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An Evidential Fractal Analytic Hierarchy Process Target Recognition Method

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ABSTRACT

Target recognition in uncertain environments is a hot issue, especially in extremely uncertain situation where both the target attribution and the sensor report are not clearly represented. To address this issue, a model which combines fractal theory, Dempster-Shafer evidence theory and analytic hierarchy process (AHP) to classify objects with incomplete information is proposed. The basic probability assignment (BPA), or belief function, can be modelled by conductivity function. The weight of each BPA is determined by AHP. Finally, the collected data are discounted with the weights. The feasibility and validness of proposed model is verified by an evidential classifier case in which sensory data are incomplete and collected from multiple level of granularity. The proposed fusion algorithm takes the advantage of not only efficient modelling of uncertain information, but also efficient combination of uncertain information.

Keywords*:* Fractal theory; Dempster-Shafer evidence theory; AHP; Belief function; Target recognition

1. INTRODUCTION

In recent years, target recognition is paid great attention in military applications. Many methods have been proposed to classify objects[1-5] and target recognition[6-8]. The information gathered in sensors fusing system exists uncertainty due to its incomplete, inconsistency and possibly imprecise[3-5]. Many methods have been proposed to classify objects, such as K-Nearest neighbour (KNN)[6], Bayes Classifier (BCL), principle component analysis (PCA)[8], linear discriminant analysis (LDA)[9], Gauss mixture model (GMM)[10]. However, there are some drawbacks limiting them for a wider applications. For instance, the KNN method is simple and valid in big-volume samples case, however, when the different classes are unbalanced which means that some classes are bigger than others or some classes are extremely small, this kind of methods will result non-negligible deviations. In addition, another important issue is these methods fail to handle both discord uncertainty and non-specificity uncertainty of the collected sample. The main purpose of this paper is to acquire a precise result using fractal modelling of belief function in Dempster-Shafer theory[9-11] combined with AHP[12-14].

The Dempster-Shafer evidence theory (evidence theory)[30,31] can handle both discord uncertainty and non-specificity uncertainty. Due to its superiority, evidence theory has been widely applied in information fusion[32-34], fault diagnosis[35-38], game theory, multicriteria decision-making, etc. AHP, served as a common method to rank objects according to their comparative superiority and select alternatives against a set of selected criteria, is first proposed by Saaty[15]. It has been applied in management fields.

From all we have discussed above, we could categorise identifying uncertain objects into multi-criterion decision-making (MCDM) problems. The existing methods are limited in 3 perspective:

(i) How to represent the characteristic of obtained object considering their uncertainty?
(ii) How to extract vital information effectively thus experts could conduct approximate reasoning and decision-making process?
(iii) Facing with multiple information acquired from multiple sensors, we should fuse them and get a comprehensive result.

To the best of our knowledge, this issue is not well addressed yet. Therefore, a hybrid evidential AHP model extended by fractal theory is presented. Fractal theory is capable to represent uncertain object considering their inherent origins. AHP is utilised to extract effective information and D-S evidence theory to fuse multiple information from various sensors.

2. PRELIMINARIES

2.1 Dempster-Shafer Evidence Theory

Dempster-Shafer evidence theory is also known as evidence theory which has excellent ability to handle uncertain information. Besides, it is capable to combine pairs of evidence bodies using combining rules to derive a new evidence body and belief function. Because of these features, the evidence theory has been widely used into target recognition for decision-making[4,22-24], supplier selection[25-26], information fusion and classify[27-30]. It should be addressed that there are various open issue in evidence theory such as how to measure correlation between two evidence[31-32], evidence combination[33-34], conflict management[35-36]. And the concrete knowledge of D-S evidence

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theory will be detailed in the following.

Definition 2.1 (framework of evidence theory). The framework of discernment θ is a set of mutually exclusive and collectively exhaustive events, denoted as:

$$\theta = \{E_1, E_2, E_3, ..., E_i, ..., E_N\}$$

where set θ is called a frame of discernment. The power set of θ is denoted by 2^θ,

$$2^\theta = \{\{E_1\}, \{E_2\}, \{E_3\}, ..., \{E_1, E_2\}, \{E_1, E_3\}, ..., \{E_1, E_2, E_3\}, \{E_1, E_2, E_4\}, ...\theta\}$$

Definition 2.2 (BPA). The basic possibility assignment (BPA) is a mass function mapping $2^\theta \rightarrow [0,1]$ observing that:

$$b(\varnothing) = 0$$

$$\sum_{A \subseteq 2^\theta} b(A) = 1 \text{ and } 0 \le b(A) \le 1$$

The $\varnothing$ is the empty set. If $b(A) > 0$, A is called a focal element, there is no other restriction restrictions on A, the focal element doesn't need to be disjoint, their integration and the integration unnecessarily cover the whole framework.

Definition 2.3 (combining rules) the combining rules of two BPAs b_1, b_2 is denoted as $b = b_1 \oplus b_2$

$$b(A) = \frac{1}{1-K} \sum_{B \cap C = A} b(A) = b_1(B) b_2(C) \text{ for } A \ne \varnothing$$

$$B, C \subseteq 2^\theta$$

$$b(A) = 0 \text{ for } A = \varnothing$$

where

$$K = \sum_{B \cap C = \varnothing} b_1(B) b_2(C)$$

K is called conflict coefficient. Notice that the D-S combining rules is applicable only when k<1.

2.2 Fractal Theory

Fractals theory was first prompted by Mandelbrot[20]. The fractal sets[37] had been developed in the early of century. Irregular continuous sets and sets with discontinuities (gaps) are two types of fractal sets. The size and density of fractal sets could be effectively evaluated by non-fractal sets which is often referred to regular sets with topological characteristic.

2.3 Analytic Hierarchy Process

Analytic hierarchy process is a multi-criteria decision-making method which can be used quantify relative weights for a given set. It was first proposed by Saaty[15]. The utilisation of AHP always require a hierarchy structure and could objectively assign weight to different objects in this structure according to our criteria.

Definition 2.4 (Pairwise Comparison matrix) assume $\{r_1, r_2, r_3, ...r_n\}$ are the n alternatives for decision making, the pairwise comparison matrix denoted as $M = (M_{ij})_{n \times n}$

$$m_{ij} = \frac{1}{m_{ji}} \text{ for } i \ne j$$

$$m_{ij} = 1 \text{ for } i = j$$

where m_{ij} represents the relatively importance r_1 over r_j. The numerical rating in AHP is as shown in Table 1.

Table 1. Numerical rating in AHP

Scale	Meaning
1	Equal importance
2	Moderate importance
5	Strong importance
7	Demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate importance

Definition 2.5 (Consistency ratio) for a pairwise comparison matrix $M_{n \times n}$, assume λ_{MAX} represents the largest eigen value of the matrix, the consistency index CI is defined as:

$$CI = \frac{\lambda_{MAX} - n}{n-1}$$

Based on the CI, random consistency ratio (CR) is defined as:

$$CR = \frac{CI}{RI}$$

where RI is the random consistency index related to the size of matrix, the concrete number of RI is listed in the Table 2

Table 2. Random consistency index

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46

If $CR \le 0.1$, the constructed pairwise comparison matrix is regarded rational and the alternatives' weight is obtained through the following definition. Otherwise the pairwise comparison matrix should be reconstructed.

Definition 2.6 (Eigen vector) for a pairwise comparison matrix $M_{n \times n}$, assume $\vec{w} = [w_1, w_2, ..., w_n]^T$ is the eigen value of M, the w_i denotes the weight assigned to i_{th} alternatives, and it could be calculated by:

$$M\vec{w} = \lambda_{MAX} \vec{w}$$

3. THE PROPOSED METHOD

3.1 Extracting Main Characteristics using Modified Fractal Model of Belief Function

3.1.1 Entropy within Belief Functions

Given a belief function $X = [\{Q\}; \{M\}]$,Q represent the set of all focal elements (i.e $Q = \{q_i\}$)and M represents all of basic possibility assignment (BPA) allocated to focal elements (i.e $M = \{m_i\}$). There are two source of entropy within belief function:

(1) The first kind of entropy within belief function is core entropy. The core entropy derives from the selected distribution of belief function in the set space. It has the following form:

$$h(q_i)=\frac{q_i}{\sum_j |q_j|}$$

$$H_q(x)=\sum H_{q_i}=-\sum h(q_i)\log h(q_i)$$

$h(q_i)=\dfrac{q_i}{\sum_j |q_j|}$ the $|q_i|$ indicates the cardinality of q_i.

(2) The second kind of entropy within belief function is belief entropy. It has the following form:

$$H_m(X)=\sum H_{m_i}=-\sum m(q_i)\log m(q_i)$$

where $m(q_i)$ means the basic probability assignment of q_i.

The existence of entropy in belief function is caused by information gaps. And gaps could be characterised by two type of distance

(i) distance between elements of set
(ii) distance between sets

Based on the concept of core entropy and belief entropy, a new type of entropy called within set entropy (WSE) which could measure ranges between zero and infinity are obtained:

$$H_w=H_m(X)+H_q(X)$$

These three kind of entropy submit a reasonable method quantifying uncertainty in focal elements itself. Besides the separate focal elements, the divergence among focal elements in a belief function also shows uncertainty, we denoted the entropy derived from dissimilarity between focal elements as focal divergence (D):

$$D(X)=\sum\sum d(q_i,q_j)$$

$d(q_i,q_j)$ is the divergence between two focal element, it can be calculated by:

$$d(q_i,q_j)=\frac{(1-sim(q_i,q_j))e^{|m(q_i)-m(q_j)|}}{e}$$

$$sim(q_i,q_j)=\frac{|a(q_i)\cap a(q_j)|}{|a(q_i)\cup a(q_j)|}$$

$a(q_i)$ is the set of all attributes of q_i. It can be easily proven formula of d has the two important characteristic:

(i) Having upper bound and lower bound (The range of $sim(q_i,q_j)$ is $[0,1]$) and
(ii) Being symmetric.

3.1.2 Entropy within the Set of Belief Functions

The set of belief functions has uncertainty as well as belief function because it is the aggregation of various belief functions. So the concept of entropy can be used to measure the information gap within different belief functions in a same set. And the entropy in its inner structure is not simply accumulation of every part of belief function. Assume $S=\{X_1,X_2,...,X_n\}$ is a set of belief functions.

$$H_w(S)=\sum H_{in}(X_i)$$

Given two different belief function:

$$X_2=\left[\{r_1,r_2,...,r_i,...\};\{m(r_1),m(r_2),...,m(r_i),...\}\right]$$

$$X_1=\left[\{q_1,q_2,...,q_i,...\};\{m(q_1),m(q_2),...,m(q_i),...\}\right]$$

$$X_2=\left[\{r_1,r_2,...,r_i,...\};\{m(r_1),m(r_2),...,m(r_i),...\}\right]$$

The divergence of two BPAs is denoted as *Div* which could be calculated with:

$$Div(X_1,X_2)=Div(X_2,X_1)=ddiv(X_1+X_2)+ddiv(X_2+X_1)$$

ddiv is defined as:

$$ddiv=(X_1,X_2)=\sum\frac{|m(q_i)-m(r_j^*)-\max_{sim(q_i,r_j)}+1|}{2}$$

$$ddiv=(X_2,X_1)=\sum\frac{|m(r_j)-m(q_i^*)-\max_{sim(r_j,q_i)}+1|}{2}$$

In the equation presented above, j^* (or i^*) refers to the focal element which makes $sim(q_i,r_j)$ (or $sim(r_j,q_i)$) the maximum among all of them.

The internal divergence of belief function set S is:

$$D(S)=\sum\sum div(x_i,x_j)$$

$$D(S)=\sum\sum div(x_i,x_j)\quad \text{for } i>j$$

The inner entropy of S is

$$H_{in}(S)=H_w(S)+D(S)$$

3.1.3 The Fracture Dimension, Capacity, Projection and the Density of Belief Functions

3.1.3.1 Fractal Dimension

Fractal dimension reveals the spread of a set in space. The belief function own this attribute due to the cavities between focal elements. Fractal dimension provides a tool to measure the belief function's scale and size. In the fractal model of belief functions, the fractal dimension is defined as the ratio of within entropy by inner entropy:

$$\dim(X)=\frac{H_W(X)}{H_{in}(X)}$$

For the set of belief functions, we also used the above equation to measure its size.

3.1.3.2 Capacity and Projection

The capacity of a belief function is defined as:

$$cap(X)=\frac{1}{H_{in}(X)}$$

And according to the capacity, we derived another concept projection of belief function. The projection of dimension s_1 of belief function X_1 on belief function X_2 of dimension s_2 ($s_2<s_1$) is

$$proj\left[\dim(X_1)\right]=\dim(X_1)cap(X_2)$$

3.1.3.3 Density of Belief Function

Density is a concept on the basis of capacity and within entropy for belief functions in a set. Let $S=\{X_1,X_2,...,X_n\}$ be a set of different belief functions, the density of any belief function in it is defined as:

$$dens(X_i) = \frac{Hin(X_i)}{cap(X_i)}$$

The density in the fractal world with various uncertainties as well as information gaps reveals its level of existence in its world. In other word, the density of a specific belief function depends on itself and its belonged set. Thus a belief function may have different densities in different sets. The density of a belief function can show its level of affiliation to its environment.

3.1.3.4 The Conductivity between Two Belief Function

Considering two different belief function X and Y, we defined two belief functions of fractal dimension as:

$$\sigma(X,Y) = \frac{Hin(Y) + Hin(X)}{Hin(Y) + Hin(X) + Div(X,Y)}$$

Two different belief function could interact with each other, this kind of interaction could be regarded as a kind of similarity to some extent.

3.2 Multiple Decision-making based Fractal Model and Evidence Theory

Step 1 : Firstly, we transformed the collected samples into the form of belief functions, and extract the main characteristics from the transformed belief functions. Since there are lots of uncertainties, we analysed the samples to reduce the noises caused by other factors. In this case, the priority of different samples should not be viewed as equal. Assigned different weight to the samples. Let $S = \{X_1, X_2, ..., X_n\}$ is n different evidences we have collected in uncertainty environment. $W = \{w_1, w_2, ..., w_n\}$ indicates their weight separately, we calculated it with:

$$w_i = \frac{\sum_{j=1, j\neq i}^{n} \sigma(X_i, X_j)}{\sum_{l=1}^{n} \sum_{j=1, j\neq l}^{n} \sigma(X_l, X_j)}$$

Through assigned different weight to the collected samples, we can know the credibility of each evidence. One evidence gets its weight by other evidences' support. The more support it is supported by other evidences, the more weight it will be allocated. Thus the belief function will produce more conductivity σ with other belief functions due to fractal characteristic.

Step 2 : Besides the fuzzy and uncertainties in the collected evidence, the limitation of our understanding for the knowledge base infects our judgements as well. We assigned different weight to different prototypes due to their own density in the class via AHP method. More specifically, for the class that has least three prototypes, we used the density of each belief function as an indicter to construct the pairwise comparison matrix and get the importance of each prototype. And for the class which consists only two prototypes, each prototype is regarded equal (assigned 0.5 separately) and only-one-prototype class, the weight is 1.

Step 3 : To know the level of match between samples and prototypes, we calculated the conductivity between a sample and every prototypes in the class. We considered the weight of the prototypes obtained from Step 2, then add all the conductivities to get conductivity between the samples and class.

Example 1 : Assume Y is a sample from robot sensor. There is a class $C = \{p_1, p_2, p_3\}$ which has three prototypes, and importance shown as $\vec{w} = [w_{p_1}, w_{p_2}, w_{p_3}]^T$ The conductivity between samples and the class C is calculated by:

$$\sigma(Y,C) = \sum_{i=1}^{3} wp_i \sigma(Y, p_i)$$

Step 4 : After conducting Step 2, we have all the conductivities of each samples and every class. Let $Y = \{Y_1, Y_2, Y_3, ...Y_n\}$ is all the samples we acquired which contains diversified uncertainties, and $C = \{C_1, C_2, C_3, ...C_m\}$ is all the class that samples may belong to. We transformed the evidences to belief functions to show the support of evidence for each class. We have the conductivity of samples and classes. In particular, we define the conductivity between sample Y_i and whole set T as:

$$\sigma(Y_i, T) = 1 - \max(\sigma(Y_i, C_j))$$

$\max(\sigma(Y_i, C_j))$ refers to the largest conductivity Y_i having with class. This value shows some other cased beyond our exception, in other words, the conductivity represent our ignorance to the evidences.

Example 2 : We have the conductivity of samples and classes from the Step 2. Taking Y_n as an example, $\sigma = \{(\sigma(Y_n, C_1), (\sigma(Y_n, C_2), ...(\sigma(Y_n, C_m), (\sigma(Y_n, T)\}$ is the conductivities between Y_n and each class, the basic probability assignment produced by Y_n is defined as:

$$b(C_i) = \frac{\sigma(Y_n, C_i)}{\sum_{j=1}^{m} \sigma(Y_n, C_j) + \sigma(Y_n, T)}$$

$$b(T) = \frac{\sigma(Y_n, T)}{\sum_{j=1}^{m} \sigma(Y_n, C_j) + \sigma(Y_n, T)}$$

In this step, we obtained n BPAs because we collected n samples Y.

Step 5 : On the basis of Step 1 and Step 4, we combined these n BPAs with their own weight by using the combining rules of D-S evidence theory, then get a comprehensive BPA, we denoted it as CBPA, we make the final accurate decision through this CBPA:

$$CBPA = [w_1 \times b(Y_1)] \oplus [w_2 \times b(Y_2)] \oplus$$
$$[w_3 \times b(Y_3)] ...... \oplus [w_n \times b(Y_n)]$$

4. A CASE TO STUDY

Here authors used a case from Erkmen & Stephanou[38] to verify the validness of our new method. Figure 1 shows four object classes: pyramid, L-shape, handle and cylinder, and it can be denoted by:

CLASS={*pyramid*, *L−shape*, *handle*, *cylinder*}

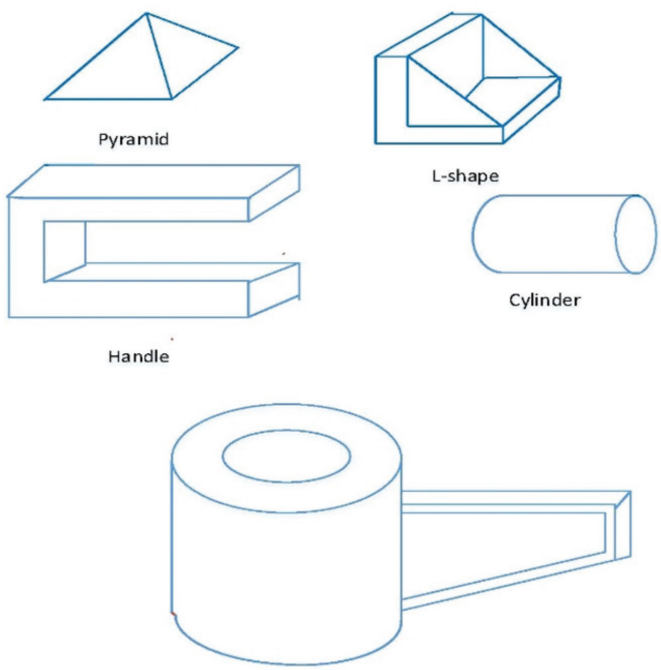

Figure 1. The typical knowledge base and collected evidence.

Part 1 : Object Representation

All these primitives can be represented as three aspects: face, edge, and vertices. Considering the aspect of plane, the characteristic of a plane is as shown by its number, type and curvature of faces. It can be expressed by a group which is triplet (consisted by its number, type and curvature of faces). For a cube, it has six squire planes, thus it could be represented as {6, *s*, *p*}. According to the practical situation, the number of face (*fnum*), face type (*ftype*) and face curative (*fcur*) have observed following rules:

$1 \le f_{num} \le 8$

ftype{T(triangle),S(square),R(rectangular),B(bracket),C(circle),D(disk)}

fcur ∈ *{ P(planer), C(curved)}*

Thus, the whole set FF contains $4 \times 3 \times 8 = 96$ different combinations with 15 attributes. In existed knowledge base, every class contains many prototypes expressed by belief function, *F* contains all the possibilities, which represents uncertainties. The knowledge is as shown in the Table 3.

Table 3. The knowledge base

Class	Prototypes
Pyramid A	A_1=[{(4,T,P),(1,S,P),F};{0.5,0.3,0.2}]
	A_2=[{(4,T,P),(1,R,P),F};{0.5,0.25,0.25}]
	A_3=[{(4,T,P),F};{0.7,0.3}]
L-shape B	B_1=[{(4,T,P),(2,L,P),(5,R,P),(1,S,P),F}; {0.2,0.4,0.1,0.2,0.1}]
	B_2=[{(2,T,P),(1,S,P),(5,R,P),F};{0.4,0.2,0.1,0.3}]
	B_3=[{(6,R,P),F};{0.5,0.5}]
Handle C	C_1=[{(2,B,P),(6,R,P),(2,S,P),F};{0.4,0.2,0.2,0.2}]
	C_2=[{(2,C,P),(1,R,Q),F};{0.4,0.4,0.2}]
Cylinder D	D_1=[{(2,C,P),(1,R,Q),F};{0.4,0.4,0.2}]

Part 2 : Sensor Evidence

Some belief functions are collected from sensors:

$$EV1 = [\{(1, R, P); FF\}; \{0.8, 0.2\}]$$

$$EV2 = [\{(1, S, P); FF\}; \{0.6, 0.4\}]$$

$$EV3 = [\{(2, S, P); FF\}; \{0.7, 0.3\}]$$

$$EV4 = [\{(2, T, P); FF\}; \{0.5, 0.5\}]$$

Part 3 : Multiple Decision-making using our Method

Now we have the information both from evidences and prototypes in knowledge base. We can make the judgement. Since the four evidence are expressed by belief functions. According to its fractal characteristic and formula we have mentioned above, the credibility derived from other evidences' support is calculated:

$$w_{EV1} = 0.2421 \quad w_{EV2} = 0.2604$$

$$w_{EV3} = 0.2631 \quad w_{EV4} = 0.2344$$

Then we calculated the importance of each prototypes in their own class.

For class pyramid A, three prototypes A_1, A_2, A_3 importance priority is:

$$w_{A_1} = 0.2421 \quad w_{A_2} = 0.2604 \quad w_{A_3} = 0.2631$$

For class L-shape *B*, three prototypes B_1, B_2, B_3 importance priority is:

$$w_{B_1} = 0.5578 \quad w_{B_2} = 0.3600 \quad w_{B_3} = 0.0822$$

And for the class Handle *C*, the two prototype C_1, C_2 are assigned 0.5 equally while the only one in the class cylinder is 1.

Next we calculated the conductivity and acquire the results shown as Table 4.

Table 4. The conductivity between the evidences and each class

	Class A	Class B	Class C	Class D	Whole set T
EV1	0.6315	0.6015	0.5960	0.6146	0.3685
EV2	0.6330	0.6873	0.5822	0.5737	0.3127
EV3	0.5965	0.6183	0.6475	0.6332	0.3525
EV4	0.6254	0.0628	0.6185	0.6218	0.3372

According to the conductivity between evidence and each class we obtained four BPAs using the rules mentioned by Step 4, For $Class = \{A, B, C, D, T\}$

$$EV1: \{0.2246, 0.2139, 0.2119, 0.2198, 0.1310\}$$

$$EV2: \{0.2270, 0.2464, 0.2088, 0.2057, 0.1121\}$$

$$EV3: \{0.2094, 0.2171, 0.2274, 0.2223, 0.1238\}$$

$$EV4: \{0.2182, 0.2313, 0.2158, 0.2170, 0.1177\}$$

Finally, we combined theses four BPAs with their weight calculated above through D-S combing rules and get comprehensive BPA (CBPA):

$$CBPA = \{0.2444, 0.2776, 0.2277, 0.2278, 0.0225\}$$

Thus the result proves that obtained object belongs to Class B.

5. DISCUSSIONS

As shown in the Section 4, the results of identification is that object is a L-shape model, this result is reasonable since the descriptions of evidence have many similarity with L-shape displayed in Fig. 1. Even it is different from Erkmen's[38] results, the results calculated from proposed model is more credible.

6. CONCLUSIONS

In this study, authors presented a novel model to solve the pattern classification using the fractal model of belief function combining with information theory, evidence theory and AHP. The novel method considers the multiple uncertainties within collected samples as well as our knowledge base. Thus it could decrease error as much as possible. In the future, we should focus on the rapid algorithms of this model, some concise and effective format will be investigated, so it could be applied in parallel computing and other complex system in practice.

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