

Recent Trends in Computational Electromagnetics for Defence Applications

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ABSTRACT

Innovations in material science, (nano) fabrication techniques, and availability of fast computers are rapidly changing the way we design and develop modern defence applications. When we want to reduce R&D and the related trial-and-error costs, virtual modelling and prototyping tools are valuable assets for design engineers. Some of the recent trends in computational electromagnetics are presented highlight the challenges and opportunities. Why researchers should equip themselves with the state-of-the-art tools with multiphysics and multiscale capabilities to design and develop modern defence applications are discussed.

Keywords: Electromagnetic modelling; Simulation; Algebraic topology; Multiscale; Multiphysics; Numerical methods; Modelling

1. INTRODUCTION

Prior to the 1960s, electromagnetic applications were developed mostly using analytical methods. Using those methods, one can easily derive closed-form expressions for calculating electromagnetic field quantities. As the applications advanced, the material properties and geometries became more complex to be modelled using analytical methods. With the advent of computers came a new area of research, namely computational electromagnetics (CEM), which brought sophisticated algorithms and tools to the design development process. Though problems involving complex geometries and material properties still pose a great challenge to researchers, new computational methods are continuously being developed to overcome these difficulties. The demand for improved accuracy, speed, and efficiency are keeping this domain evergreen. Over the last few decades, CEM has emerged into a prominent field of research. Various advanced methods were developed to solve complex real-world problems. But still there isn't a single method that we can call as the best method for all kinds of engineering challenges. It often comes to the expertise of the design engineers, which plays a critical role in choosing the most suitable method for a given problem.

Defence applications span a broad electromagnetic frequency range. Radio frequency (RF)^{1,2}, microwave antennas³⁻⁵, radars and spaceborne imaging^{6,7}, terahertz and optical applications⁸⁻¹¹ are some of the major applications. An emerging area combining radio and optical frequencies, namely RF photonics, shows great promise for defence applications^{12,13}. CEM tools play a crucial role in improving compactness, robustness, efficiency, and cost of the end product. Design of

advanced applications demand attention to fine details while modelling. One such example in antenna engineering is as shown in Fig. 1. Various design specifications of an actual Archimedean spiral antenna were carefully modelled in the virtual prototype capturing all design details^{14,15}. We will quickly review some mainstream, non-mainstream, and recent CEM methods, which are of interest to defence research.

2. MAIN TIME-DOMAIN CEM METHODS

In the world of computational electromagnetics, the finite-difference time-domain (FDTD) method^{16,17} and the finite-element method (FEM)¹⁸⁻²⁰ are extensively used for practical problem solving. The FDTD method is popular among engineers and physicists primarily due to its simple algorithm and implementation. These factors propelled big developments in this method over the years. However, the classical (standard) FDTD formulation is limited to structured spatial grids. These structured grids inherently suffer from staircasing errors while modelling complex curved or irregular geometries. As most of the advanced real-world problems fall in this category, the FDTD method is not an ideal choice to model those problems. If we still want to use only the FDTD method for such problems, we may have to unnecessarily mesh the entire domain with tiny FDTD cells in order to accurately capture the fine details in the problem. One cannot selectively refine the mesh only in regions where it is required in the standard FDTD method. There are some advanced FDTD methods, which use subgridding techniques to address this issue, however, at the cost of increasing the complexity of the resulting advanced FDTD method²¹⁻²³.

Another important time-domain method worth mentioning is the transmission line matrix (TLM) method²⁴⁻²⁷. This method uses the Huygen-Fresnel wave propagation principle to model

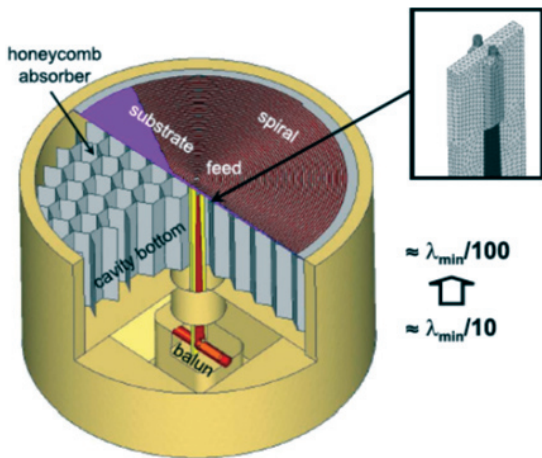


Figure 1. A cut-view of an Archimedean spiral antenna modelled as a virtual prototype showing complex geometries and materials involved in the design^{14,15}.

field propagation. The TLM method uses a structured mesh of transmission lines, which are interconnected at nodes. One has to build the scattering matrix for modelling wave propagation. Inhomogeneous and lossy media can be modelled by adapting the scattering matrix. Different boundaries can be simulated by adjusting respective connection equations. Like in the case of standard FDTD method, the TLM method also is limited to mainly structured transmission lines grids. Local adaptation of grids requires more complex algorithms with special treatments to calculate the scattering matrix.

Selective grid refinement is characteristic to all multiscale methods. This feature allows us to easily adapt the size of the cells only in areas of interest without unnecessarily refining the cell size in the entire domain. A practical example employing unstructured conformal tetrahedral mesh is shown in the Fig. 2. Notice the size of cells used in the feeding balun region of the horn antenna compared to side flare region. Multiscale modelling examples using unstructured triangular (2D) and tetrahedral (3D) grids are as shown in Fig. 3. Many practical problems have layered structures with high dielectric (refractive index) contrast as shown in Fig. 4. In these layered structures, the electro-magnetic wavelengths and velocities of wave propagation vary according to the dielectric constant or refractive index of the material. For example, inside high refractive index materials, the wavelengths are smaller compared to low refractive index materials. In other words, the wave propagates faster inside a low refractive index material than inside a high refractive index material. For accurately

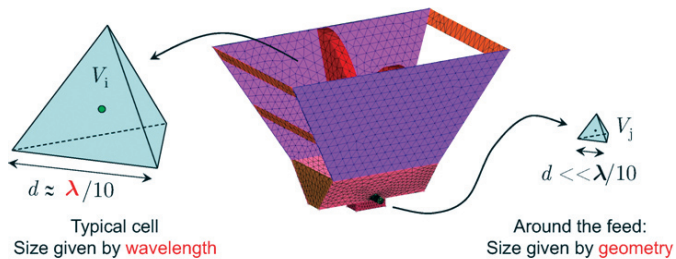


Figure 2. Multiscale modelling where design features can go from $\lambda_{\min}/10$ to $\lambda_{\min}/100$.

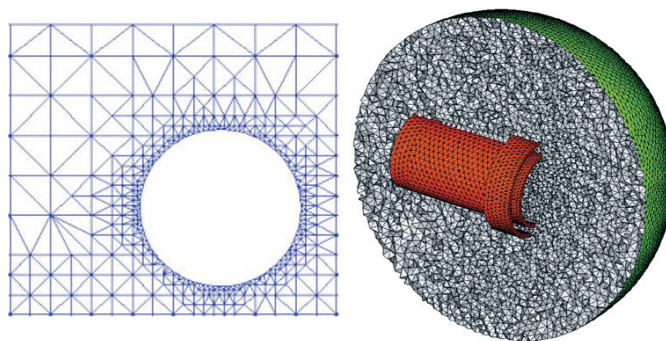


Figure 3. Typical unstructured triangular 2D (left) and tetrahedral 3D (right) used in FEM, ATM, DGTD, and FVTD methods.

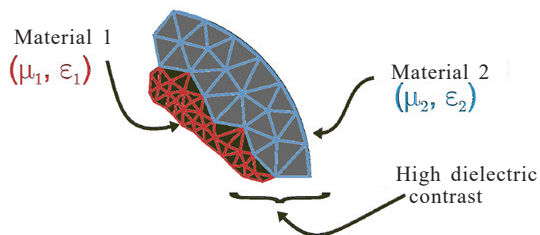


Figure 4. Layered structures with high dielectric contrast demanding different discretisations to match wave propagation velocities (wavelengths) in different media.

modelling a slow propagating wave (short wavelengths), we need to use small cells inside high refractive index materials and vice versa.

Using standard FDTD method for such problems again requires unnecessary refining of grid in the entire domain so as to properly resolve short wavelengths. The real strength of the FEM lies in its flexibility to employ unstructured mesh as shown in Fig. 4. This feature enables us to easily refine the mesh only in regions where it is really needed. This way we avoid global mesh refining, which we normally end up doing in the case of standard FDTD method. Furthermore, methods employing conformal unstructured mesh can be adapted naturally to slanted or curved boundaries and hence, they avoid any special treatments in those boundary cells.

Though FEM gives maximum flexibility to model complex structures employing conformal unstructured mesh, the standard FEM is predominantly used as a frequency-domain tool. If you are interested in modelling broadband response of your technology, you require several individual simulations to cover the entire frequency range of interest. Moreover, in FEM, the memory requirement varies disproportionately with the total number of cells in the computational domain. For example, if we double the number of cells in the computational domain, then memory required to simulate the problem increases by more than a factor of two.

We mainly use the classical FEM in its frequency-domain formulation. This is because when we model FEM in the time-domain, we normally get an implicit time-stepping scheme^{28,29}. Normally, methods employing implicit time-stepping schemes are computationally heavier than explicit time-stepping

counterparts. This is due to the global matrix inversion required at each time-step for implicit schemes. The sparseness of the matrix controls the speed of matrix inversion operation. The related computational effort grows in a nonlinear manner for implicit schemes. This limits the size of problems that can be solved within a given computational resource (time and memory). With the availability of cheap computer memory, one can still use implicit time-stepping methods for certain problems where the total number of cells in the computational domain is rather small. Such implicit schemes will become, however, prohibitively expensive for majority of practical problems. Because of these constraints, FEM is mostly used in frequency-domain formulation. The search for an explicit conformal time-domain method that has the flexibility of using unstructured mesh continued.

3. CONFORMAL EXPLICIT TIME-DOMAIN METHODS

To precisely model all the fine structural features with complex (curved) geometries and material properties, we need to use unstructured mesh. This is one of the highly desirable features in a computational method. Several research efforts are continuously being made to extend the standard FDTD method to model conformal geometries. One of the earliest efforts to expand the FDTD for generalised conformal mesh led to the development of the finite integration technique (FIT)^{30,31}. The FIT is a state-of-the-art method for modelling complex geometries using conformal mesh, which can reduce or completely avoid stair casing errors.

There were several efforts in last two decades to develop other time-domain conformal methods. One such non-mainstream method well studied by the author and collaborators is the finite-volume time-domain (FVTD). The FVTD method was originally developed for computational fluid dynamics (CFD) and was later adapted to model electromagnetic problems^{32,33}. In FVTD method we can get the best of both worlds - FEM and FDTD. Like in FEM, we can employ unstructured mesh and like in FDTD we can employ a fully explicit time-stepping scheme⁵. This explicit time-stepping provides broadband frequency response just from a single simulation run. This is obtained through post-processing of the time-domain results with Fast Fourier Transform. Moreover, multiscaling fits naturally within the FVTD framework, which allows for detailed modelling of electromagnetic structures as shown in Fig. 2. As in the case of FEM, it is straightforward to model materials with high dielectric-contrast and curved geometries in the FVTD method.

Though FVTD offers these highly desirable features, there is one serious limitation to this method. The FVTD method suffers from high numerical dissipation and hence, cannot be applied for long distance wave propagation problems. This led to the development of methods, which extend the positive features of the FVTD method and overcomes the bottleneck of numerical dissipation as discussed in the next section.

4. STATE-OF-THE-ART CEM METHODS

In traditional FEM the tangential-continuity condition is kept across cell boundaries. However, if we are able to relax

this condition, we can get to a new class of method called the Discontinuous Galerkin Method (DGM). Here, instead of forcing the tangential fields on the cell interface, we impose the continuity constraints on the computed flux components. This is similar to the FVTD based method, however, with a few advantages over FVTD and FETD. The main advantage of DGM over conventional FETD is due to the resulting block-diagonal linear system of equations to be solved. This greatly reduces the computational load by requiring only a single inversion of K square matrices of $N \times N$ elements. Here, K and N denote the number of elements and the number of basis functions per element, respectively. As we need this information only once, we can easily do this in the pre-processing stage. The additional computational load due to doubling the number of unknowns in the cell interfaces is tolerated because we can substantially improve the computational efficiency and accuracy of the resulting scheme. Furthermore, we get the well-deserved explicit time-domain formulation³⁴⁻³⁶.

Let h and p denote the size of the spatial element (cell) and the order of the basis function inside each element, respectively. Then, the numerical error of DGM is of the order h^{2p+1} . If we set the order of the basis function $p = 0$ inside each cell, this corresponds to constant value inside each cell. Then the order of numerical error will be h , which corresponds to the FVTD method discussed earlier. The DG time-domain (DGTD) is increasingly used in the recent times and it is certainly one of the state-of-the-art methods in CEM.

5. NON-MAINSTREAM ALGEBRAIC TOPOLOGICAL METHOD

Another unconventional approach in CEM was developed by the author and others using the tools of algebraic topology. The domain of algebraic topology is still not widely known to majority of engineering communities. Algebraic topological method (ATM) emerges from a radically different way of thinking about modelling electromagnetic problem. In ATM we do not use vector calculus and differential equations. Literatures^{37,38} provides historical development of ATM.

We have developed a more intuitive and meaningful way to model electromagnetic problems using ATM³⁹. For electromagnetic modelling using ATM, we start by describing the physical quantities only using physically measurable scalar variables and hence, avoid vector calculus completely. This is counter intuitive. However, in retrospect we can understand by examining all the quantities in electromagnetics that can be physically measured. These are voltage, current, electric and magnetic fluxes, charge content and charge flow, etc., which are only scalar quantities. We have demonstrated that there is absolutely no need for vectors like electric and magnetic fields and the respective field densities to model an electromagnetic problem. In addition, we can also represent all the relationships between these scalar quantities only using discrete algebraic summation. Hence, there is also no need for differential equations. It is important to note that the algebraic formulation of the underlying physical problem gives an exact discrete representation of the continuous differential (Maxwell-Heaviside) equations. For more details on the mathematics involved, readers refer to literature³⁹.

The power and elegance of ATM lie in two inter-related tools, namely boundary and coboundary operators⁴⁰. Let us first explain the boundary operator. The boundary operator is a mathematical tool, which operates on the underlying topological object, which could be lines, surfaces, or volumes. Note that there is no boundary operation possible on a point because the boundary of boundary does not exist^{41,42}. It is worth noticing that the boundary operator reduces the dimensionality of the topological objects by one. That is, when operated on a surface or a volume, we get the enclosing lines or surfaces, respectively as results. The coboundary operator operates on the cochains, which are physical quantities explained in the previous section. The coboundary operator operates on the node potentials to give the potential difference between the nodes (electromotance). When it operates on the potential difference on a chain of lines forming a contour, then we get the flux passing through the surface enclosed by the contour. In that sense, the coboundary operator does the opposite of what the boundary operator does - increases the dimensionality of the cochains by one. For more discussion on the ATM framework, refer to^{39,43-45}.

We will briefly describe the ATM formulation for a simple electrodynamic problem. The power of the ATM framework lies in the relationship between the boundary and coboundary operators acting on chains and cochains, respectively. This relationship creates a direct discrete framework to describe underlying physics close to the experimental principles^{46,47}. The 4+1 equations describing an electrodynamic problem written using ATM framework are as follows^{39,48}.

$$\Phi(\partial s^3, \tilde{t}) = 0 \quad (1)$$

$$\Psi(\partial s^3, t) = Q_c(\tilde{s}^3, t) \quad (2)$$

$$\mathcal{V}(\partial s^2, \tilde{\tau}) = \Phi(s^2, \tilde{t}^-) - \Phi(s^2, \tilde{t}^+) \quad (3)$$

$$\mathcal{U}(\partial s^2, \tau) = Q_f(\tilde{s}^2, \tau) + \Psi(\tilde{s}^2, t^+) - \Psi(\tilde{s}^2, t^-) \quad (4)$$

$$Q_f(\partial s^3, \tau) = Q_c(\tilde{s}^3, t^-) - Q_c(\tilde{s}^3, t^+) \quad (5)$$

For simplicity, we use the same notations introduced in³⁹. One can relate Eqns. (1) and (2) to the Gauss magnetic and electric divergence equations, respectively. Similarly, Eqns. (3) and (4) correspond to ATM formulation of the Faraday and Ampere laws, respectively. Lastly, Eqn. (5) is the ATM formulation of the electric charge continuity equation. It is important to note that we are not using any field vectors or differential equations in deriving the above ATM formulations. One can derive the above 4+1 ATM equations directly from the experimental principles. In doing this, we only use physically measurable quantities such as potential ϕ , electromotance impulse V , magnetomotance impulse U , electric flux Ψ , magnetic flux Φ , electric charge content Q_c and charge flow Q_f . We can use the same approach to also derive the ATM formulation for other multiphysics problems as briefly described in the next section.

6. MULTIPHYSICS CAPABILITIES AND ACCURATE BOUNDARY CONDITIONS

We have seen so far that there is a major push for developing flexible multiscale CEM tools, which can provide explicit time-stepping formulation to capture complex geometries and phenomena without heavy computational memory requirements. The recent breakthroughs in material science and engineering, (nano) fabrication techniques, and 3D printing are allowing us to develop new applications. Advanced terahertz devices, wearable antennas, graphene, novel meta- and nanomaterial based devices are some of the recent trends. These new applications demand multiphysics features in addition to explicit time domain formulation and unstructured multiscale capabilities.

These multiphysics phenomena include electrodynamic, thermodynamic, photoelectric, electrochemical aspects of the problem in quantum- and macro-levels. Major efforts are done to incorporate multiphysics capabilities in standard CEM tools. To achieve this, we first need a method that can naturally interface different physics in a single underlying model. For example, while modelling nanoscale device like a quantum-tunnelling diode, we need to study not only the electrodynamic aspects, but also the impact of thermodynamic and thermoelectric effects¹⁰.

All practical CEM tools need two major features, namely, perfectly matched layer (PML) and absorbing boundary conditions. These two features are beyond the scope of this paper. The PML is introduced as a new class of boundary truncation technique for the FDTD-based applications⁴⁹. The author and various collaborators have extensively studied PMLs⁵⁰⁻⁵² for conformal time-domain methods. Apart from the standard PML formulation, which involves field-splitting, there are a couple of other important formulations and complex-space stretching approach is used⁵⁴. Corner reflections and special treatment for different orientations of the classical PML was overcome by implementing a radial PML with single formulation for all orientations⁵⁵. Instability issues in certain PML implementations attracted many mathematicians to rigorously study PML for stability and error limits⁵⁶. The author and collaborators compared different PMLs for application in FVTD and other conformal time-domain methods⁵⁷.

Several efforts are ongoing to develop advanced CEM tools in the Radical Innovations Group. We are incorporating multiscale, multiphysics, and explicit time-stepping features in our tools, which results in powerful capabilities for CEM modelling and simulation. These advanced capabilities are going to elevate the standard of modelling and rapid virtual prototyping for advanced defence applications. Interested readers are encouraged participate in one of the world's largest online learning and certification programme on CEM through the Government of India's National Programme on Technology Enhanced Learning (NPTEL) platform⁵⁸.

7. CURRENT DEFENCE APPLICATION DEVELOPMENTS

Some of the major defence applications are in design development of microwave, terahertz and optical communications devices. Recent breakthroughs in developing

compact, affordable, high precision radars have led to a renaissance of the radar technology with many new applications in defence stealth and signal jamming. Development of radar systems in the millimeter-wave range is catching up as they are ideal for surveillance tasks in the immediate environment, particularly when visibility is poor^{59,60}. Many space borne satellites are now equipped with state-of-the-art synthetic aperture radars (SAR) with full polarimetric features to monitor land and ocean surfaces^{7,61}. Accurate modelling of ocean and land surface demands complex material models with dynamically changing geometries like ocean surface due to wind and water currents. Such models for space borne imaging radar are made using the scattering theory and can be adapted to patterns such as oil spills, vegetation or defence deployments as shown in Fig. 5.

Modelling based on full polarimetric SAR imaging is as shown in Fig. 6 where a ship contaminating the ocean surface with oil spill is captured using SAR. This research was done by the author in collaboration with the European Commission, Joint Research Centre (JRC), Italy.

The radar cross-section (RCS) in different polarimetric channels can be extracted from the total power intensity radar image. In fact, total intensity image is an ensemble of information from all 4 channels namely HH , HV , VH and VV . Each channel consists of two polarisations - receive and transmit. For a (quasi)monostatic case of spaceborne radar imaging, reciprocal condition $HV \approx VH$ holds.

Radar backscatter from various imaged objects is described using the target scattering or Sinclair matrix S . All elements in the scattering matrix S are dimensionless. The first and second subscripts of each element represent the received and transmitted radar polarisation, respectively. For example, consider one element SHV , where the first H and second V subscripts represent horizontal receive and vertical transmit polarisations, respectively. Elements of S are functions of frequency, incidence angle, and scattering angle of the radar incident wave.

The covariance matrices can be exploited to study the reflectivity (RCS) variations in different channels. A generic model is described here to understand the behaviour of co-to-

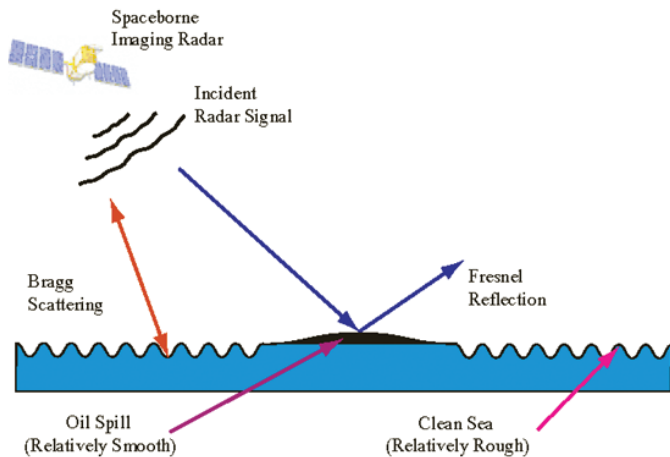


Figure 5. Sea modelling along with oil spills based on scattering theory.

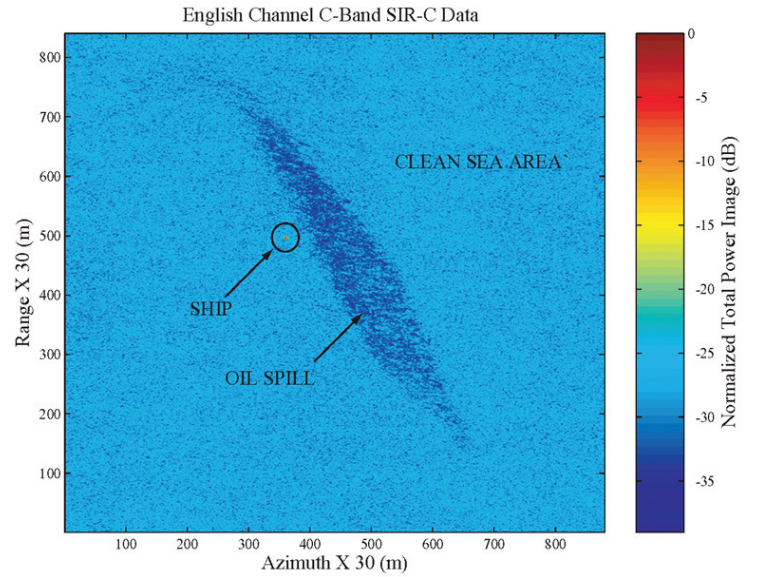


Figure 6. Full polarimetric space borne radar image capturing the oil spill along with the polluting ship in the middle of the English Channel.

cross covariances from ocean surface. The polarimetric radar covariance matrix for a (quasi) monostatic case has 3 real and 3 complex covariance elements. This matrix provides a complete set of measurements from an ocean surface. Variation in the reflected signals from the surface features can be examined using various covariance parameters for different transmit and receive polarisations⁶². Let us define a few parameters for identification, which will be used in the polarimetric synthesis⁶³.

Total Power Image contains data fused from all channels. For the reciprocal (quasi) monostatic case we have,

$$P' = S_{HH}S_{HH}^* + 2S_{HV}S_{HV}^* + S_{VV}S_{VV}^* \quad (6)$$

where $*$ represents complex conjugate of the respective quantities.

Reflectivity corresponds to the radar backscattered power in a particular channel. For the (quasi) monostatic and full-polarimetric system we have the three standard linear reflectivity measurements corresponding to HH , HV , and VV channels as below,

$$R_{HH} = 10 \log(|S_{HH}|^2) \quad (7)$$

$$R_{HV} = 10 \log(|S_{HV}|^2) \quad (8)$$

$$R_{VV} = 10 \log(|S_{VV}|^2) \quad (9)$$

where R_{HH} , R_{HV} and R_{VV} represent reflectivity in HH , HV and VV channels, respectively. A typical example of these parameters extracted from the total power image is as shown in Fig. 7. This is the variation of reflectivity parameters along the azimuth direction for the same image as shown in Fig. 6.

Co-polarised differential reflectivity is the ratio of radar backscattered power from two different co-polarised channels (HH and VV) written as,

$$R_{HH-VV} = 10 \log \left(\frac{|S_{HH}|^2}{|S_{VV}|^2} \right) \quad (10)$$

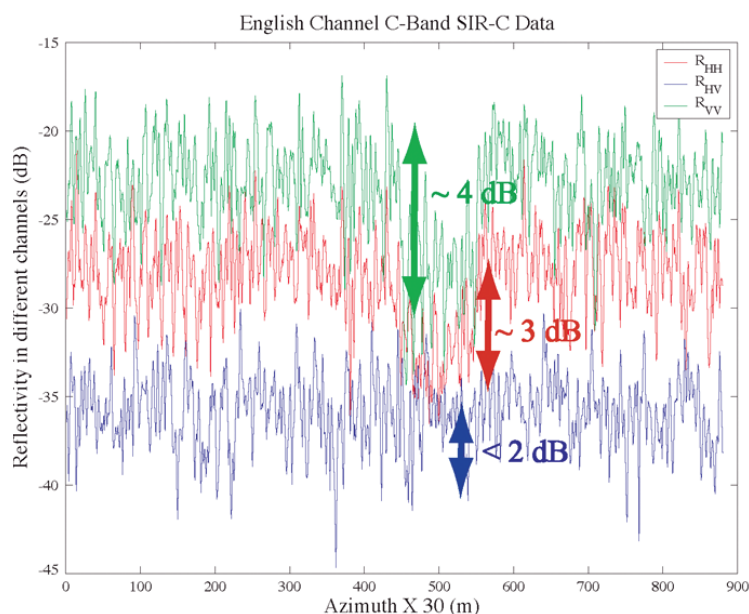


Figure 7. Reflectivity variations in different channels along the azimuth direction for the image captured in the English Channel (see Fig. 6).

For more detailed description of the RCS measurements using full-polarimetric data⁶³.

Dynamically controlling radiated phase-fronts without mechanical motion using electronically reconfigurable apertures is becoming more popular. This enables in fast beam-forming. Normally antennas are designed for far-field applications. This is also the case for electronically scanned antennas. Conventional design of electronically scanned antennas relies on the distribution of a large array of antennas, each backed by an active phase-shifter.

Such designs were successfully used for beamforming and beam-steering applications such as radars, sensor arrays, long-distance communication systems, etc. In spite of having excellent high-fidelity beam patterns, phased array technologies have several limitations. For example, each antenna in a phased array aperture requires a separate phase shifting circuit. A significant number of phase shifters are required even for a moderately-sized antenna aperture. This makes the overall system complex and expensive. Furthermore, these systems suffer from high insertion losses, which must be offset using power amplifiers. This means such systems are high power consuming. Some of these limitations can be overcome by employing meta-surfaces. These metasurfaces are typically arrays made of sub-wavelength elements. The electromagnetic properties of these metasurfaces can be fine-tuned to achieve required electromagnetic response. There has been an increased interest in static metasurface, which shows great potential wavefront shaping applications^{64,65}. Stealth technologies using microwave absorbers demand complex modelling capabilities to accurately capture material behaviours under different conditions⁶⁶. Tools used to predict the scattered fields and compute RCS of complex structures require multiscale modelling capabilities.

Among other developments in CEM, the IEEE initiative to validate and standardise CEM tools is worth mentioning⁶⁷.

The IEEE Standard 1597.2-2010 stipulates various criteria to validate CEM simulation codes. Many engineering applications need such a standard approach to meaningfully compare methods and results for their efficiency and accuracy claims. A method can be validated by comparing various data set obtained through experiments, simulations, analytical processes, etc. This will certainly help users of CEM tools to make informed decision about the choice of a tool for a particular problem.

These are some of the recent trends in defence and aerospace applications. It is an interesting period to venture into R&D in these topics. It is paramount to update our knowledge and knowhow about the state-of-the-art CEM tools. We have argued why the old tools for designing engineering applications will not be enough for the future needs in this domain. A new set of CEM tools with multiscale and multiphysics capabilities are needed to model and simulate advanced functional materials like graphene, meta- and nanomaterials. These materials are increasingly introduced for different defence applications. The future of defence application developments is going to strongly depend on design engineers' mastery of these advanced CEM tools.

8. SUMMARY

Recent trends in the domain of computational electromagnetics for defence application development have been reviewed. Innovations in material science and (nano) fabrication techniques and availability of fast computers are rapidly changing the way we design and develop modern defence applications. When we want to reduce R&D and the related trial-and-error costs, virtual modelling and prototyping tools are invaluable assets for design engineers. We have argued why defence researchers should update their knowledge and know-how about the state-of-the-art CEM tools with multiphysics and multiscale capabilities to design and develop modern defence applications. Some of the recent innovations in advanced materials like graphene, meta- and nanomaterials are leading to new applications in microwave, terahertz, and photonics. The future CEM tools should have multiphysics, multiscale and explicit time-stepping features. Multiphysics features like electrodynamics, thermodynamics, and thermoelectric modules are becoming a minimum requirement for various advanced applications. In addition, these tools have to be computationally efficient and accurate to model large complex real-world problems.

REFERENCES

1. Riihonen, T.; Korpi, D.; Rantula, O.; Rantanen, H.; Saarelainen, T. & Valkama, M. Inband full-duplex radio transceivers: A paradigm shift in tactical communications and electronic warfare? *IEEE Commun. Magazine*, 2017, **55**(10), 30–36. doi: 10.1109/MCOM.2017.1700220
2. Zhang, Bing-Bing; Yang, Q; Wang, Hong-Yi & Li, Jian-Cheng. Realization of china military RFID air interface protocol based on software-defined radio. *In 11th IEEE International Conference on Solid-State and Integrated*

- Circuit Technology, 2012.
3. Aerospace and defense supplement. *Microwave Journal*, 2018, **61**(06 Sup). doi: 10.1109/ICSICT.2012.6467601
 4. Fumeaux, C.; Almpanis, G.; Sankaran, K.; Baumann, D. & Vahldieck, R. Finite-volume time-domain modeling of the mutual coupling between dielectric resonator antennas in array configurations. *In 2nd European Conference on Antennas and Propagation (EuCAP), 2007*, pp. 1–4. doi: 10.1049/ic.2007.0909
 5. Fumeaux, C.; Baumann, D.; Sankaran, K.; Krohne, K.; Vahldieck, R. & Li, E. The finite-volume time-domain method for 3-D solutions of Maxwell's equations in complex geometries: A review. *In Proceedings of the European Microwave Association (EuMA) 3, 2007*, pp. 136–146.
 6. Zhao, Z.; Ji, K.; Xing, X.; Zou, H. & Zhou, S. Ship surveillance by integration of space-borne SAR and AIS – Further research. *J. Navigation*, 2014, **67**, 295-309. doi: 10.1017/S0373463313000702
 7. Sankaran K. & Fortuny-Guasch, J. Radar remote sensing for oil spill classification (optimization for enhanced classification). *In Proceedings of the 12th IEEE Mediterranean Electrotechnical Conference, 2004*, pp. 511–514. doi: 10.1109/MELCON.2004.1346979
 8. Mayes, P. E. Frequency-independent antennas and broadband derivatives thereof. *In Proceedings of the IEEE 80, 1992*, pp. 103–112. doi: 10.1109/5.119570
 9. Weiss, S. & Dahlstrom, R. Rotman lens development at the army research lab. *In Proceedings of the IEEE Aerospace Conference, 2006*.
 10. Sankaran, K. & Sairam, B. Modelling of nanoscale quantum tunnelling structures using algebraic topology method. *In AIP Conference Proceedings 1953, 2018*, pp. 140105–1 to 4. doi: 10.1063/1.5033280
 11. Jensen, J. O. & Cui, Hong-Liang (Editors). Terahertz for military and security applications. *In Proceedings of SPIE 6549, 2007*.
 12. Choe, J. Y. Defense RF systems: Future needs, requirements, and opportunities for photonics. *In International Topical Meeting on Microwave Photonics IEEE, 2005*. doi: 10.1109/MWP.2005.203600
 13. Zach, S. & Singer, L. RF photonics - why should defense take notice? *In 24th IEEE Convention of Electrical & Electronics Engineers in Israel, 2006*. doi: 10.1109/IEEEI.2006.321121
 14. Fumeaux, C.; Baumann, D. & Vahldieck, R. Finite-volume time-domain analysis of a cavity-backed Archimedean spiral antenna. *IEEE Tran. Antennas Propagation*, 2006, **54**(3), 844–851. doi: 10.1109/TAP.2006.869935
 15. Sankaran, K. Accurate domain truncation techniques for time-domain conformal methods. 2007. PhD thesis, ETH Zürich, Switzerland. doi: 10.3929/ethz-a-005514071
 16. Yee, K. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Trans. Antennas Propag.*, 1966, **14**(3), 302–307.
 17. Tafflove, A. & Brodwin, M. E. Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations. *IEEE Trans. Microwave Theory Technol.*, 1975, **23**(8), 623–630. doi: 10.1109/TAP.1966.1138693
 18. Jin, J. M. The finite element method in electromagnetics. John Wiley & Sons, 2nd Edn, 2002.
 19. Bossavit, A. Computational electromagnetism: Variational formulations, complementarity, edge elements. Academic Press, 1998.
 20. Volakis, J.; Sertel, K. & Usner, B. C. Frequency domain hybrid finite element methods for electromagnetics. Morgan & Claypool Publishers, 2006.
 21. Okoniewski, M.; Okoniewska, E. & Stuchly, M. Three-dimensional subgridding algorithm for FDTD. *IEEE Trans. Antennas Propag.*, 1997, **45**(3), 422–429. doi: 10.1109/8.558657
 22. Denecker, B.; Olyslager, F.; Knockaert, L. & Zutter, D. De. Generation of FDTD subcell equations by means of reduced order modeling. *IEEE Trans. Antennas Propag.*, 2003, **51**(8), 1806–1817. doi: 10.1109/TSP.2003.815439
 23. Xiao, K.; Pommerenke, D. & Drewniak, J. A three-dimensional FDTD subgridding algorithm with separated temporal and spatial interfaces and related stability analysis. *IEEE Trans. Antennas Propag.*, 2007, **55**(7), 1981–1990. doi: 10.1109/TAP.2007.900180
 24. Johns, P. B. & Beurle, R. L. Numerical solutions of 2-dimensional scattering problems using a transmission-line matrix. *In Proceedings of the IEEE, 1971*, **118**(9), 1203–1208. doi: 10.1049/piee.1971.0217
 25. Hoefer, W. Numerical techniques for microwave and millimeter wave passive structures. *In The transmission line matrix (TLM) method*. Wiley, 1989, pp. 451–496.
 26. Krumpholz, M. & Russer, P. A field theoretical derivation of TLM. *IEEE Trans. Microwave Theory Tech.*, 1994, **42**(9), 1660–1668. doi: 10.1109/22.310559
 27. Christopoulos, C. The transmission-line modeling method: TLM. IEEE Press, 1995.
 28. Lee, J. F.; Lee, R. & Cangellaris, A. Time-domain finite-element methods. *IEEE Trans. Antennas Propagation*, 1997, **45**(3), 430–442. doi: 10.1109/8.558658
 29. Yioultsis, T. V.; Kantartzis, N. V.; Antonopoulos, C. S. & Tsiboukis, T. D. A fully explicit Whitney element-time domain scheme with higher-order vector finite elements for three-dimensional high frequency problems. *IEEE Trans. Magnetics*, 1998, **34**(5), 3288–3291. doi: 10.1109/20.717772
 30. Weiland, T. A discretization method for the solution of Maxwell's equations for six component fields. *Electron.*

- Commun. AEU*, 1977, **31**(3), 116–120.
31. Weiland, T. Time domain electromagnetic field computation with finite difference methods. *Int. J. Numerical Modelling: Electron. Networks, Devices Fields*, 1996, **9**(4), 295–319.
 32. Madsen, N.K. & Ziolkowski, R.W. A three-dimensional modified finite volume technique for Maxwell's equations. *Electromagnetics*, 1990, **10**(1-2), 147–161.
 33. Holland, R.; Cable, V. P. & Wilson, L. C. Finite-volume time-domain (FVTD) techniques for EM scattering. *IEEE Tran. Electromagnetic Compatibility*, 1991, **33**(4), 281–294.
doi: 10.1109/15.99109
 34. Alvarez, J.; Angulo, L. D.; Bretones, A. R. & Garcia, S. A spurious free discontinuous Galerkin time-domain method for the accurate modeling of microwave filters. *IEEE Trans. Microwave Theory Techniques*, 2012, **60**(8), 2359–2369.
doi: 10.1109/TMTT.2012.2202683
 35. Hesthaven, J. S. & Warburton, T. Nodal discontinuous galerkin methods: Algorithms, analysis, and applications. Springer Publishing Company, 2007.
 36. Chen, J. & Liu, Q. H. Discontinuous Galerkin time-domain methods for multiscale electro-magnetic simulations: A review. *In Proceedings of the IEEE 101*, 2013, pp. 242–254.
doi: 10.1109/JPROC.2012.2219031
 37. Branin Jr., F.H. Problem analysis in science and engineering. *In The network concept as a unifying principle in engineering and the physical sciences*. Academic Press, 1977, pp. 41–111.
 38. Tonti, E. Finite formulation of the electromagnetic field. *Progress Electromagnetics Research*, 2001, **32** pp. 1–44.
doi: 10.2528/PIER00080101
 39. Sankaran, K. Beyond DIV, CURL and GRAD: Modelling electromagnetic problems using algebraic topology. *J. Electromagnetic Waves Applications*, 2017, **31**, 121–149.
doi: 10.1080/09205071.2016.1257397
 40. Grady, L. J. & Polimeni, J. R. Discrete Calculus - Applied analysis on graphs for computational science. Springer-Verlag, 2010.
 41. Fomenko, A. Visual geometry and topology. Springer-Verlag, 1994.
 42. Langefors, B. Algebraic topology and networks. Technical report, Svenska Aeroplan Aktiebolaget, 1959. Technical Report TN 43.
 43. Aakash; Bhatt, A. & Sankaran, K. How to model electromagnetic problems without using vector calculus and differential equations? *IETE J. Education*, 2018, **59**(2), 85-92.
doi: 10.1080/09747338.2018.1554456
 44. Aakash; Bhatt, A. & Sankaran, K. Transcending limits: Recent trends & challenges in computational electromagnetics. *In IEEE-INAE Workshop on Electromagnetics - IIWE*, 2018, Thiruvananthapuram, India.
 45. Aakash; Bhatt, A. & Sankaran, K. Algebraic topological method: An alternative modelling tool for electromagnetics. *In URSI Asia Pacific Radio Science Conference AP-RASC*, 2019, New Delhi, India.
 46. Pohl, R.W. Physical principles of electricity and magnetism. Blackie & Son Ltd., 1933.
 47. Prytz, K. Electrodynamics: The Field-Free Approach (Electrostatics, Magnetism, Induction, Relativity and Field Theory). Springer, 2015.
 48. Tonti, E. The mathematical structure of classical and relativistic physics - A general classification diagram. Birkhauser, Basel, 2013.
 49. Berenger, J.P. Three-dimensional perfectly matched layer for the absorption of electromagnetic waves. *J. Computational Phy.*, 1996, **127**(2), 363–379.
doi: 10.1006/jcph.1996.0181
 50. Sankaran, K.; Fumeaux, C. & Vahldieck, R. Hybrid PML-ABC truncation techniques for finite-volume time-domain simulations. *In Proceedings of the Asia-Pacific Microwave Conference APMC*, 2006, pp. 949–952.
doi: 10.1109/APMC.2006.4429569
 51. Sankaran, K.; Fumeaux, C. & Vahldieck, R. Split and unsplit finite-volume absorbers: Formulation and performance comparison. *In Proceedings of the European Microwave Conference (EuMA)*, 2006 pp. 17–20.
doi: 10.1109/EUMC.2006.281170
 52. Sankaran, K.; Fumeaux, C. & Vahldieck, R. An investigation of the accuracy of finite-volume radial domain truncation technique. *In Workshop on Computational Electromagnetics in Time-Domain IEEE*, 2007, pp. 1–4.
doi: 10.1109/CEMTD.2007.4373520
 53. Roden, J. A. & Gedney, S. D. Convolution PML (CPML): An efficient FDTD implementation of the CFS-PML for arbitrary media. *Microwave Optical Technol. Letters*, 2000, **27**(5), 334–339.
doi: 10.1002/1098-2760(20001205)27:5<334::AID-MOP14>3.0.CO;2-A
 54. Teixeira, F.L. & Chew, W.C. Complex space approach to perfectly matched layers: a review and some new developments. *Int. J. Numerical Modelling: Electron. Networks, Devices Fields*, 2000, **13**, 441–455.
doi: 10.1002/1099-1204(200009/10)13:5<441::AID-JNM376>3.0.CO;2-J
 55. Sankaran, K.; Fumeaux, C. & Vahldieck, R. Radial absorbers for conformal time-domain methods: A solution to corner problems in mesh truncation. *In IEEE/MTT-S International Microwave Symposium*, 2007, pp. 709–712.
doi: 10.1109/MWSYM.2007.380019
 56. Abarbanel, S. & Gottlieb, D. On the construction and analysis of absorbing layers in CEM. *Applied Numerical Mathematics*, 1998, **27**(4), 331–340.
doi: 10.1016/S0168-9274(98)00018-X
 57. Sankaran, K.; Kaufmann, T.; Fumeaux, C. & Vahldieck, R. Different perfectly matched absorbers for conformal time-domain method: A finite-volume time-domain perspective. *In 23rd Annual Review of Progress in Applied Computational Electromagnetics ACES*, 2007, pp. 1712–1718.
 58. Sankaran, K. Computational electromagnetics &

- applications, NPTEL online programme. <https://onlinecourses.nptel.ac.in> (Accessed on 28 December 2018).
59. Caris, M.; Stanko, S.; Palm, S.; Sommer, R. & Pohl, N. Synthetic aperture radar at millimeter wave-length for UAV surveillance applications. *In Proceedings of the 1st International Forum on Research and Technologies for Society and Industry Leveraging a better tomorrow (RTSI) IEEE, 2015.*
doi: 10.1109/RTSI.2015.7325145
 60. Tessmann, A.; Kudszus, S.; Feltgen, T.; Riessle, M.; Sklarczyk, C. & Haydl, W.H. Compact single-chip W-band FMCW radar modules for commercial high-resolution sensor applications. *IEEE Trans. Microwave Theory Tech.*, 2002, **50**, 2995–3001.
doi: 10.1109/TMTT.2002.805162
 61. Ghanmi, H.; Khenchaf, A. & Comblet, F. Numerical simulation of bistatic electromagnetic scattering by contaminated sea surface. *In International Radar Conference, 2014*, pp. 1–4.
doi: 10.1109/RADAR.2014.7060310
 62. Hubbert, J. C. & Bringi, V. N. Studies of the polarimetric covariance matrix. part II: Modeling and polarization errors. *J. Atmospheric Oceanic Technol.*, 2003, **20**(7), 1011–1022.
doi: 10.1175/1456.1
 63. Ulaby, F. T. & Elachi, C. Radar polarimetry for geoscience applications. Artech House, Inc., 1990.
 64. Brookner, E. Metamaterial advances for radar and communications. *In Proceedings of the IEEE International Symposium on Phased Array Systems and Technology (PAST) IEEE, 2016.*
doi: 10.1109/ARRAY.2016.7832577
 65. Yurduseven, O.; Marks, D. L.; Gollub, J. N. & Smith, D. R. Design and analysis of a reconfigurable holographic metasurface aperture for dynamic focusing in the Fresnel zone. *Access IEEE, 2017*, **5**, pp. 15055–15065.
doi: 10.1109/ACCESS.2017.2712659
 66. Xin, W.; Binzhen, Z.; Wanjun, W.; Junlin, W. & Junping, D. Design and characterization of an ultrabroadband metamaterial microwave absorber. *IEEE Photonics J.*, 2017, **9**(3), 1-13.
doi: 10.1109/JPHOT.2017.2700056
 67. IEEE recommended practice for validation of computational electromagnetics computer modeling and simulations. *IEEE Std 1597.2-2010, 2011*, pp. 1–124.
doi: 10.1109/IEEESTD.2011.5721917

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