

Fuzzy Logic-based Adaptive Extended Kalman Filter Algorithm for GNSS Receivers

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ABSTRACT

Designing robust carrier tracking algorithms that are operable in strident environmental conditions for global navigation satellite systems (GNSS) receivers is the discern task. Major contribution in weakening the GNSS signals are ionospheric scintillations. The effect of scintillation can be known by amplitude scintillation index S_4 and phase scintillation index σ_ϕ parameters. The proposed fuzzy logic based adaptive extended Kalman filter (AEKF) method helps in modelling the signal amplitude and phase dynamically by Auto-Regressive Exogenous (ARX) analysis using Sugeno fuzzy logic inference system. The algorithm gave good performance evaluation for synthetic Cornell scintillation monitor (CSM) data and real-time strong scintillated PRN 12 L1 C/A data on October 24th, 2012 around 21:30 h, Brazil local time collected by GNSS software navigation receiver (GSRN'x). Fuzzy logic algorithm is implemented for selecting the ARX orders based on estimated amplitude and phase ionospheric scintillation observations. Fuzzy based AEKF algorithm has the capability to mitigate ionospheric scintillations under both geomagnetic quiet and disturbed conditions.

Keywords: Ionospheric scintillations; Adaptive extended Kalman filter; GSNR; Fuzzy logic

Nomenclature

A_k	Received amplitude
C_k	Original amplitude
$\rho_{s,k}$	Amplitude (with scintillations)
θ_k	Total phase
$\theta_{d,k}$	Phase (with dynamics)
$\theta_{s,k}$	Phase (with scintillations)
S_4	Amplitude scintillation index
σ_ϕ	Phase scintillation index
S	Signal strength
ϕ	Detrended phase
μ_A	Membership function
$f_{d,k}$	Doppler frequency shift
T_s	Sampling time
k	Discrete time step
$f_{r,k}$	Doppler frequency rate
$\psi_k(l)$	Time series input
β_{k-i}	Exogenous AR coefficients
η_k	Gaussian variance
K	Overall transition matrix
Q_k	Process noise covariance
H_k	Measurement vector
R_k	Measurement noise
$\sigma_{n_0}^2$	Noise variance
P_k	Model covariance
KG	Kalman gain

1. INTRODUCTION

One of the major sources of GNSS signal degradation are ionospheric scintillations. There will be severe disturbance in signal amplitude and phase while passing through trans-ionospheric region that holds irregular plasma densities¹. The PLL carrier tracking technique available in global navigation satellite system receivers comes under fixed bandwidth which cannot subsisted a tradeoff between phase dynamics and noise reduction². The conventional GNSS receivers are unable to take care of these two problems i.e. dynamics and mitigating the erroneous scintillations that comes with received signal.

The raw pre-processed data in complex In and Quadrature-phases (I and Q) form get disturbed by the ionospheric scintillations³. The amplitude scintillation index S_4 and phase scintillation index σ_ϕ parameters are defined to measure the influence of ionospheric scintillation on received signal⁴. Modelling ionospheric scintillations is possible by fixing the coefficients of auto-regressive (AR) method for amplitude and phase of a signal as given in with adaptive Kalman filter⁵.

Fuzzy Logic tool understands system to think in a human perspective⁶. The tool well establishes a relation between the scintillation indices and orders for AR method. The sequence of steps that involve in FIS is fuzzification, logic and de-fuzzification. Mao⁷, *et al.* gave a successful method to implement fuzzy logic in design of intelligent PLL for GNSS receivers.

The phase dynamics are additional phase variations that occur because of changes in the position of satellite and receiver. The available PLL cannot withstand with the effects on phase

by dynamics and scintillations⁸. Prolific research taken place in improving the carrier tracking algorithm to make GNSS receiver self-sufficient to work even in stressful propagation conditions. Lopez-Salcedo⁹, *et al.* furnishes a detail survey on advanced carrier tracking algorithms.

Kalman Filter (KF) methodology took a huge knock in design of GNSS carrier tracking algorithms. It involves the process of prediction and correction through a sequence of steps¹⁰. A quality research takes place in including phase dynamics, amplitude and phase scintillation parameters in algorithm of KF, Extended Kalman Filter (EKF) and Adaptive Kalman Filter (AKF) using AR method^{5,8}. Harsha¹¹, *et al.* also notifies the AEKF technique with ARX modelling that holds variable orders for amplitude and phase scintillations. In the view of designing robust carrier tracking algorithm addition of fuzzy logic controller in choosing variable ARX orders based on effect of scintillation from S_4 and σ_ϕ indices using a Sugeno FIS can make an additional benefit. So, a complete approach in effective carrier tracking and mitigation of ionospheric scintillations adaptively by varying measurement noise using ARX analysis is proposed¹².

The proposed method is tested for both simulated datasets as well as real-time data. The Cornell Scintillation Monitor (CSM) is a simulated toolbox developed to understand equatorial scintillations with adjusted carrier-to-noise ratio, amplitude and phase scintillations, time resolution, length of data record and correlation coefficient parameters and provide In-phase and quadrature-phase components respectively. Three data sets with 10 ms time resolution are generated from CSM for weak, moderate and strong scintillation conditions. The Real-Time data collected from GNSS Software Navigation Receiver (GSNR'x) located at Rio De Janeiro, Brazil configured with both L1 C/A and L2C signals acquisition capability. It has 1 ms time resolution and the data under strong scintillation is observed and chosen for our study to know the better applicability of algorithm¹³.

The paper is presented as follows:

- (i) Implementing Sugeno FIS to model GNSS signal using ARX modelling
- (ii) Effective carrier tracking and mitigation of scintillations is noticed
- (iii) A new way using fuzzy logic based AEKF algorithm is showcased
- (iv) Worked out proposed technique for CSM and Real-Time ionospheric GNSS data and the results including their performance is presented in this paper.

2. GNSS SIGNAL MODELLING

The general base band signal can be modelled as given in Eqn. (1),

$$Y_k = A_k e^{j\theta_k} + \eta_k \quad (1)$$

where A_k , the received amplitude may include the scintillation effects such that signal amplitude C_k and scintillated amplitude $\rho_{s,k}$ given by $A_k = C_k \rho_{s,k}$ ⁵. The received phase is an additive of phase dynamics $\theta_{d,k}$ and scintillations $\theta_{s,k}$ termed as $\theta_k = \theta_{s,k} + \theta_{d,k}$. The noise at the output of the correlators is considered additive Gaussian with variance

$\eta_k \sim \mathbb{N}(0, \sigma_{s,k}^2)$. The amplitude scintillation index S_4 in Eqn. (2) is the normalised RMS deviation of signal strength³ S and the phase scintillation index σ_ϕ in Eqn. (3) is the standard deviation for the de-trended phase ϕ .

$$S_4 = \sqrt{\frac{\langle S^2 \rangle - \langle S \rangle^2}{\langle S \rangle^2}} \quad (2)$$

$$\sigma_\phi = \sqrt{\langle \phi^2 \rangle - \langle \phi \rangle^2} \quad (3)$$

2.1 Sugeno Fuzzy Logic Inference System

Fuzzy Logic uses degree of truth as a mathematical model of vagueness⁶. It uses linguistic variables that can be expressed in the form of defined rules and truth. For a fuzzy set A, the value μ_A is termed as a membership function (MF) and $\mu_A(x)$ is the degree of membership with element x in the fuzzy set, where $\mu_A(x) \Rightarrow 0$ means x is not a member and $\mu_A(x) \Rightarrow 1$ means x is a member of fuzzy set. The process of fuzzification involves mapping input values to fuzzy membership functions. Sugeno FIS differs in output membership functions, either the output MF's are crisp or constant values. The trapezoidal membership functions (TRAPMF) are used to define the input range of amplitude and phase scintillation indices. The logic control involves three different membership functions for each input according to the Weak (W), Moderate (M) and Strong (S) ionospheric scintillations. The output membership functions are chosen as a constant ARX orders increasing respective to effect of ionospheric scintillation.

2.2 ARX Time Series Data Modelling

Modelling amplitude and phase of received signal that are affected by ionospheric scintillation and with high dynamics is possible through auto-regressive exogenous analysis. ARX is a signal processing tool helps in fitting the empirical data that generates current input based on previous input with properly fixed coefficients. The generalised equation for ARX modelling is given by Eqn. (4),

$$\Psi_k(I) = \sum_{k=1}^N \beta_{k-i} \Psi_{k-i}(I) \quad (4)$$

where $\Psi_{k-i}(I)$ gives the past model input values, β_{k-i} are the variable coefficients of modeled time series data¹², and η_k is the mean square error calculated from observed and fitted value by the model.

3. GNSS CARRIER TRACKING AND IONOSPHERIC SCINTILLATION EFFECTS MITIGATION METHOD

Kalman Filter involves the mechanism of prediction and correction¹⁰. The KF approach comes under variable bandwidth carrier tracking technique⁹. A standard KF is formulated using phase dynamics as its state input. The phase dynamics given in Eqn. (5) is approximated to third-order Taylor series expansion that includes constant θ_0 , phase doppler frequency shift $f_{d,k}$ and doppler frequency rate $f_{r,k}$ as other state inputs for discrete time k for sampling time T_s ⁵

drivers such as solar and geomagnetic index parameters. The number of fuzzy membership functions in FIS is chosen to three in the proposed technique. The amplitude and phase ionospheric scintillations are broadly divided into three categories based on the effect of scintillation i.e., weak, moderate and strong. Based on this criterion we confined to three membership functions in the FIS. The trapezoidal membership function provides almost equal weights to the range of scintillation indices as shown in Table 1 in the fuzzification process. Altogether, six trapezoidal membership functions are utilised i.e., three for S_4 index and three for σ_ϕ index as given in Table 1. The output membership functions are fixed as 1, 2 and 3 for each respective input membership functions in the Sugeno FIS. The standard deviations for amplitude and phase values for both CSM and real-time data sets are as shown in Table 2.

The algorithm implemented for synthetic CSM data and real-time Brazil data. CSM data under three scintillation conditions is generated through CSM toolbox. The adjusted parameters are S_4 , T_o (correlation coefficient) and C/N_o (carrier to noise ratio) for 10 ms of time resolution. The

Table 2. Standard deviation value for CSM and Real-time Brazil data sets under strong ionospheric scintillation condition.

Data	Standard deviation	
	CSM	Real-time Brazil data
Amplitude (volts)	0.4297	1.112
Phase (rad)	0.7442	1.091

CSM creates a synthetic complex I and Q data that is useful to calculate amplitude and phase values³. The real-time Brazil data is collected from a GSNR'x receiver on October 24th, 2012 located at Rio De Janeiro, Brazil and is severely scintillated with PRN 12 L1 C/A around 21.30 h (local time) with 1 ms time resolution¹³. The doppler parameters chosen are aeronautical user specifications $f_{d,k} = 2625.7$ Hz, $f_{r,k} = 107$ Hz/s and initial dynamic phase $\theta_0 = (-\Pi, \Pi)$ ⁵.

Figure 2 clearly explains the fitting of ARX analysis to the periodogram power spectral density (PSD) estimate. The applicability of Sugeno fuzzy logic to different scintillation

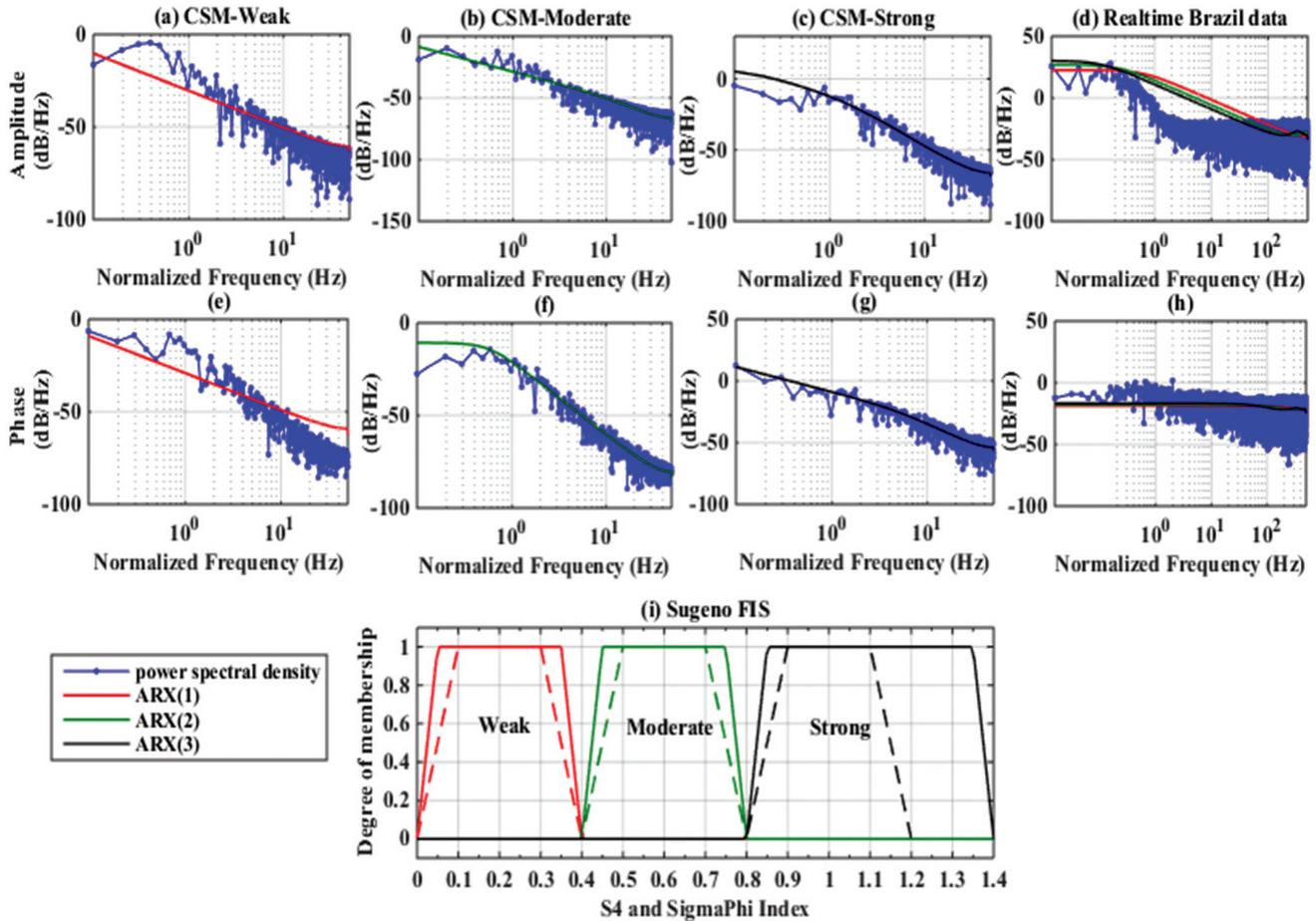


Figure 2. Modelling ionospheric scintillations for Synthetic CSM data (a)-(g) and Real-Time Brazil data (d) and (h) using Sugeno FIS (i). (a) & (e) Periodogram estimates of PSD (blue), ARX(1) (red) for weak CSM amplitude and phase scintillation ($0 < S_4 \leq 0.4$) & ($0 < \sigma_\phi \leq 0.4$), respectively. (b)&(f) Periodogram estimates of PSD (blue), ARX(2) (black) for moderate CSM amplitude and phase scintillation ($0.4 < S_4 \leq 0.8$) & ($0.4 < \sigma_\phi \leq 0.8$) respectively. (c) & (g) Periodogram estimates of PSD (blue), ARX(3) (green) for strong CSM amplitude and phase scintillation ($0.8 < S_4 \leq 1.2$) & ($0.8 < \sigma_\phi \leq 1.4$), respectively. (d) & (h) Periodogram estimates of PSD (blue), ARX(1), (2) & (3) orders for severe ionospheric scintillated amplitude and phase signals of real time GNSS data. (i) Degree of membership (DOM) for both S_4 (dotted lines) and σ_ϕ (solid lines) under weak (W), moderate (M) and strong (S) scintillation conditions.

regions is also included. The available real-time GNSS data is severely scintillated ($0.6 < S_4 < 1.04(\text{max})$) and the possible ARX orders (1), (2) and (3) is shown to choose the best suitable order. It clearly explains that ARX(3) is suitable for real-time amplitude and phase scintillations so that three coefficients are chosen to transition matrix K i.e., ($p1 \Rightarrow 3, p2 \Rightarrow 3$) (Eqn (8)). The power spectral density with respect to ARX modelling results for both CSM and real-time data sets can be seen from Fig. 2. It is observed that ARX (1) is appropriate to model the weak ionospheric scintillations (Figs. 2(a) & 2 (e)). Similarly, ARX(2) acts good for moderate scintillations (Figs. 2 (b) & 2 (f)) and ARX(3) is suitable for strong scintillations (Fig. 2 (c) & 2 (g)), respectively. The chosen real-time GPS data is under strong scintillations at 21.30 h and ARX(3) is well followed the power spectral density for both amplitude and phase of received GPS L1 PRN 12 C/A signal. The fuzzy logic is established based on effect of scintillation based on S_4 and indices also given in Fig. 2 (i).

The measurement vector is linearised with priori state-input $H_k = h_k(x_{k/k-1})$. The measurement noise Eqn. (12) helps in making the algorithm adaptive by varying C/N_0 estimates for each sampling time¹². The AEKF state space formulation helps in proper carrier tracking and mitigation of scintillation effects. The CSM carrier tracking and mitigation can be seen with mitigated phase transition in Fig. 3.

A strong ionospheric scintillated amplitude and phase data samples are chosen to analyse the proposed AEKF algorithm capability. The measurement noise vector R_k is kept constant for EKF and adaptive as shown in Eqn. (13) for the proposed AEKF algorithm. This makes a huge improvement in phase scintillation mitigation that is clearly seen from Fig. 3. At 62.4 s to 62.5 s of duration the original phase gave a sudden transition from 1 rad to -3 rad. The EKF algorithm succeeded in mitigating this phase transition and estimated the phase as 4 radians. The proposed AEKF algorithm gave effective scintillation mitigation and estimated it as 1 rad and removed the phase transition that can cause loss-of-lock to the receiver as shown in Fig. 3. This clearly showcase the usefulness of proposed algorithm and its optimal estimation. It is also observed that carrier amplitude tracking is well established by AEKF than given by EKF. The overestimation of EKF is adaptively adjusted by AEKF by varying measurement noise vector R_k for each instant of time. In the case of strong ionospheric scintillations there will be drop of C/N_0 value that increases the R_k value and decreases the kalman gain such that state input depends on previous state input rather than present measurements that are scintillated. This makes an effective mitigation

of phase transition at that discrete time step.

A similar analysis is carried out for a real-time scenario. The AEKF acts well in carrier amplitude tracking than EKF that is clearly seen from Fig. 4. Phase transitions are observed between 6.75 s to 6.8 s of time. The original phase gave deep transition from 0 radian to -3.5 radian. The AEKF algorithm gave 5.2 rad of value by mitigating the phase transition followed by EKF approach as shown in Fig. 4. Thus, the proposed fuzzy logic based AEKF also successful to mitigate strong ionospheric scintillations for real-time data sets. The algorithm gave good performance evaluation for real-time data in mitigation can be seen in Fig. 4.

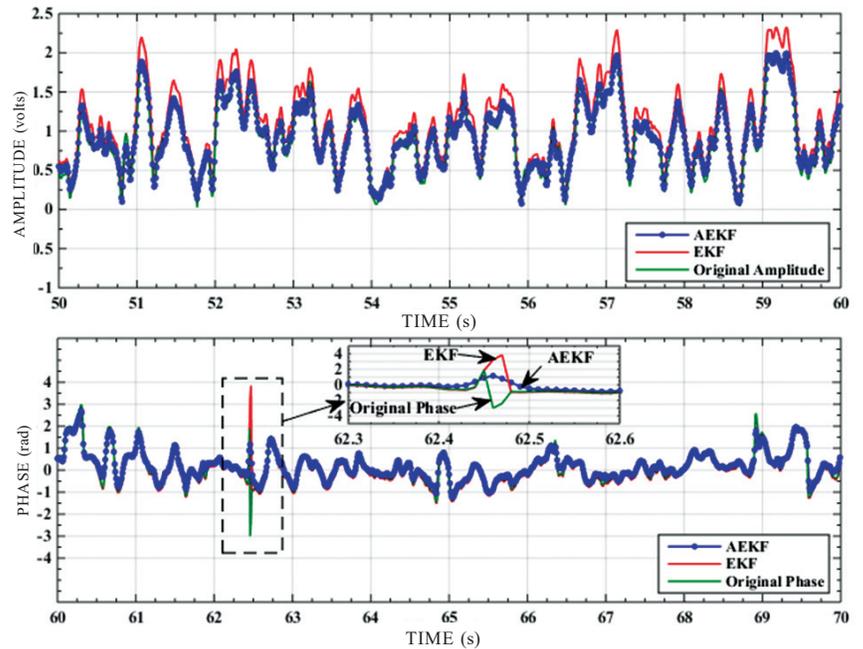


Figure 3. CSM carrier tracking and mitigation of amplitude and phase scintillations data ($S_4=0.8, T_0=0.3, C/N_0=35$ dB).

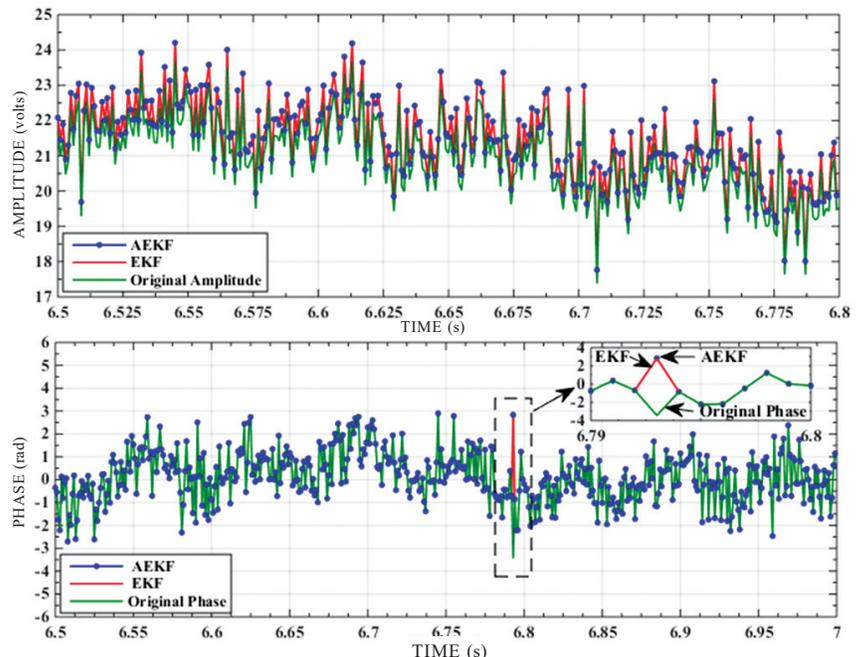


Figure 4. Real-Time Brazil data tracking and mitigation of amplitude and phase scintillations (data under severe scintillation).

5. CONCLUSIONS

The proposed algorithm succeeded in effective mitigation of amplitude and phase scintillations even in catastrophic conditions. The variable ARX orders p_1 (phase) and p_2 (amplitude) can be chosen based on the effect of scintillation using a fuzzy logic controller makes an automated carrier tracking algorithm. This approach is useful in effective tracking and mitigation of scintillations adaptively that helps for future GNSS receivers. The measurement noise vector R_k is fixed in EKF and is adaptive in the proposed AEKF algorithm for each discrete time step makes a significant improvement even under strong scintillation condition. The proposed AEKF algorithm acts well than EKF in the mitigation of amplitude ionospheric scintillations for both CSM and real-time data. The proposed fuzzy logic based AEKF outperforms EKF in phase scintillation mitigation for CSM data. For real-time data AEKF and EKF gave almost equal for phase scintillation mitigation needs further investigation is necessary. The carrier frequency tracking capabilities needs to be included in the proposed algorithm.

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In the current study, he wrote matlab programs for GPS and CSM simulation ionospheric scintillations data analysis and Fuzzy based Kalman filter and drafted manuscript.

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