_1)\Pr(A_4 / A_3) \tag{1}$$

where $\Pr(A_3|A_2,A_1)$ represents conditional probability of A_3 given A_2 and A_1, and $\Pr(A_4|A_3)$ represents conditional probability of A_4 given A_3. Two state Bayesian network is considered, where components can be either functioning or failure.

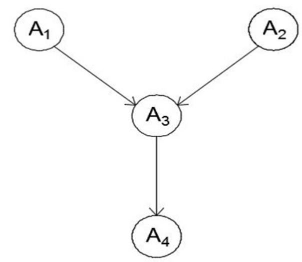

Figure 1. Simple Bayesian network

3. COMPUTATION OF CONDITIONAL PROBABILITIES USING WEIGHTED SUM ALGORITHM

Weighted sum algorithm is a method to generate conditional probabilities using the expert's judgmental strategy. For a small system, it is easy for the expert to give opinion about the conditional probabilities based on his experience and expertise, but when the system become more and more complex the expert fails to give the conditional probability of certain incompatible parent configuration. Hence there is a need for the method "Weighted Sum Algorithm" proposed by Das[8]. In this method, the expert and user ask the probabilities of only compatible combinations. Expert will have idea about the probability or failure times or failure rates or parameters of the failure of specific compatible combination. A simple BN is as shown in Fig. 2, where *S* represents the system and C_1, C_2 and C_1 are the components of system.

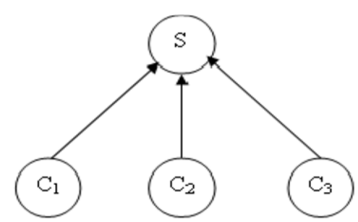

Figure 2. Conditional probability calculation in Bayesian network.

Let us consider system S and components and each have two states, say failure or success. Now we can ask the expert, “What is the probability of system failure given component C_1 is in success state?” It is easy to answer for the expert as it was the compatible parental combination. The example for incompatible parental configuration is “What is the probability of the system failure given C_1 is in failure state and C_2 is in failure state and C_3 is in success state?” It will become difficult for an expert to answer. There are two type of data that can be collected from the expert to generate conditional probabilities:

- Probabilities of compatible parental configuration
- Weights of the components

Finally using these inputs, the conditional probability can be calculated from Eqn. (2)

$$P(S / C_1, C_2,\ and\ C_n) = W_1 * P(C_1 / Comp(S)) + ... + Wn * P(Cn / Comp(S)) \tag{2}$$

where W_1, $\ldots W_n$, are the weights of the components. Comp(S) represents the compatible parent configuration of system S. The weights are assigned by the expert opinion. Sum of the weights should be unity and the weights are given such that how much each component is affecting the system.

4. CASE STUDY: RELIABILITY ESTIMATION OF DACS

In this paper, a typical divert attitude control system of launch vehicle is considered for reliability estimation. It consists of eight sub systems.

The reliability block diagram of DACS is shown in Fig. 3, in which all components of system connected in series configuration. The fault tree of DACS is as shown in Fig. 4 and represented by OR Gate.

Bayesian networks can be used as direct generalisation of fault trees. The translation of fault tree to Bayesian network is simple, with the basic events that contribute to the independent event is represented as parents and child.

The BN model for DAC is as shown in Fig. 5. Node DAC represents divert altitude and control system and the AB, PV, PR, FC, DBV, LT, MT, ST nodes are representing its components.

Figure. 3 Reliability block diagram o DACS system.

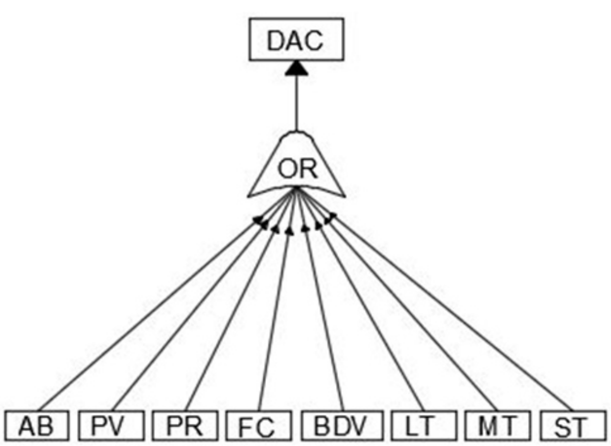

Figure 4. Fault tree of DAC system.

Qualification test data, Acceptance test data, ground test data and flight test data of DACS was collected systematically from individual component designers and as given in Table 1. The conditional distributions and unknown posterior marginal distributions of P(AB), P(PV), P(PR), P(FC), P(BDV), P(LT), P(MT) and P(ST) are computed from the compatible parental configurations as given in Table 2. These values are obtained systematically from expert opinion. Unavailable marginal probabilities of the sub- systems are computed by Win-BUGS software using pass /pail data mentioned as in Table 1.Code developed for the computation of marginal probabilities .Non informative uniform prior (0,1) was used as the conjugate prior when prior test data is not available.

Table 1. Test data of DAC sub systems

Sub system	Previous data	Flight test data	No. of failures	No. of sub system passed
Air bottle (AB)	288	12	0	300
Gas Pyro valve (PV)	194	6	1	199
Pressure regulator (PR)	34	6	1	39
Fuel chamber (FC)	8	12	1	19
BDV	68	12	1	79
Large thrusters (LT)	0	20	0	20
Medium thrusters (MT)	0	40	0	40
Small thrusters (LT)	0	20	0	20

Table 2. Compatible parental configurations of DAC systems

Prob. distribution over system P(S|C)	Success	Failure
P(S=1|AB)	0.92	0.08
P(S=0|AB)	0.08	0.92
P(S=1|PV)	0.98	0.02
P(S=0|PV)	0.02	0.98
P(S=1|PR)	0.98	0.02
P(S=0|PR)	0.02	0.98
P(S=1|FC)	0.96	0.04
P(S=0|FC)	0.04	0.96
P(S=1|BDV	0.9	0.1
P(S=0|BDV)	0.1	0.9
P(S=1|LT)	0.96	0.04
P(S=0|LT)	0.04	0.96
P(S=1|MT)	0.97	0.03
P(S=0|M)	0.03	0.97
P(S=1|ST)	0.95	0.05
P(S=0|ST)	0.05	0.95

Conditional probabilities of DACS was computed by Weighted Sum Algorithm and as given by the Eqn. (3)

$$\begin{aligned} &P(DAC / AB, PV, PR, FC, BDV, LT, MT, ST) \\ &= W_1 * P(AB / Comp(DAC)) \\ &+ ... + W_8 * P(ST / Comp(DAC)) \end{aligned} \tag{3}$$

where W_1, W_2 …. W_8 represents the effective weights of individual components on the system and as given in Table 3.

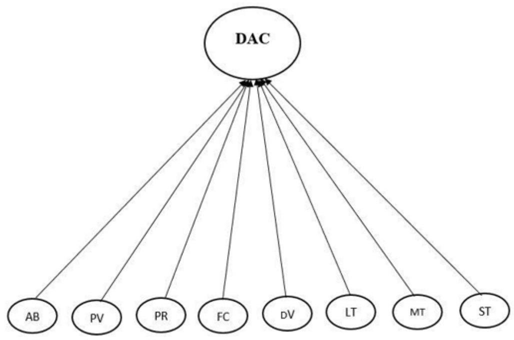

Figure 5. Bayesian network of DAC system.

Table 3. Weights of the components

Sub system	Weight of each component on system
Air bottle	0.1
Gas Pyro valve	0.1
Pressure regulator	0.1
Fuel chamber	0.05
BDV	0.05
Large thrusters	0.2
Medium thrusters	0.2
Small thrusters	0.2

The joint probabilities of DACS represented by the following Eqn. (4)

$$P(DAC) = \sum_{i \Rightarrow p=0}^{1} P\begin{pmatrix} (S=1/\,AB_i, PV_j, RV_k, FC_l, BDV_m, LT_n, MT_o, ST_p) \\ *\,P(AB_i, PV_j, RV_k, FC_l, BDV_m, LT_n, MT_o, ST_p) \end{pmatrix} \tag{4}$$

Marginal posterior distribution of the DAC components as shown in Fig. 6. Posterior distribution summary is as given Table 4. Non-informative prior was used when data is not available in a particular component.

Reliability of DACS was estimated from computed conditional probabilities, sub system posterior probabilities in previous steps using Eqn. (4). MATLAB code generated for calculations of reliability of DAC system. The estimated the reliability of DACS is 0.925.

To validate the results obtained from BN model, Reliability of DAC System was calculated Bayesian Hybrid Method[11]. Bayesian Hybrid method is a combination of Bayesian method and variance propagation method.

Table 4. Numerical summary of posterior distributions of DACS

Sub system	Mean	2.5 %	97.5 %	No. of samples
p[AB]	0.9968	0.9882	0.9999	1000
p[PV]	0.9901	0.9721	0.9988	1000
p[PR]	0.9527	0.8766	0.9952	1000
p[FC]	0.9104	0.7676	0.9903	1000
p[BDV]	0.9759	0.934	0.997	1000
p[LT]	0.9539	0.8324	0.9988	1000
p[MT]	0.9754	0.9105	0.9995	1000
p[ST]	0.9551	0.8441	0.9987	1000

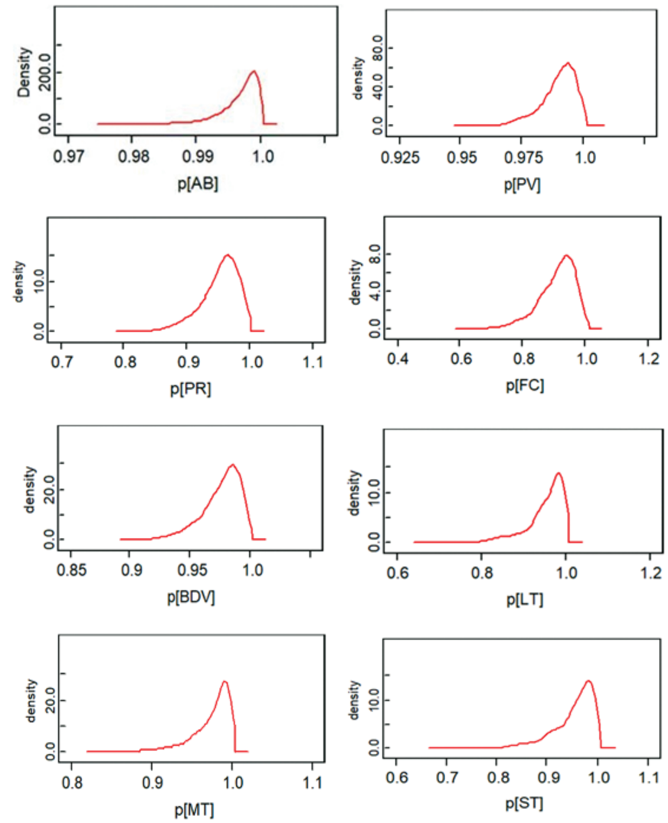

Figure 6. Computed posterior distribution of DACS sub systems.

Table 5. shows the results obtained from two BN methods and Bayesian Hybrid method. The estimated reliability using MCMC method is 0.925 whereas from Junction Tree algorithm is 0.934. It can be observed that, there is a small difference of 0.009 observed between two BN methods. Estimated reliability using Bayesian Hybrid Method is 0. 932.Results of JTA and Bayesian Hybrid methods are comparable.

Table 5. Comparison of reliability estimation results

Method	Estimated reliability with 95% confidence level
Bayesian network using MCMC	0.925
Bayesian network using junction tree algorithm (JTA)	0.934
Bayesian hybrid method	0.932

5. CONCLUSION

Reliability of DAC system was estimated using Bayesian networks considering the dependencies between the components and the system. System/component discrete test data was obtained from historical records, simulation results, and expert opinion was used for reliability estimation. Win-BUGS software was used for finding the component posterior probabilities. Weighted sum algorithm is used for finding the conditional probabilities of all the combinations and the MATLAB code has been developed for calculations. Results of two BN methods compared with Bayesian Hybrid method. Therefore, the methodology has been established for the reliability estimation of Divert Attitude Control System considering dependency among system and components.

The proposed approach can be extended to reliability estimation of Launch vehicles or one-shot systems when limited test data is available.

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