Reliability Analysis of Complex Systems with Failure Propagation

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ABSTRACT

Failure propagation is a critical factor for the reliability and safety of complex systems. To recognise and identify failure propagation of systems, a deep fusion model based on deep belief network (DBN) and Bayesian structural equation model (BSEM) is proposed. The deep belief network is applied to extract features between status monitoring data and the performance degradation in different failure components. To calculate the path weight of failure propagation, the Bayesian structural equation model is proposed to study the relationship among different fault modes. After getting the performance degradation of each fault through DBN and calculating the path weight of fault propagation by BSEM, it is available to get the overall reliability of the system. The aircraft landing gear system with 19 fault patterns is selected to evaluate the feasibility of the proposed deep fusion model. The results demonstrate that the overall reliability of the system can be obtained by analysing the fault propagation of multiple fault patterns, and the proposed model has a lower deviation than traditional back propagation neural network.

Keywords: Failure propagation; Performance degradation; Deep belief network; DBN; Bayesian structural equation model; BSEM; Reliability analysis

1. INTRODUCTION

Reliability analysis is the key point of ensuring the efficiency, reliability and safety of complex systems. The findings of related studies are applied in civil aviation, electric systems and nuclear power stations to provide support for maintenance decision and support management. Traditional reliability analysis methods include mathematical and physical models. Bansal & Sinha1 applied edgeworth-gamma class of priors to analyse the robustness of Bayes estimate of reliability function and the reliable life. Arora2 used the Markov renewal processes theory to express the distribution of the time to first system failure, and obtained the reliability of a modular standby redundant system. Choi & Chang3 built a fault tree framework to assess the reliability and availability of tanks. Chojaczyk4, et al. applied artificial neural network models in structural reliability analysis. Lee and Pan5 presents a reliability analysis scheme for complex systems by combining discrete time Markov chain models with Bayesian network (BN) model.

In recent years, intelligent algorithms have been received increasing research attention. The current artificial intelligent methods include machine learning (ML) and deep learning (DL). Machine learning methods are mainly based on shallow network models, which have poor performance on feature extraction. Deep learning is a significant and particular part of machine learning. It uses multiple neural network structure to extract deep abstract features of samples, which can effectively overcome the problem of traditional ML as mentioned above. Classical deep learning models include deep belief network (DBN), convolutional neural network (CNN), recurrent neural network (RNN), and Stacked Auto Encoder (SAE). In terms of performance degradation analysis, artificial intelligent methods have been preliminarily applied. Ma6, et al. proposed a novel method based on discriminative deep belief networks (DDBN) and ant colony optimisation (ACO) for predicting health status of machine. Jiang7, et al. defined a new comprehensive feature index based on locally linear embedding to quantify rolling bearing performance degradation, and a continuous deep belief network (CDBN) is applied to model vibration signals. Shao8, et al. presented a bearing performance degradation assessment method based on HMM and nuisance attribute projection (NAP).

With the in-depth research on the operating mechanism of systems, it can be found that the system has complex failure modes, and slight performance degradation may cause severe failure. Therefore, the failure propagation is a critical research content for system reliability. Recent publications propose research progress concerning failure propagation. Sierla9, et al. built an analysis framework of functional failure identification and propagation to study simulation-based functional failure propagation. Levitin10, et al. performed reliability evaluation after considering the propagated failures of a phased mission system. Wang11, et al. proposed an algorithm for researching the reliability of non-repairable binary systems, which are subject to competing propagated failures and failure isolation events with global and selective failure effects. Al-Begain12, et al. applied multi-dimensional continuous-time Markov chain to describe the propagated failures of a queuing system. Xing and Levitin13 evaluated the reliability of multi-state
systems (MSSs) subject to propagated failures with global effect and failure isolation effect. Mo et al. presented a novel analytical method based on multi-valued decision diagrams for the reliability analysis of network systems with dependent propagation effects. Gong et al. studied the effect of selective failure propagation and the reliability of MSSs using four different propagation mechanisms. Huang et al. performed a reliability assessment on cross-strapped redundant systems with potential propagating failure modes. The above mentioned analysis of propagated failures focus on specific systems or single parts. The systems with multiple components, failure modes and incomplete information are not considered.

Based on the above summary of related methods about reliability analysis and fault propagation. A deep fusion model based on DBN and Bayesian structural equation model (BSEM) is proposed for reliability analysis of systems with failure propagation. The DBN is applied to study the performance degradation under different failure modes. The BSEM is used to calculate the path weight of failure propagation. After performance degradation analysis and path weight solution, the overall reliability of the system can be obtained for further research.

2. THE FRAMEWORK OF RELIABILITY ANALYSIS

In this paper, the framework of reliability analysis with failure propagation is proposed. The main tasks are summarised as follows:
(a) The performance degradation process for each fault mode
(b) The dynamic relationship between failure propagation and operational reliability
(c) The path weight of failure propagation.

The specific process is as shown in the Fig. 1. First, after studying the failure mechanism and prior knowledge of reliability, it is feasible to choose the appropriate algorithm for determining the latent relationship between monitoring parameters and performance degradation, and dynamically describe failure propagation. Second, in order to solve the questions of multiple fault modes, the expert system is employed to obtain a dataset, which can be used for calculating the path weight of failure propagation. Finally, to study the dynamic relationship between failure propagation and operational reliability, an integrated algorithm is proposed to determine system reliability over time by putting monitoring data into the model.

3. MODELS FOR THE ANALYSIS OF FAILURE PROPAGATION

The analysis of failure propagation includes process analysis and path weight analysis. The DBN algorithm is used to explore the performance degradation process of failure propagation, and the BSEM is applied to solve the path weight of failure propagation.

3.1 Deep Belief Network

As shown in Fig. 2, DBN is composed of several restricted Boltzmann machines (RBM), which can extract the implicit features of observed data by unsupervised learning. A typical RBM structure consists of three parts: the visible layer v, the hidden layer h, and the weight vector w. The visible layer and hidden layer, which contain several nodes, are input and output, respectively. These layers are independent of each other. The weight value represents the relationship between the visible layer and the hidden layer. The training process of DBN can be described as follows:

3.1.1 Establishment of Deep Belief Network

The number of layers and nodes in the DBN is determined according to the dimension of the input and output samples. The bias vector of the visible layer and bias vector of the hidden layer are defined as a and b. For the RBM model, the energy function \( E(v, h) \) between the visible-layer vector \( v \) and the hidden-layer vector \( h \) is expressed as:

\[
E(v, h) = -\sum_{i=1}^{m} a_i v_i - \sum_{j=1}^{n} b_j h_j - \sum_{i=1}^{m} \sum_{j=1}^{n} v_i h_j w_{ij}\]

The function of joint probability is:

\[
P(v, h) = \frac{1}{\sum_{v, h} e^{-E(v, h)}} e^{-E(v, h)}
\]

Figure 1. Reliability analysis framework of complex systems with failure propagation.

Figure 2. The structure diagram of DBN.
According to (1) and (2), the probability of activation of the visible layer and the hidden layer can be obtained as

\[ P(h_j = 1 | v) = \frac{P(h_j = 1, v)}{P(h_j = 1, v) + P(h_j = 0, v)} = \text{sigmoid}(\sum_{i=1}^{q} a_i + w_j v_i) \]  (3)

\[ P(v_i = 1 | h) = \frac{P(v_i = 1, h)}{P(v_i = 1, h) + P(v_i = 0, h)} = \text{sigmoid}(\sum_{j=1}^{p} b_j + w_h v_j) \]  (4)

\[ \text{sigmoid}(x) = \frac{1}{1 + e^{-x}} \]  (5)

### 3.1.2 Sample Training and Parameter Updating

By inputting the sample and network parameters, the updated \( w, a \) and \( b \) take the form \(^{18}\)

\[ w = w + \varepsilon (P(h_1^{(k)} = 1 | v^{(k)}) - v_1^{(k)(+1)}) - P(h_1^{(k+1)} = 1 | v^{(k+1)(+1)}) \]  (6)

\[ a = a + \varepsilon (v_1^{(k)} - v_1^{(k)(+1)}) \]  (7)

\[ b = b + \varepsilon (P(h_1^{(k+1)}) = 1 | v^{(k+1)}) - P(h_1^{(k+2)} = 1 | v^{(k+2)(+1)}) \]  (8)

### 3.1.3 Reverse Fine-tuning

After the training the RBMs, the loss between the sample label solved by the DBN model and the known sample label can be defined as

\[ \text{loss} = \frac{1}{K} \sum_{k=1}^{K} \| x_k - \tilde{x}_k \| \]  (9)

where vector \( x \) is the solved sample label, vector \( \tilde{x} \) is the original sample label, and \( K \) is the number of samples.

According to the loss function, the stochastic gradient descent method and back propagation algorithms can be used to fine-tune the model parameters reversely.

### 3.2 Bayesian Structural Equation Model

BSEM is a combination of the structural equation model (SEM) and Bayesian method. Structural equation model can deal with expert datasets and build the path nodes of datasets. The Bayesian method can estimate the parameters of path nodes after Gibbs sampling. The model details are:

#### 3.2.1 Building the SEM

The SEM includes measurement and structural equations. The measurement equation describes the relationship between latent variable \( \omega \), and measurement variable \( v \), as \(^{19}\)

\[ v_i = \Lambda \omega_i + \varepsilon_i, i = 1, \ldots, n \]  (10)

where \( \Lambda \) is the component matrix, which represents the relationship between measurement indicators and potential variables. \( \varepsilon_i \) is an error term subject to a distribution of \( N(0, \Psi_\varepsilon) \).

Structural equation is developed to illustrate the relationship between exogenous and endogenous latent variables. The structural equations for evaluating their relationships are as follows:

\[ \eta_j = \Pi_1 \eta_1 + \Gamma \xi_j + \delta_j \]  (11)

where \( \omega \) contains exogenous latent variable \( \xi_j \) and endogenous latent variable \( \eta_1 \), \( \Pi \) and \( \Gamma \) represent the structure matrix. \( \delta_j \) is residual matrix of the structural model. The distribution of \( \xi_j \) is \( N(0, \Phi) \), and \( \delta_j \) is subject to \( N(0, \Psi_\delta) \).

Assume \( v = \{x, y\} \) and \( p = x = p - r \geq 0 \), \( x \) represents observable continuous measurements, which can be defined as \( x = \{x_1, \ldots, x_n\} \). \( y = \{y_1, \ldots, y_s\} \) is the unobservable variable.

Exogenous latent variables \( \xi \) corresponds to \( x \), and endogenous latent variables \( \eta \) corresponds to \( y \). The information of \( y \) is given by observable ordered classified variable \( z \) defined as \( z = \{z_1, \ldots, z_n\} \). Therefore, any latent variable can use a continuous variable or an ordered categorical variable as its external index. The relationship between \( y \) and \( z \) is defined by a set of thresholds as

\[ \alpha_{k,z_1} < \alpha_{k,z_2} < \cdots < \alpha_{k,z_n} \]  (12)

where, \( z_k \) is an integer with a range of \( \{0,1,\ldots,b_k\} \), \( k = 1,2,3,\ldots,n \), and \( \alpha_{k,0} < \alpha_{k,1} < \cdots < \alpha_{k,b_k} < \alpha_{k+1,b_k} \).

#### 3.2.2 Gibbs Sampling of Datasets

Assuming that \( \theta = \{\alpha, \Psi, \Upsilon, \Gamma, \Phi, \Psi_\Delta\} \), the Gibbs sampling method is used to extract the number of samples from the posterior distribution \( [\alpha, \theta, \omega, y | x, z] \) \(^{20}\). The given variable \( m \) is the number of iterations. The extraction process is as follows:

(a) \( \omega^{(m+1)} \) is extracted from the distribution of

\[ p(\omega | \theta^{(m)}, \alpha^{(m)}, y^{(m)}, x, z) \]

(b) \( \theta^{(m+1)} \) is extracted from the distribution of

\[ p(\theta | \omega^{(m+1)}, \alpha^{(m)}, y^{(m)}, x, z) \]

(c) \( \alpha^{(m+1)}, y^{(m+1)} \) is extracted from the distribution of

\[ p(\alpha, y | \omega^{(m+1)}, \alpha^{(m)}, y^{(m)}, x, z) \]

#### 3.2.3 Bayesian Estimation of Structural Equation Model

Assume that \( \alpha^{(j)}, \theta^{(j)}, \omega^{(j)}, j = 1,2,\cdots,J \) are extracted from samples. Therefore, the Bayesian estimation of \( (\alpha, \theta, \omega) \) can be described as \(^{21}\)

\[ \overline{\alpha} = J^{-1} \sum_{j=1}^{J} \alpha^{(j)}, \overline{\theta} = J^{-1} \sum_{j=1}^{J} \theta^{(j)} \]

\[ \overline{\omega} = J^{-1} \sum_{j=1}^{J} \omega^{(j)}, \overline{y} = J^{-1} \sum_{j=1}^{J} y^{(j)} \]  (13)

The covariance matrix estimation of \( \overline{\alpha}, \overline{\theta}, \overline{\omega} \) can be obtained as

\[ \text{Var}(\alpha | x, z) = (J-1)^{-1} \sum_{j=1}^{J} (\alpha^{(j)} - \overline{\alpha})(\alpha^{(j)} - \overline{\alpha})^T \]  (14)

\[ \text{Var}(\theta | x, z) = (J-1)^{-1} \sum_{j=1}^{J} (\theta^{(j)} - \overline{\theta})(\theta^{(j)} - \overline{\theta})^T \]  (15)

\[ \text{Var}(\omega | x, z) = (J-1)^{-1} \sum_{j=1}^{J} (\omega^{(j)} - \overline{\omega})(\omega^{(j)} - \overline{\omega})^T \]  (16)
4. **DEEP FUSION MODEL FOR RELIABILITY ANALYSIS**

The deep fusion model for the reliability analysis model is illustrated in Fig. 3. On the one hand, after collecting information of monitoring parameters and performance degradation of system, a DBN model can be built to get the density function of failure probability. On the other hand, through analysing the dataset of multiple components and multiple failure modes by BSEMs, the path weight of failure propagation can be calculated. Then the overall reliability of system can be obtained by weighted summation.

Assume that \( n \) is the number of components. Thus, \( f(x_i) \) is the failure probability density function, and \( p_i \) is the path weight of failure propagation. The reliability \( R(t) \) can then be expressed as

\[
R(t) = 1 - \left( p_1 \int_0^t f(x_1) dt + \cdots + p_n \int_0^t f(x_n) dt + \cdots + p_n \int_0^t f(x_n) dt \right) \sum_{i=1}^n p_i = 1 \quad (17)
\]

5. **CASE STUDY**

An aircraft landing gear system as shown in Fig. 4 is a typical complex system with numerous components\(^2^{22}\). The landing gear failure of ground support function will lead to airframe touchdown and severe hard landing, which will damage the structure of fuselage. Thus, the aircraft landing gear system is selected as the object of this study. The failure components include shock absorber, side stay, and lock stay and sealing structure. The components and corresponding failure modes are listed in Table 1\(^2^{23}\).

<table>
<thead>
<tr>
<th>Components</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock absorber</td>
<td>(F01) Outer cylinder fracture of damper strut</td>
</tr>
<tr>
<td></td>
<td>(F02) Pin connection breaking of damper strut</td>
</tr>
<tr>
<td></td>
<td>(F03) Connection splitting of damper strut and side stay</td>
</tr>
<tr>
<td></td>
<td>(F04) Piston rod breach of damper strut</td>
</tr>
<tr>
<td></td>
<td>(F05) Stationary barrier fracture of lock</td>
</tr>
<tr>
<td></td>
<td>(F06) Connection bolt rupture of damper strut and fuselage</td>
</tr>
<tr>
<td></td>
<td>(F07) Connection bolt rupture of side stay and outer cylinder</td>
</tr>
<tr>
<td>Side stay</td>
<td>(F08) Main body cracking of lower side stay</td>
</tr>
<tr>
<td></td>
<td>(F09) Main body cracking of upper side stay</td>
</tr>
<tr>
<td></td>
<td>(F10) Connection bolt rupture of side stay</td>
</tr>
<tr>
<td></td>
<td>(F11) Connection bolt rupture of side stay and fuselage</td>
</tr>
<tr>
<td>Sealing structure</td>
<td>(F12) Leakage of fuel feeding valve</td>
</tr>
<tr>
<td></td>
<td>(F13) Seal crack</td>
</tr>
<tr>
<td></td>
<td>(F14) Leakage of charge valve</td>
</tr>
<tr>
<td></td>
<td>(F15) Seal failure of oil-gas chamber</td>
</tr>
<tr>
<td>Lock stay</td>
<td>(F16) cracking of lower lock stay</td>
</tr>
<tr>
<td></td>
<td>(F17) cracking of upper side stay</td>
</tr>
<tr>
<td></td>
<td>(F18) Connection bolt rupture of lower lock stay and upper stay</td>
</tr>
<tr>
<td></td>
<td>(F19) Function failure of lock spring</td>
</tr>
</tbody>
</table>

5.1 **The Propagated Process of Failure**

The performance degradation of sealing structure is taken as an example. The monitoring parameters include initial stowing pressure, initial gas volume, compressed gas area, piston rod area, polytropic index of gas, oil pressure area, main oil hole area, flow coefficient of the main oil hole, and liquid oil density. Every sample has 20 groups of monitoring data, which are collected every 50 hours of working time. 50 samples are collected and put into the DBN network to obtain the estimated value of performance degradation. Figure 5 shows result of one sample. Comparing the target values and estimated values, the mean absolute error (MAE) is \(1.46 \times 10^{-3}\), which is small enough to satisfy the requirements.

Suppose that the initial performance degradation of systems is denoted as \( D_0 \) and the cumulative degradation of systems for time \( t \) can be expressed as \( w(t) = D(t) - D(t_0) \). Assume that the reliability is \( R(t) = P[T > t] = P[w(t) < \varepsilon] \) and \( \varepsilon \) is the threshold of performance degradation. Therefore, the failure density function can be deduced through the performance degradation. In this study, we input the monitoring
data into the DBN trained and obtain the value of performance degradation. Figure 6 shows the probability density curve of sealing structure.

Similarly, in order to visualise the degradation process, Fig. 7 and Fig. 8 plot the fault probability density curves and fault cumulative distribution curves of four components in landing gear system.

5.2 Solving the Path Weight of Propagation

Maintenance history data can be collected from many landing gear systems, and the datasets can be obtained according to expert systems. The datasets range from 1 to 5 (1 = “extremely slight failure”; 2 = “slight failure”; 3 = “medium failure”; 4 = “major failure”; 5 = “severe failure”). The F01-F19 are the fault modes of system, and F20 is defined as the overall state of system. Assume that \( \omega = (\eta_1, \xi_1, \xi_2, \xi_3, \xi_4) \).

Thus, the measurement equation and structural equation can be respectively described as:

\[
\begin{align*}
Y_i &= \Lambda_0 + e_i \\
\eta &= \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_3 + \gamma_4 \xi_4 + \delta
\end{align*}
\]

The WINBUGS is used to simulate the model. Assume that hyperparameter \( \lambda \) is 0.8. Thus, the hyperparameters of \( \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} \) are \( \{0.6, 0.6, 0.4, 0.4\} \). According to experience, the model runs 2000 time have more stable results and less operation time. Therefore, the number of operations is set as 2,000 time. The path weight can be obtained after the operation of WINBUGS, Fig. 9 shows the path weight graph of fault propagation.

After experiment, the path weight values of aircraft landing gear system can be obtained and listed in Table 2. The estimation of the structural equation takes the form

\[
\eta = 0.34\xi_1 + 0.25\xi_2 + 0.23\xi_3 + 0.18\xi_4 + 0.48
\]

By comparing the weights of paths, it is concluded that the major component is the shock absorber, and the most likely failure mode is F02 (pin connection breaking of damper strut).

5.3 Reliability Analysis of Aircraft Landing Gear System

For the aircraft landing gear system, any component failure will lead to the overall failure of the system. Therefore,
the reliability analysis model for the landing gear system is logically connected in series.

General reliability based on experience is as shown in the Fig. 10. Since the samples studied in this paper are all degraded from the initial state, the reliability threshold is set to 0.8 based on experience, and first maintenance and inspection are carried out before the corresponding working time of the threshold.

By taking the calculated failure density function and path weight of each component into the proposed reliability analysis model, the estimated reliability index can be obtained. To demonstrate the effectiveness of the proposed method, the general reliability is compared with the reliability of the proposed model and that of the BP neural network. As shown in Fig. 11, the MAE between the general reliability and the reliability solved by the proposed model is $2.47 \times 10^{-4}$, which is lower than the $1.74 \times 10^{-2}$ of BP neural network.

### Table 2. Path weight values of aircraft landing gear system

<table>
<thead>
<tr>
<th>Path weight of components</th>
<th>Path weight of fault modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock absorber ($\xi_1$) 0.34</td>
<td>(F01) 1.00</td>
</tr>
<tr>
<td></td>
<td>(F02) 1.06</td>
</tr>
<tr>
<td></td>
<td>(F03) 0.81</td>
</tr>
<tr>
<td></td>
<td>(F04) 0.67</td>
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<tr>
<td></td>
<td>(F05) 0.89</td>
</tr>
<tr>
<td></td>
<td>(F06) 0.31</td>
</tr>
<tr>
<td></td>
<td>(F07) 0.51</td>
</tr>
<tr>
<td>Side stay ($\xi_2$) 0.25</td>
<td>(F08) 1.00</td>
</tr>
<tr>
<td></td>
<td>(F09) 1.01</td>
</tr>
<tr>
<td></td>
<td>(F10) 1.13</td>
</tr>
<tr>
<td></td>
<td>(F11) 1.12</td>
</tr>
<tr>
<td>Sealing structure ($\xi_3$) 0.23</td>
<td>(F12) 1.00</td>
</tr>
<tr>
<td></td>
<td>(F13) 0.54</td>
</tr>
<tr>
<td></td>
<td>(F14) 1.66</td>
</tr>
<tr>
<td></td>
<td>(F15) 1.61</td>
</tr>
<tr>
<td>Lock stay ($\xi_4$) 0.18</td>
<td>(F16) 1.00</td>
</tr>
<tr>
<td></td>
<td>(F17) 0.69</td>
</tr>
<tr>
<td></td>
<td>(F18) 0.67</td>
</tr>
<tr>
<td></td>
<td>(F19) 0.91</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

In this paper, a novel deep fusion model based on DBN and BSEM is proposed for the reliability analysis of complex systems. Based on the concept of fault propagation, the DBN is used to explore the process of failure propagation through by mining hidden features between state parameters and performance degradation. The BSEM is applied to solve the path weight of failure propagation among multiple fault modes. After obtaining the probability density function and the corresponding path weight of each fault mode, the overall reliability can be calculated by weighted summation. Take landing gear system as an example, the experimental results show that the proposed model have lower error than traditional
method, and it is an effective means to prevent system failure and monitor the health status of the system in time.

**Conflict of Interest**

The authors declare no conflict of interest.

**REFERENCE**


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Mr Changchang Che received his Masters from Nanjing University of Aeronautics and Astronautics. Currently, his research focuses on aircraft reliability analysis and aeroengine health management. He is pursuing a PhD degree in Nanjing University of Aeronautics and Astronautics. Contribution in the current study, he has initiated this study, implemented the algorithm, carried out the experimentation, checked and written the paper.

Prof. Huawei Wang received her Bachelor’s in School of Management from the University of National Defense Science and Technology in 2003. Currently, she is a professor in college of civil aviation, Nanjing University of Aeronautics and Astronautics, China. Her research is mainly focussed on reliability analysis of aircraft, aircraft fault diagnosis and health management. Contribution in the current study, she has guided the overall work.

Mrs Xiaomei Ni received her Masters from Nanjing University of Aeronautics and Astronautics. Currently, she is pursuing a PhD in Nanjing University of Aeronautics and Astronautics. Contribution in the current study, she has supported the data processing of aircraft landing gear system in the study.