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# Rapid Transfer Alignment of SINS with Measurement Packet Dropping-based on a Novel Suboptimal Estimator

Hongde Dai[#], Juan Li[*], Liang Tang[$], and Xibin Wang[#]

[#]*School of Basic Sciences for Aviation, Naval Aviation University, Yantai - 264 001, China*
[*]*College of Mathematics and Statistics, Ludong University, Yantai - 264 025, China*
[$]*Laishan Airport, Yantai - 264 007, China*
[*]*E-mail: daidaiquanquan123@126.com*

**ABSTRACT**

Transfer alignment (TA) is an important step for strapdown inertial navigation system (SINS) starting from a moving base, which utilises the information proposed from the higher accurate and well performed master inertial navigation system. But the information is often delayed or even lost in real application, which will seriously affect the accuracy of TA. This paper models the stochastic measurement packet dropping as an independent identically distributed (IID) Bernoulli random process, and introduces it into the measurement equation of rapid TA, and the influence of measurement packet dropping is analysed. Then, it presents a suboptimal estimator for the estimation of the misalignment in TA considering the random arrival of the measurement packet. Simulation has been done for the performance comparison about the suboptimal estimator, standard Kalman filter and minimum mean squared estimator. The results show that the suboptimal estimator has better performance, which can achieve the best TA accuracy.

**Keywords:** Transfer alignment; Strapdown inertial navigation system; Estimation; Kalman filter; Packet dropping

## 1. INTRODUCTION

Inertial navigation system (INS) applies the inertial properties of inertial sensors mounted aboard the vehicle to execute the navigation function, which is capable of continuous determination of the vehicle position, velocity and attitude without using of external information. This property provides the INS with obvious advantages, not only for military applications but also for civil applications[1]. Recent advances in computer technology combined with the development of suitable sensors have allowed the exploitation of the strapdown inertial navigation system (SINS), which rigidly connects all inertial sensors to the host vehicle, obtains the potential benefits about lower cost, reduced size and enhanced reliability[2]. Initial alignment is the process whereby the orientations of the axes of SINS are determined with respect to the reference axis system, which is vital to the starting of the SINS. It is easy to complete the initial alignment process on a statistic basis such as on the ground utilising the gyro compass theorem, but it is difficult to successfully complete the initial alignment on a moving base such as a ship or an aircraft.

Transfer alignment (TA) is an efficient initial alignment procedure for the slave SINS on moving base, which utilises the precise information from a high accurate INS (usually called as master INS or SINS). The speed and accuracy of the TA directly affects the rapid reaction and accuracy of the slave SINS. So rapid TA has gained much attention in the past few decades, which was first researched by Kain and Cloutier[3]. They presented a rapid TA approach which allowed alignment in less than ten seconds with an accuracy of less than one *mrad* without the requirement for aircraft's lateral manoeuvres. The measurements of this TA approach combine velocity and attitude from the master and slave INS, and this approach applies standard Kalman filter to estimate the misalignment of the slave SINS. This rapid TA has been further studied by scholars all over the world[4-8]. A simple pre-filter for Kain's rapid TA method was introduced, which results in a new rapid TA filter which is more efficient in terms of throughput and data transfer rate requirement, and has higher performance with accuracy of 1 *mrad* and rapid convergence less than six seconds are maintained[4]. Shortelle etc. carried out a series of laboratory and F-16 flight tests at Eglin AFB, the results of which also demonstrated that the rapid TA can achieve sub-milliradian alignment accuracy in less than ten seconds[5]. The system model and measurement model introduced in Ref 6 are derived from Ref 3, and gave a simulation and analysis of the model. The author derived Kain's rapid TA model from the aspect of quaternion[9]. To solve the large misalignment problem, Hao[11] introduced a nonlinear model for the rapid TA, and the misalignment was estimated by unscented Kalman filter[10]. The authors applied the quaternion instead of Euler angle in the system and measurement model, the rapid TA model is extended to nonlinear error model. The measurement model was then changed to quaternion plus velocity, and the results showed high accuracy and rapid performance under large misalignment[12].

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The works above all assume that the measurement transferred from the master SINS can arrive at the slave SINS on time, but measurement time-delay or even measurement packet dropping frequently occur in real engineering applications. Lim concentrated on this problem first, and derived an error compensation method based on delay state augmentation[13]. They analysed the effect of time-delay on velocity matching TA[14], it is found that the effect of time-delay on TA by velocity matching is more severe under swerves and continuance manoeuvres conditions. The authors of this paper studied a method based on the prediction of the attitude matrix for solving the problem of measurement time-delay of master SINS in velocity plus attitude matching rapid TA, which can effectively improve the accuracy of TA[15]. Developed an effective correction method, using a polynomial fitting interpolation to calculate the slave SINS information based on the master SINS time mark Ref 16. The effect of time-delay in attitude angle matching TA is discussed in Ref 17, and compensated the transmission time-delay by delaying the salve SINS's information, and compensated the random time-delay by the time state augmentation method. Recently Lyu and Chen[18] analysed the time-delay in TA process, and proposed a H∞ filter method with delay compensation. This adaptive H∞ filter with time-delay compensation can adaptively adjust the value of the robustness factor according to the dynamic external environment. The TA accuracy and the pure inertial navigation accuracy can be dramatically improved.

From the perspective of control theory, significant delay is equivalent to loss, because the data needs to arrive to its destination in time which is to be used for control[19,20]. According to the analysis in Ref 13-18, when the measurement information from the master SINS is dropped, the performance of TA will be reduced. However, TA under measurement packet dropping has not been studied elsewhere, and it is of significant interest to study how will the packet dropping affect the performance of the TA estimator.

Considering the estimation problem of measurement packet dropping, the arrival of the measurement data is modelled as an IID sequence. Nahi[21] derived a minimum mean squared estimator (MMSE), which is of recursive form similar to the standard Kalman filter utilising the statistics of the unobserved binary uncertainty sequences. Based on the discrete Kalman filtering formula, Sinopoli[20] studied the measurement packet dropping problem and modelled the arrival of the observation as a stochastic process, the statistical convergence of the estimation error covariance is studied, and the existence of a critical value for the arrival rate of observed values is proved under a new performance index. A suboptimal estimator (SoE) is presented by Zhang[22] to solve the estimation problem with measurement packet dropping which can improve the performance of MMSE and standard Kalman filter.

Motivated by the above works to solve the TA problem when the measurement packet is dropping, we modeled the arrival of the information from the master SINS as a statistic process, and introduced it into the measurement model of the TA. Then, we studied the SoE for the application in estimating the misalignment of TA, and compared it with standard Kalman filter and MMSE.

## 2. PROBLEM STATEMENT ABOUT THE TRANSFER ALIGNMENT WITH MEASUREMENT PACKET DROPPING

In the research of the TA, the system model and the measurement model are the most important issues[23], and the models always come from the navigation system error equations. Kain[3] introduced a rapid TA model for the initial alignment of air-to-ground weapon, which allows the process of alignment to be finished in less than ten seconds with accuracy of less than one *mrad* without the requirement for aircraft lateral manoeuvres. This work has attracted worldwide attention, Shortelle[5] reported the application of this rapid TA method in the F-16, and verified the effectiveness of this method through actual flight tests. In addition, the authors also studied the rapid TA problem of the SINS[7,24,25]. In this section, the concept and implementation procedure of TA are introduced first, and then followed by the system model and the measurement model, based on these, measurement packet dropping is described in mathematical.

### 2.1 Transfer Alignment of SINS

Directly copy position, velocity and attitude data from the master SINS to the slave SINS is the simplest TA technique, which is called as 'one shot' alignment process[7,26]. It is clear that angular displacement between the master and slave SINS will appear as an alignment error in the slave SINS. Furthermore, the slave and master SINS are always positioned at distance from each other, and there will be relative motion between them while the vehicle turns or manoeuvres, this will introduce additional errors into the TA. The TA method based on inertial measurement matching have received extensive attention in recent years[18,27-40], which can be depicted in Fig. 1.

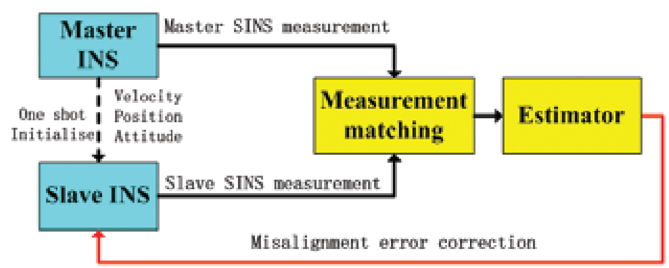


**Figure 1. Scheme about the measurement matching TA.**

### 2.2 System and Measurement Models for Rapid Transfer Alignment

The design of the estimator is an important issue in TA. But the transmission of the state and measurement presented by the state should be determined first[41], which is called as system model and measurement model presented as:

$$X_{k+1} = A_k X_k + W_k \tag{1}$$

$$Y_k = H_k X_k + V_k \tag{2}$$

where the state vector of system at time instant $k$ is $X_k$, the system dynamics matrix is $A_k$, and the measurement vector is $Y_k$, the measurement matrix is $H_k$, which expresses the relation between the measurement and the state, $W_k$ and $V_k$ are white, zero-mean, uncorrelated noise processes, the covariance matrices of $W_k$ and $V_k$ are $Q_k$ and $R_k$ respectively. In standard Kalman filter, the dimension of the process noise vector is sometimes smaller than the dimension of the state vector. In our study, the process noise vector contains gyro drift errors, accelerator bias errors and flexure errors in three directions respectively, so in our study the dimension of the process noise vector is equal to the dimension of the state vector.

The system model describes the system dynamics. In the TA, the most important state is the misalignment, and the traditional system model is converted from the inertial error model. The rapid TA model is presented in Eqn. (3):

$$\dot{\psi}_a = \eta_a \tag{3}$$

The physical misalignment between the master and slave SINS is define as $\psi_a$. $\eta_a$ is the white noise process with strength given by the diagonal matrix $Q_a$, the values along the diagonal of $Q_a$ were selected by the analysis of engineering process.

$$\dot{\psi}_m = (\psi_m - \psi_a)\times\hat{\omega}_{nsr}^{sr} + \omega_{fs}^{sr} + \varepsilon^{sr} \tag{4}$$

The computed misalignment between the master and slave SINS is define as $\psi_m$. $\hat{\omega}_{nsr}^{sr}$ is the rotation rate between the real SINS and the navigation frame, described in the real SINS frame, which is deduced from the output of the slave gyroscopes. The flexible rotation rate of the body is define as $\omega_{fs}^{sr}$. $\varepsilon^{sr}$ and $\nabla^{sr}$ are the gyro drift and accelerator bias, respectively.

$$\delta\dot{V} = C_{sc}^{n}(\psi_m - \psi_a)\times\hat{f}_{sr}^{sr} + C_{sc}^{n}(f_f^{sr} + \nabla^{sr}) \tag{5}$$

where $\delta V$ is the computed difference between master and slave velocities. $C_{sc}^{n}$ is the estimated direction cosine transformation matrix from body to navigation frame. $\hat{f}_{sr}^{sr}$ is the measured slave acceleration vector in the slave body axes, $f_f^{sr}$ is the flexible body acceleration stochastic process.

Rapid TA of SINS on moving base, the system state in (1) is $X = [\psi_{mx}, \psi_{my}, \psi_{mz}, \delta V_x, \delta V_y, \delta V_z, \psi_{ax}, \psi_{ay}, \psi_{az}]$. The initial state of $\psi_m$ and $\delta V$ are identically zero since the slave SINS is initialised with the master SINS data. The initial state of $\psi_a$ corresponds to the uncertainty in the physical misalignment. The system matrix $A_k$ is determined according to Eqns. (3) - (5).

The measurements consist of velocity and attitude difference between the master and slave SINS. The computed attitude error measurement $\psi_m$ can be obtained by:

$$\left(C_{sc}^{n}\right)^T * C_m^n = I - \psi_m \times \tag{6}$$

Minus the velocity from master and slave SINS can obtain the velocity error measurement $\delta V$:

$$\delta V = V_{sc}^{n} - V_m^n \tag{7}$$

According to Eqns. (6) and (7), the measurement matrix in (2) is

$$H = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{8}$$

### 2.3 Description of the Measurement Packet Dropping

The measurement packet from matching step may be dropped because of unreliable transmission link, under this circumstance, the measurement can be given by

$$\begin{aligned} Y_k &= H_k X_k + V_k \quad \text{with probability } p \\ &= V_k \quad \text{with probability } 1-p \end{aligned} \tag{9}$$

The probability of false alarm is defined as $1-p(k)$ in (9), the probability that the measurement vector contains information about the state is $p(k)$, and the above measurement equation can be defined equivalently by

$$Y_k = \gamma_k H_k X_k + V_k \tag{10}$$

The arrival of the measurement can be considered as a Bernoulli process, that is $\gamma_k$ indicating the arrival of the measurement, which is a scalar quantity taking on values of 0 and 1.

$$\begin{aligned} &\Pr\{\gamma_k = 1\} = p \\ &\Pr\{\gamma_k = 0\} = 1-p \end{aligned} \tag{11}$$

where $0 < p < 1$, and $\gamma_k$ is assumed to be observed at every time instant by the time stamp technique, that is $\gamma_k$ together with measurement are arrived at the estimator, then the TA as shown in Fig. 1 can be changed to Fig. 2.

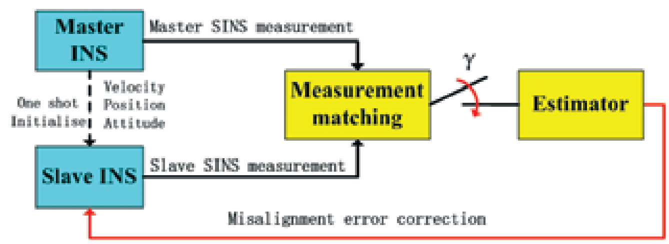


**Figure 2. Scheme about the measurement matching TA with measurement packet dropping.**

## 3. SUBOPTIMAL ESTIMATOR DESIGN FOR THE TRANSFER ALIGNMENT WITH MEASUREMENT PACKET DROPPING

### 3.1 Standard Kalman Filter

The standard Kalman filter (KF) is a powerful tool for the estimation problems as shown in Eqns. (1) and (2), under general assumption[41], which contains two steps: predict and correct as following:

**Predict**: propagate the state and the covariance with time according to the system equation beginning with the initial condition.

$$\hat{X}_{k+1/k} = A_k \hat{X}_k \tag{12}$$

$$P_{k+1/k} = A_k P_k A_k^T + Q_k \tag{13}$$

**Correct**: in the correct step, first calculate the Kalman filter gain, then the predicted prior state is corrected by plus the innovation weighted by the Kalman filter gain, additionally, the posterior covariance is recursively update:

$$K_k = P_{k+1/k} H_k^T \left(H_k P_{k+1/k} H_k^T + R_k\right)^{-1} \tag{14}$$

$$\hat{X}_{k+1} = \hat{X}_{k+1/k} + K_k\left(Y_k - H_k \hat{X}_{k+1/k}\right) \tag{15}$$

$$P_{k+1} = \left(I - K_k H_k\right) P_{k+1/k} \left(I - K_k H_k\right)^{-1} + K_k R_k K_k^{-1} \tag{16}$$

It is clear that in the standard Kalman filter, the measurement should be arrived sequentially on time.

### 3.2 Minimum Mean Square Estimator

If the measurement packet is dropped randomly as described in Eqn. (10), how it will affect the performance of the estimator? Early work modelled the uncertainty as a sequence of IID. The binary random variables indicate the measurement arrival[21], applied the minimum mean squared estimation theorem, and derived a Kalman filter liked recursive estimator utilising the statistics of the arrival of the measurement sequence. The algorithm is as follows:

$$\hat{X}_k = F_{1(k-1)}\hat{X}_{k-1} + F_{2(k-1)}Y_{k-1} \tag{17}$$

where

$$F_{1(k-1)} = A_k - pF_{2(k-1)}H_k \tag{18}$$

$$F_{2(k-1)} = pA_k P_{(k-1)} H_k^T \{R_k + p^2 H_k P_{k-1} H_k^T + p(1-p)H_k S_{k-1} H_k^T\}^{-1} \tag{19}$$

$$S_k = A_k S_{k-1} A_k^T + Q_k \tag{20}$$

The covariance equation is given by

$$P_k = [A_k - pF_{2(k-1)}H_k]P_{k-1}A_k^T + Q_k \tag{21}$$

where $p$ is the probability about the arrival of the measurement defined in Eqn. (11). This optimal estimator is called as minimum mean square estimator (MMSE). $S_0 = P_0 = E[X_0 X_0^T]$ is the initial condition.

### 3.3 Suboptimal Estimator

Suboptimal estimator is derived by minimising the mean squared estimation error over a packet dropping condition[22], which is designed by solving a deterministic Riccati equation. The newly presented estimator has a smaller error covariance compared with the MMSE.

$$\hat{X}_k = \left[A_k - \gamma_{k-1}K_{p(k-1)}H_k\right]\hat{X}_{k-1} + K_{p(k-1)}Y_{k-1} \tag{22}$$

where

$$\begin{aligned} K_{p(k-1)} &= pA_k P_{k-1} H_k^T M_{k-1}^{-1} \\ M_{k-1} &= pH_k P_{k-1} H_k^T + R_k \end{aligned} \tag{23}$$

The covariance is

$$P_k = A_k P_{k-1} A_k^T + Q_k - K_{p(k-1)} M_{k-1} K_{p(k-1)}^T \tag{24}$$

where $p$ is the probability about the arrival of the measurement as defined in Eqn. (11).

## 4. EXPERIMENT ANALYSIS ABOUT THE ESTIMATORS

### 4.1 Experiment settings

Simulations have been performed in this sub-section for the evaluation of the introduced estimator. Figure 3 shows the block diagram of TA simulation system. Followed by Ref[12], the simulation system is developed, where the vehicle motion is simulated first, which can set the motion manner of the vehicle; then, the IMU of the master and slave SINS is simulated respectively, which can sense the motion of the vehicle. the IMU is composed of three axes of gyros and accelerometers, only the output of slave IMU contain the flexure of the body and lever arm effect; then applying the strapdown inertial navigation algorithm to obtain the navigation information, which are the velocity, attitude and position of the vehicle. The accuracy of the master SINS is much higher than the slave SINS, then transfer the navigation parameters of the master SINS to the slave SINS, by comparing the information from master SINS and slave SINS, the measurement message of TA can be obtained. But the measurement packet may be dropped randomly during the transmission, this estimator is then used to estimate the misalignment of the slave SINS.

In simulation, we choose one classically utilised condition of the inertial navigation. The initial position is latitude $\varphi_0 = 37.9°$ and longitude $\lambda_0 = 121.7°$, the initial velocity is $V_x = V_y = 25$ m/s, the vehicle is considered as a large ship on the ocean, the initial attitude of the ship is [5,-5,30]°, the time period for SINS data update is 10 ms, and the alignment filter period is 50 ms, set the initial misalignment of X, Y and Z as [1,1,2]°.

### 4.2 Experiment Analysis

It is well known that the rapid TA can achieve the accuracy about 1 $mrad$ in 10 s. Simulations have been done under the condition that the arrival probability is $p$ = 0.9, 0.7, and 0.5 respectively. TA performances under different measurement arrival rates are as shown in Fig. 4 – Fig. 6, which are

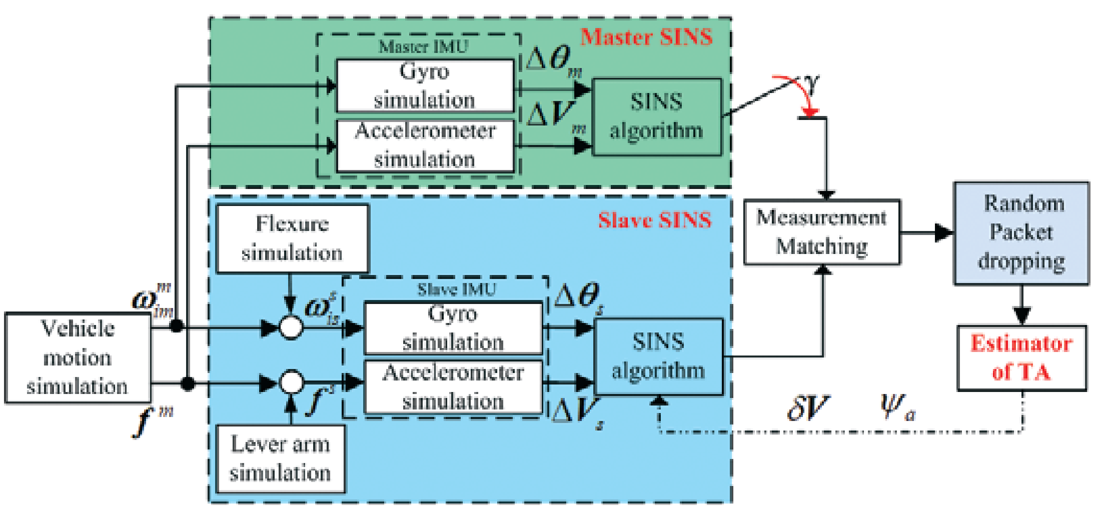


Figure 3. Block diagram of TA simulation system.

accompanied with the arrival indicator of the measurement. The numerical accuracies at the end of the TA are as shown in Table 1.

We can see from the simulation results that the estimation accuracies of the misalignment in TA are degrading while the measurement packet is dropping. Even the arrival rate is $p(\gamma)$=0.9, the tradition Kalman filter can't reach the accuracy less than 1 *mrad* within 10 s, and the accuracy will even be worse with the decline of the measurement arrival rate. The accuracy of the MMSE is better than the standard Kalman filter, but it can't reach the accuracy less than 1 *mrad* with 10 s. However, the suboptimal estimator can perform well even if the measurement arrival rate is $p(\gamma)$=0.5, the accuracy can reach 1 *mrad* with 10 s, which verifies the improved efficiency of the suboptimal estimator in rapid TA.

## 5. CONCLUSIONS

Rapid TA of SINS with measurement packet dropping is studied in this paper, the TA process and rapid TA model are introduced first, and the measurement packet dropping is modelled as an IID Bernoulli process, then a suboptimal

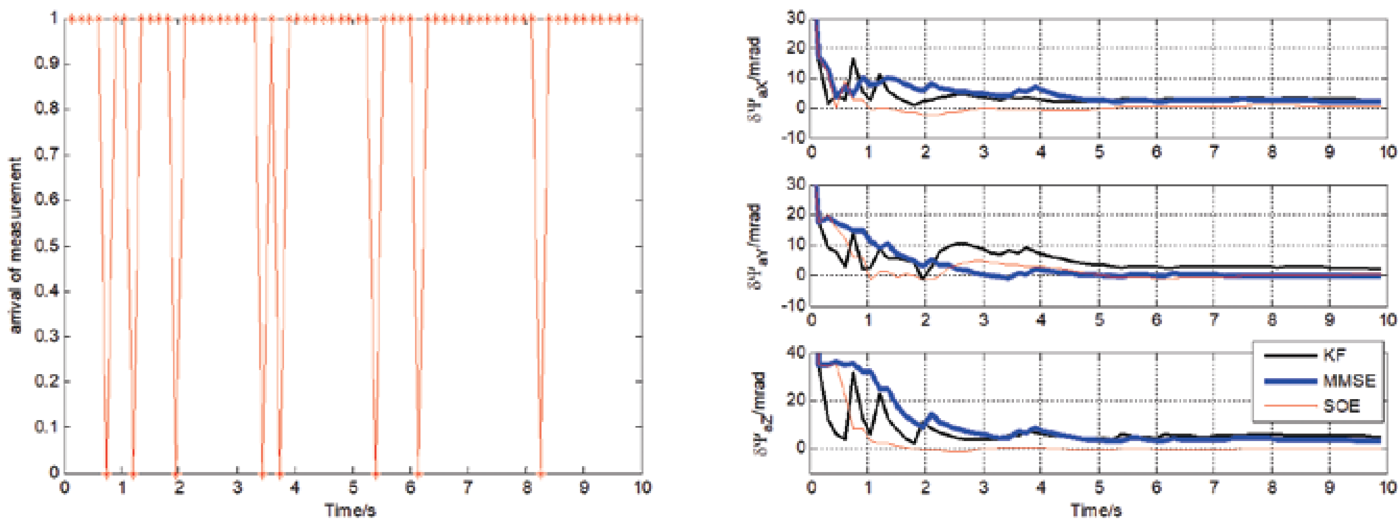


**Figure 4. Measurement packet dropping indicator and misalignment estimation of TA applying standard Kalman filter, minimum mean squared estimator and suboptimal estimator, with measurement arrival rate *p* (γ)=0.9.**

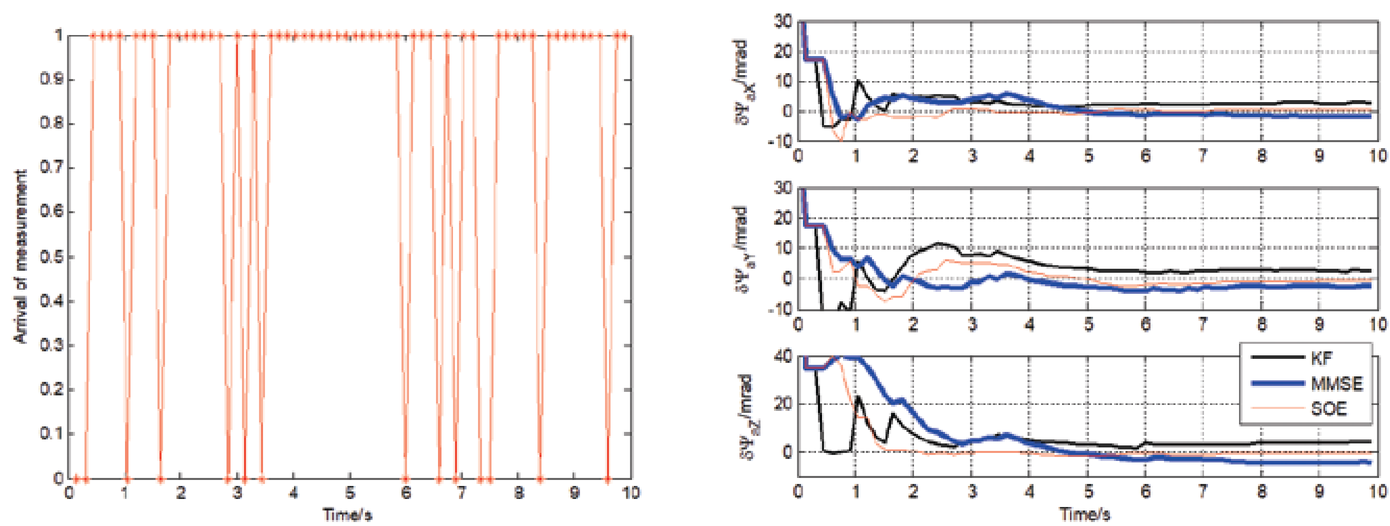


**Figure 5. Measurement packet dropping indicator and misalignment estimation of TA applying standard Kalman filter, minimum mean squared estimator and suboptimal estimator, with measurement arrival rate *p* (γ)=0.7.**

**Table 1. TA accuracy under different measurement arrival rates (*mrad*)**

| **Rate** | ***p* (γ) =0.9** | | | ***p*(γ) =0.7** | | | ***p* (γ)=0.5** | | |
|---|---|---|---|---|---|---|---|---|---|
| **Mis** | $\Psi_{ax}$ | $\Psi_{ay}$ | $\Psi_{az}$ | $\Psi_{ax}$ | $\Psi_{ay}$ | $\Psi_{az}$ | $\Psi_{ax}$ | $\Psi_{ay}$ | $\Psi_{az}$ |
| KF | 2.592 | 2.247 | 5.215 | 2.876 | 2.577 | 4.320 | 7.851 | 8.523 | 15.644 |
| MMSE | 2.110 | 0.025 | 3.511 | -1.643 | -2.419 | -4.057 | 5.616 | 1.411 | 9.728 |
| SOE | 0.609 | 0.158 | 0.314 | 0.363 | -0.619 | -0.522 | 0.398 | 0.920 | 0.732 |

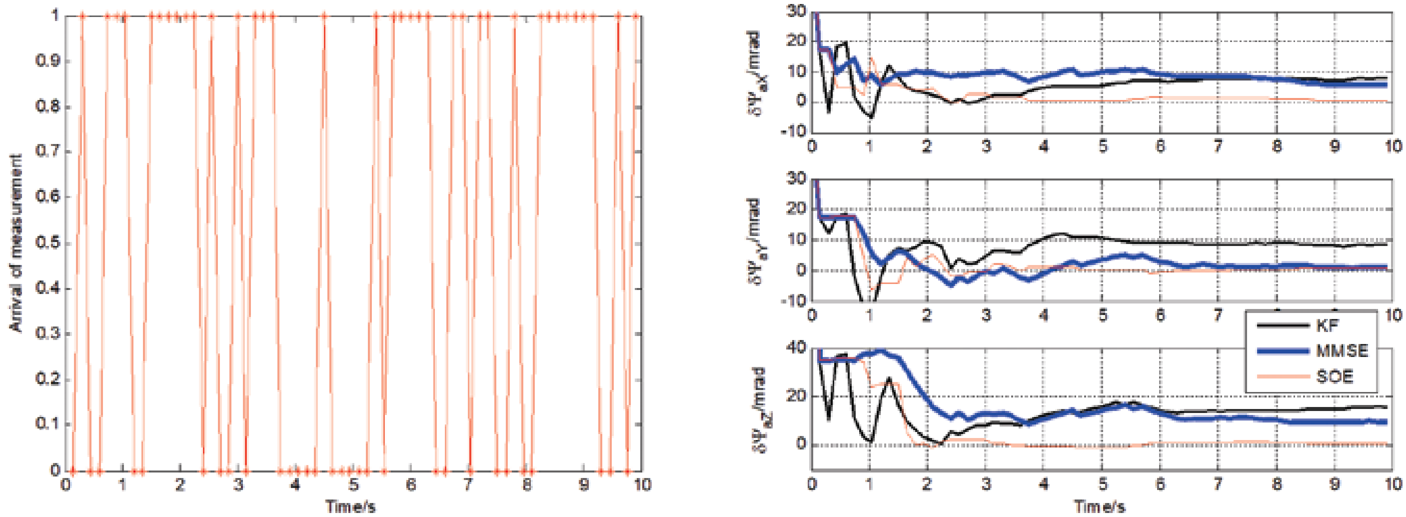


**Figure 6. Measurement packet dropping indicator and misalignment estimation of TA applying standard Kalman filter, minimum mean squared estimator and suboptimal estimator, with measurement arrival rate *p* (γ)=0.5.**

estimator is introduced for the estimation problem, as well as standard Kalman filter and minimum mean squared estimator. A simulation system is designed for the evaluation of the proposed suboptimal estimator. The simulation results are compared with the standard Kalman filter and MMSE, indicated that under the measurement packet dropping conditions, the suboptimal estimator can achieve an accuracy of the TA less than 1 *mrad* with 10 seconds, while the other two estimators are failed to complete the TA mission.

## REFERENCES


1. Britting, K.R. Inertial navigation systems analysis. John Wiley and Sons, Inc: New York, 2010.
2. Titterton, D.; Weston, J.L. Strapdown inertial navigation technology (2ed) . The Institution of Electrical Engineers, Stevenage Herts, 2004. doi: 10.1049/PBRA017E
3. Kain, J.E. & Cloutier, J.R. In Rapid transfer alignment for tactical weapon applications. *In* AIAA Guidance, Navigation and Control Conference, 1989, pp. 1290-1300. doi: 10.2514/6.1989-3581
4. Spalding, K. An efficient rapid transfer alignment filter. AIAA-92-4598-CP, 1992, pp. 1276-1286. doi: 10.2514/6.1992-4598
5. Shortelle, K.J.; Graham, W.R. & Rabourn, C. In F-16 flight tests of a rapid transfer alignment procedure. *In* Position Location and Navigation Symposium, IEEE 1998, pp. 379-386. doi: 10.1109/PLANS.1998.670185
6. Wang, D.; Zhang, H. & Dai, R. A method for rapid transfer alignment. *Aerospace Control,* 2000, **4,** 8-16. doi: 10.16804/j.cnki.issn1006-3242.2000.04.002
7. Dai, H.; Dai, S.; Cong, Y.; Zhao, G. & Wu, G. Rapid transfer alignment of laser sins using quaternion based angular measurement. *Optik,* 2013, **124**, 4364-4368. doi: 10.1016/j.ijleo.2013.01.057
8. Gao, S.; Wei, W.; Zhong, Y. & Feng, Z. Rapid alignment method based on local observability analysis for strapdown inertial navigation system. *Acta Astronautica* 2014, **94**, 790-798. doi: 10.1016/j.actaastro.2013.10.003
9. Shaowu, D.; Juan, L.; Hongde, D. & Shaolei, Z. Study of rapid transfer alignment error model. *J. Astronautics* 2009, **30**, 942-946. doi: 10.1109/CLEOE-EQEC.2009.5194697
10. Julier, S.; Uhlmann, J.; Durrant-Whyte, H.F. A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Trans. Automatic Control,* 2001, **45**, 477-482. doi: 10.1109/9.847726
11. Hao, Y.; Xiong, Z.; Wang, W. & Sun, F. Rapid transfer alignment based on unscented Kalman filter. *In* American Control Conference, 2006, pp. 2210-2215. doi: 10.1109/ACC.2006.1656548
12. Dai, H.; Zhou, S. & Chen, M. Quaternion based nonlinear error model for rapid transfer alignment. *J. Astronautics* 2010, **31**, 2328-2334. doi: 10.3873/j.issn.1000-1328.2010.10.015
13. Lim, Y.-c. & Lyou, J. An error compensation method for transfer alignment. *In* Proceedings of IEEE Conference on Electrical and Electronic Technology, TENCON, 2001, **2**, pp. 850-855. doi: 10.1109/TENCON.2001.949715
14. Liu, H.G.; Chen, Z.G. & Gang, C. Time-delay's effect on velocity matching transfer alignment. *J. Chinese Inertial Technol.,* 2012, **20**, 544-551. doi: 10.3969/j.issn.1005-6734.2012.05.009
15. Huang, G.; Dai, H. & Chen, M. Measurement time delay error compensation in rapid transfer alignment. *Measurement and Control Technology,* 2009, **28**, 55-57. doi: 10.3969/j.issn.1000-8829.2009.08.016
16. Xie, C.M.; Yan, Z. &Yang, C.C. Influence and compensation of time-mark discrepancy in transfer alignment. *J. Chin. Inertial Technol.*, 2010, **18**, 414-420. doi: 10.13695/j.cnki.12-1222/o3.2010.04.017

17. Gang, C.; Chao, Z. & Liu, H.G. Influence of time delay for attitude angle matching transfer alignment. *J. Chinese Inertial Technol.,* 2014, **22**, 172-176.
doi: 10.13695/j.cnki.12-1222/o3.2014.02.006
18. Lyu, W. & Cheng, X. A novel adaptive h∞ filtering method with delay compensation for the transfer alignment of strapdown inertial navigation systems. *Sensors,* 2017, **17**, 2753.
doi: 10.3390/s17122753
19. Gao, R.; Xu, J. & Zhang, H. Receding horizon control for multiplicative noise stochastic systems with input delay. *Automatica,* 2017, **81**, 390-396.
doi: 10.1016/j.automatica.2017.04.002
20. Sinopoli, B.; Schenato, L.; Franceschetti, M.; Poolla, K.; Jordan, M.I. & Sastry, S.S. Kalman filtering with intermittent observations. *Automatic Control, IEEE Trans.,* 2004, **49**, 1453-1464.
doi: 10.1109/TAC.2004.834121
21. Nahi, N.E. Optimal recursive estimation with uncertain observation. *Information Theory IEEE Trans.,* 1969, **15**, 457-462.
doi: 10.1109/tit.1969.1054329
22. Zhang, H.; Song, X. & Shi, L. Convergence and mean square stability of suboptimal estimator for systems with measurement packet dropping distributed. *IEEE Trans. Automatic Control,* 2012, **57**, 1248-1253.
doi: 10.1109/tac.2012.2191857
23. Rogers, R.M. Applied mathematics in integrated navigation systems. American Institute of Aeronautics and Astronautics Inc: Reston, 2007.
24. Dai, H.D.; Liu, A.L.; Lu, J.H. & Dai, S.W. Optimization about the layout of imus in large ship based on fruit fly optimization algorithm. *Optik-Int. J. Light Electron Optics*, 2015, **126**, 490-493.
doi: 10.1016/j.ijleo.2014.08.037
25. Dai, H.; Chen, M.; Zhou, S. & Li, J. A new rapid transfer alignment method and the analysis of observable degree. *Journal Astronautics* 2009, **30**, 1449-1454.
doi: 10.1109/CLEOE-EQEC.2009.5194697
26. Wang, Y. Sun, F. & Zhang, Y. Central difference particle filter applied to transfer alignment for SINS on missiles [J]. *IEEE Trans. Aerosp. Electron. Sys.*, 2012, **48**, 375-387.
doi:10.1109/taes.2012.6129642
27. Gong, X.; Liu, H.; Fang, J. & Liu, G. Multi-node transfer alignment based on mechanics modeling for airborne dpos. IEEE *Sensors J.,* 2018, **18,** 669-679.
doi: 10.1109/JSEN.2017.2771263
28. Lu, Z.; Fang, J.; Liu, H.; Gong, X. & Wang, S. Dual-filter transfer alignment for airborne distributed pos based on pvam. *Aerospace Sci. Technol.* 2017, **71**, 136-146.
doi: 10.1016/j.ast.2017.09.016
29. Chen, W.; Zeng, Q.; Liu, J. & Wang, H. Research on shipborne transfer alignment under the influence of the uncertain disturbance based on the extended state observer. *Optik - Int. J. Light Electron Optics,* 2017, **130**, 777-785.
doi: 10.1016/j.ijleo.2016.10.126
30. Cao, Q.; Zhong, M. &Guo, J. Non-linear estimation of the flexural lever arm for transfer alignment of airborne distributed position and orientation system. IET Radar, *Sonar Navigation* 2017, **11**, 41-51.
doi: 10.1049/iet-rsn.2015.0541
31. Lu, J.; Xie, L. & Li, B. Analytic coarse transfer alignment based on inertial measurement vector matching and real-time precision evaluation. *IEEE Trans. Instrument. Measurement,* 2016, **65**, 355-364.
doi: 10.1109/TIM.2015.2502879
32. Lu, J.; Xie, L. & Li, B. Applied quaternion optimization method in transfer alignment for airborne ahrs under large misalignment angle. *IEEE Trans. Instrument. Measurement,* 2016, **65**, 346-354.
doi: 10.1109/TIM.2015.2502838
33. Gong, X. & Zhang, J. An innovative transfer alignment method based on federated filter for airborne distributed pos. *Measurement,* 2016, **86,** 165-181.
doi: 10.1016/j.measurement.2016.02.016
34. Gao, Z.; Mu, D.; Gao, S.; Zhong, Y. & Gu, C. Robust adaptive filter allowing systematic model errors for transfer alignment. *Aerospace Sci. Technol.,* 2016, **59**, 32-40.
doi: 10.1016/j.ast.2016.10.002
35. Chen, W.; Zeng, Q.; Liu, J. & Wang, H. Research on shipborne transfer alignment under the influence of the uncertain disturbance based on the extended state observer. *Optik - Int. J. Light Electron Optics,* 2016, **130**, 777-785.
doi: 10.1016/j.ijleo.2016.10.126
36. Chen, H.; Cheng, X.; Liu, F. & Liu, F. Robust stability analysis of h∞-sgqkf and its application to transfer alignment. *Signal Processing,* 2015, **117,** 310-321.
doi: 10.1016/j.sigpro.2015.05.019
37. Yang, G.L.; Wang, Y.Y. & Yang, S.J. Assessment approach for calculating transfer alignment accuracy of sins on moving base. *Measurement,* 2014, **52,** 55-63.
doi: 10.1016/j.measurement.2014.03.014
38. Lu, Y. & Cheng, X. Random misalignment and lever arm vector online estimation in shipborne aircraft transfer alignment. *Measurement,* 2014, **47**, 756-764.
doi: 10.1016/j.measurement.2013.09.032
39. Liu, X.; Xu, X.; Liu, Y. & Wang, L. A fast and high-accuracy transfer alignment method between m/s ins for ship based on iterative calculation. *Measurement,* 2014, **51**, 297-309.
doi: 10.1016/j.measurement.2014.02.019
40. Gong, X.; Fan, W. & Fang, J. An innovational transfer alignment method based on parameter identification ukf for airborne distributed pos. *Measurement,* 2014, **58**, 103-114.
doi: 10.1016/j.measurement.2014.08.034
41. Dan, S. Optimal state estimation: Kalman, h∞, and nonlinear approaches. *Wiley*-Interscience, 2006.
42. Jones, D.; Roberts, C.; Tarrant, D.; Yang, C. & Lin, C.F. In Transfer alignment design and evaluation environment. *In* Proceedings, Aerospace Control Systems. The First IEEE Regional Conference on, 1993, pp. 753-757.
doi: 10.1109/AEROCS.1993.721034

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## CONTRIBUTORS

**Dr Hongde Dai** received his BS, MSc and PhD in 2004 and 2008, all from Northwestern Polytechincal University, now he is an Associate Professor with School of Basic Sciences for Aviation, Naval Aviation University. His main research interests include inertial technology and integrated navigation, filtering and estimation theory, and PHM of airborne system.
He has introduced the original idea of this work, conceived and designed the experiments. He has also contributed in writing the research paper.

**Dr Juan Li**, received her MSc in 2006 from Northwestern Polytechincal University, and PhD in 2018 from Air Force Engineering University. Now she is lecturer in Lu Dong University, her main research interests include statistical modelling, prognosis and health management, remaining useful life estimation.
She has derived the estimator, analysed the experimental data, and also contributed in the paper writing.

**Mr Liang Tang,** received his MSc from Department of Control Engineering, Naval Aeronautical and Astronautical University, Now he is a director of special equipment of unit 91213, engineer, research on aviation special equipment.
He has helped to analysed the experimental data, also contributed in the paper writing.

**Dr Xibin Wang,** received his MSc in 2009 and PhD in 2012, all from Naval Aeronautical and Astronautical University. now he is a lecturer with School of Basic Sciences for Aviation, Naval Aviation University. His main research interests include filtering and estimation theory, and aviation special equipment.
He has contributed in the filter design and paper writing.