# Effect of Troposphere and Ionosphere on C-Band Radar Track Data and Correction of Tracking Parameters 

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#### Abstract

At any launch base radar tracking is critical for assessing the state of the launch vehicle for range safety function as well as for trajectory monitoring. Tracking data corrected for refraction effects is essential for estimating the flight performance. This paper presents the refraction effects of troposphere and ionosphere on the radio waves of C-Band radars in terms of errors in the measured tracking parameters. Application of mapping functions for correcting refraction errors in C-Band radar track data is studied and a comparison of the performance of mapping functions is presented. The mapping functions not only show good convergence at low elevations but also correct around 90 per cent of the refraction error in elevation.


Keywords: Astronomical refraction, mapping function, refraction error, ionospheric refraction, single layer model, ionospheric pierce point

## NOMENCLATURE

| $a, b, c$ | Coefficients of the Niell's mapping function |
| :---: | :---: |
| $e$ | Partial pressure of water vapour in millibars |
| H | Effective height of the atmosphere |
| $H_{m}$ | Height of ionospheric pierce point |
| I | Normalised zenith argument |
| $m^{\prime}\left(\xi_{0}, \mathrm{p}\right)$ | Mapping function of astronomical refraction |
| $n$ | Refractive index of the atmosphere at any field point |
| $n_{e}(x)$ | Electron density |
| $N(h)$ | Refractivity of the atmosphere at the height $h$ |
| P | Total barometric pressure in millibars |
| T | Ambient air temperature in Kelvin |
| TEC | Total electron content |
| TEC ${ }_{v}$ | Vertical total electron content |
| TEC ${ }_{s}$ | Slant total electron content |
| $\mathrm{Z}_{0}$ | Arrival zenith angle |
| $\xi_{0}$ | True zenith angle |
| $\emptyset_{i}$ | Latitude of the station |
| DOY | Day of the year |
| $v$ | Frequency of radio wave |
| $\sigma$ | Atmospheric delay |

## 1. INTRODUCTION

The atmosphere of the earth is divided into a neutral layer near the earth called troposphere and an outer ionized layer called ionosphere. Radar waves travel in rectilinear path in free space ${ }^{1}$. However, as they pass through the earth's atmosphere, they are refracted due to the variation of velocity of propagation with altitude. Refraction in troposphere does
not depend on frequency but on elevation and meteorological conditions. Refraction error in elevation grows as large as $1^{\circ}$ at horizon. Substantial work is carried out to estimate the effect of troposphere on radio wave propagation using mapping functions with continued fraction representation. Marini ${ }^{3}$, Chao, Davis ${ }^{4}$, Niells ${ }^{8}$, Yan ${ }^{5}$, Ifadis ${ }^{7}$, and Boehm ${ }^{9}$ contributed significantly towards deriving mapping functions to predict refraction error up to $3^{\circ}$ of elevation. In addition, ionosphere stretching from 50 km to 1000 km also affects radar waves. Ionospheric error is proportional to total electron content along the path and inversely proportional to the square of frequency of radar waves. Ionospheric models such as Klobuchar ionospheric model, Global Ionospheric maps, Ne Quick model and International Reference Ionosphere 2007 reasonably predict total electron content ${ }^{12-16}$. Ionospheric correction models by Klobuchar ${ }^{12}$, Stephen ${ }^{13}$, Petrie ${ }^{17}$, Brunner ${ }^{18}$ and Hoque ${ }^{19}$ denote some of the note-worthy contributions. This paper presents the effects of troposphere and ionosphere on the radio waves of radars in terms of errors in the measured tracking parameters and the efficacy of the various correction methodologies. These corrections in tracking data are essential for accurate trajectory estimation.

## 2. ATMOSPHERIC REFRACTION CORRECTION-MAPPING FUNCTION METHODOLOGIES

### 2.1 Atmospheric Profile

Complexity of the earth's atmosphere and its variability with time makes it difficult to model refraction. The most widely used model in modern space techniques is a spherically symmetric, layered atmosphere with troposphere characterised by constant lapse rate of temperature near the earth's ground

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and the stratosphere with constant temperature above troposphere ${ }^{2-3}$. Ionosphere starts from a height of 50 km and stretches to heights of 1000 km and more. Klobuchar (1996) described the layers of ionosphere and their contribution to global positioning system (GPS) signal propagation. They are denoted as $D, E, F 1$, and $F 2$ layers. $D$ region extends in height from 50 km to 90 km and it has no measurable effect on GPS frequencies. E region from 90 km to 140 km has minimal effect on GPS frequencies in normal conditions. F1 region extending from 140 km to 210 km contribute 10 per cent of the ionospheric delay. F2 region lying between 210 km to 1000 km is the most dense, highly variable region affecting GPS signal propagation. Peak electron density in the $F 2$ region occurs at heights varying between 250 km and 400 km . In nutshell, F2 layer and to some extent $F 1$ layer affect radio wave propagation at GPS frequencies.

### 2.2 Modelling of Refraction Error

The transit of a radio wave between a source in the space and a tracking station on the ground is diagrammatically represented in Fig 1. The tracking station is located at a height of $h_{0}$ from the sea level whereas the source is at a height of $h_{1}$. $R_{e}$ is the radius of the earth. The path of the ray as well as the chord joining the source and the tracking station are depicted as a dotted line and straight line respectively.


Figure 1. The signal path and the chord from source to tracking station.
The arrival zenith angle at the station is $Z_{0}$. The true zenith angle is $\xi_{0}$. The astronomical refraction $\Delta Z_{0}$ is defined as the difference between the true zenith angle $\xi_{0}$ and the arrival zenith angle $z_{0}$ at a station and it is expressed as ${ }^{5}$

$$
\begin{equation*}
\Delta Z_{0}=\xi_{0}-Z_{0}=\int_{1}^{n o} \frac{\tan z}{n} d n \tag{1}
\end{equation*}
$$

Here $n$ is the refractive index of the atmosphere at any field point on the signal path. $n_{0}$ is the refractive index at the station and $Z$ is the arrival zenith angle of the signal taken at a field point on the signal path ${ }^{5}$. The signal delay due to atmospheric refraction is the difference between optical distance and geometrical distance between source and tracking station and is given by

$$
\begin{equation*}
\Delta \sigma=\int_{l} n d l-\int_{X} d x \tag{2}
\end{equation*}
$$

where $d l$ and $d x$ are elements on the signal path and the geometrical distance.

The distribution function of refractivity in an exponential atmospheric profile $N(h)$, is written as a function of the vertical coordinate $h$ of the field point

$$
\begin{equation*}
N(h)=N(o) e^{-(h-h 0) / H} \tag{3}
\end{equation*}
$$

where the refractivity $N$ and refractive index $n$ are related as

$$
\begin{equation*}
\mathrm{N}=10^{6}(\mathrm{n}-1) \tag{4}
\end{equation*}
$$

$N(0)$ is the value measured at the station of height $h_{o}$. Refractivity $N$ for radio frequencies can be obtained from the Smith-Weintraub equation ${ }^{10}$ as

$$
\begin{equation*}
\mathrm{N}=77.6 \frac{p}{T}-12.8 \frac{e}{T}+3.776 \times 10^{5} \times \frac{e}{T^{2}} \tag{5}
\end{equation*}
$$

Here $P$ is the total barometric pressure in milli bars, e is the partial pressure of water vapour in milli bars, and $T$ is the ambient air temperature in Kelvin.

The effective height $H$ of the atmosphere expressed in terms of vertical integration function of the refractivity $N$ is

$$
\begin{equation*}
\mathrm{H}=\frac{1}{N 0} \int_{h 0}^{\infty} N(h) d h \tag{6}
\end{equation*}
$$

From Eqn (3) its differential relation is expressed as

$$
\begin{equation*}
\mathrm{dn}=-10^{-6} \frac{N(h)}{H} d h \tag{7}
\end{equation*}
$$

Then the astronomical refraction $\Delta Z_{0}$ can be expressed as an approximation

$$
\begin{equation*}
\Delta Z_{0} \approx 10^{-6} \frac{\operatorname{Sin} Z}{H} \int_{h 0}^{\infty} \frac{N(h) d h}{n \cos Z} \tag{8}
\end{equation*}
$$

According to the definition of refractive delay by Yan ${ }^{5}$, the above expression can be formally written as

$$
\begin{equation*}
\Delta Z_{0}=10^{-6} N(0) \sin \xi_{0} \mathrm{~m}^{\prime}\left(\xi_{0,} p\right) \tag{9}
\end{equation*}
$$

where the function $m^{\prime}\left(\xi_{0}, p\right)$ can be defined as the mapping function of astronomical refraction. Here $\xi_{0}$ is the true elevation and $p$ represents the meteorological and geophysical parameters such as temperature, pressure, humidity, height of the tropopause, etc. In a similar way $\Delta \sigma$ of Eqn (2) can be expressed as

$$
\begin{equation*}
\Delta \sigma=\Delta \sigma_{z} m^{\prime}\left(\xi_{0}, p\right) \text { where } \Delta \sigma_{z}=10^{-6} N(0) H \tag{10}
\end{equation*}
$$

### 2.3 Mapping Functions

Based on the continued fraction expression of the incomplete $г$-function, Marini ${ }^{3}$ developed a mapping function as a function of elevation $\xi$ and with a set of parameters $f 1, f 2, f 3$ as

$$
\mathrm{m}(\mathrm{\xi})=
$$



Davis ${ }^{4}$, et al. developed a parametrized continued fraction model CFA2.2 where the second $\operatorname{Sin}(\xi)$ was replaced by $\operatorname{Tan} \xi$ and $f 1, f 2, f 3$ were defined as functions of total surface pressure, partial pressure of water, surface temperature, temperature lapse rate and tropopause height. It is designed to achieve accuracy up to $5^{\circ}$ of elevation. He has also considered the standard atmosphere and true direction instead of arrival direction for computation of zenith angle. An improved continued fraction mapping function for astronomical refraction (UNSW931) was developed by Yan and Peng ${ }^{5-6}$ which can be described as

$$
\begin{align*}
& \mathrm{m}\left(\xi_{0}\right)= 1 \\
& \operatorname{Cos} \xi_{0}+\frac{A_{1}}{I^{2} \operatorname{Sec} \xi_{0}+\frac{A_{2}}{\left.\operatorname{Cos} \xi_{0}+\ldots A_{3}\right]}} \\
& I^{2} \operatorname{Sec} \xi_{0}+A_{4} \tag{12}
\end{align*}
$$

The meteorological parameters in $A_{1}, A_{2}, A_{3}$ and $A_{4}$ of the above equation come from the least squares adjustment procedure of the numerical integrals of equation along the paths of signals for different ground meteorological parameters and various elevations. They are functions of the ground temperature, total ground pressure and wet partial pressure respectively. $\xi_{0}$ is the true zenith angle and $I$ is the normalized zenith argument. This mapping function is designed for use up to $2^{\circ}$ of elevation.

Ifadis ${ }^{7}$ ray traced real weather profiles from geographically distributed stations all over the world and with different climatic conditions and developed a mapping function that can be used down to $2^{\circ}$ of elevation. Niell ${ }^{8}$ used globally distributed radio sonde network data and developed a global mapping function [NMF] with wide latitude and meteorological parameter coverage having high convergence up to $3^{\circ}$ of elevation.

$$
\mathrm{m}^{\prime}(\xi)=1+\frac{\mathrm{a} /(1+\mathrm{b} /(1+\mathrm{c}))}{\frac{\sin \xi^{\prime}+\ldots \mathrm{a}}{\sin \xi+\ldots-\mathrm{b} \overline{\sin \bar{\xi}+\mathrm{c}}}} \quad \begin{gather*}
\text { +Height } \\
\text { correction } \tag{13}
\end{gather*}
$$

The coefficients $a, b, c$ are totally independent of surface meteorological parameters but depend on latitude, height of the station and Day of the year (DOY). Niell constructed a $15^{\circ}$ latitude grid table and modelled the seasonal amplitude for each of these three coefficients. For example the coefficient a of the hydrostatic mapping function can be calculated as
$\mathrm{a}_{\text {hyd }}\left(\varnothing_{\mathrm{i}}, \mathrm{DOY}\right)=\mathrm{a}_{\text {avg }}\left(\varnothing_{\mathrm{i}}\right)+\mathrm{a}_{\text {amp }}\left(\varnothing_{\mathrm{i}}\right) \operatorname{COS}[2 \pi($ DOY -28$) / 365.25]$
where $\emptyset_{i}$ is the latitude of observation, DOY is the Day of the Year from January 0.0 of $U T, a_{\text {amp }}$ and $a_{\text {avg }}$ are values of the average and amplitude of $a$. Similar procedure is adopted for coefficients $b$ and $c$. The height correction is a function of height of the station above the sea level and a three term continued fraction expression similar to Eqn 13. The coefficients $a, b$, and $c$ for height correction were determined by least squares
fit to nine elevation angles ${ }^{8}$. Johannes ${ }^{9}$, et al. proposed Vienna mapping functions (VMF) and later improved upon it (VMF1) using 40 years data of European centre for medium range weather forecasts (ECMWF) for year 2001.

## 3. PLASMA EFFECTS ON RADAR TRACKING

### 3.1 Ionospheric Effect on Radio Signals

Radio signals undergo group delay as they pass through the ionosphere. In addition, the signals also experience significant refraction ${ }^{12-13}$.

The group delay of a radar pulse travelling through a plasma is

$$
\begin{equation*}
\Delta \mathrm{L}=40.3 / v^{2} \int_{0}^{l} \mathrm{n}_{\mathrm{e}} \mathrm{dl} \tag{15}
\end{equation*}
$$

where $v$ is the radar pulse frequency and $n_{e}$ stands for the electron density along the path of the ray. The integral stands for the total electron content (TEC) along the one-way signal path of length $l$. It is called slant TEC $\left(\mathrm{TEC}_{\mathrm{s}}\right)$ representing the total number of electrons in a column of length $l$ and cross section of one square metre. It is measured in the units of $10^{16}$ electrons $/ m^{2}$. Nevertheless it is the vertical TEC (TEC ${ }_{\mathrm{v}}$ ) that is generally modelled. A two dimensional Ionospheric Single Layer model is used to convert the slant TEC values to vertical TEC values using a Simple mapping function ${ }^{17}$.

This model assumes ionosphere to be compressed into a thin shell at the peak ionospheric height of 350 km as illustrated in Fig. 2. The thin shell model is used and its height is taken as the altitude of Ionospheric pierce point (IPP).


Figure 2. Ionospheric single layer model.
TECs through a given sub-ionospheric point is obtained
as $T E C_{v}=T E C_{s} \cos \xi^{\prime}$
where $\operatorname{Sin} \xi^{\prime}=\frac{R e}{R e+H m} \operatorname{Sin} \xi$
where $R_{e}$ is the mean earth radius, $H_{m}$ is the height of maximum electron density, and $\xi$ and $\xi^{\prime}$ are the zenith angles at the station and at the ionospheric pierce point respectively. Angle $\xi$ can be calculated for the source with respect to the station. In general the value of $H_{m}$ is taken as the height corresponding to the maximum electron density at the F2 peak. The peak altitude ranges from 250 km to 350 km at mid-latitudes and from 350 km to 500 km at equatorial latitudes. Typical value
for $R_{e}$ and $H_{m}$ are set to 6371 and 450 km , respectively ${ }^{17}$.
Now the group delay of the radar pulse through plasma can be expressed as

$$
\begin{equation*}
\Delta \mathrm{L}=40.3 \sec \left(\xi^{\prime}\right) / v^{2} T E C v \tag{17}
\end{equation*}
$$

where $\xi$ is the zenith angle of signal path at the Ionospheric pierce point, and $T E C v$ is the vertical electron content estimated there. For a C-Band radar tracking a launch vehicle, $\Delta L$ will have a value around a few metres depending on the state of the ionosphere and altitude of the vehicle. The transverse displacement of the radar pulse will have very similar values to group delay. Since total electron content is a key player in the estimation of ionospheric corrections, a good understanding of the plasma distribution in the ionosphere and plasma sphere is essential to arrive at the partition of total electron content between the vehicle and station. A number of models and products are available to estimate total electron content globally and round the clock. Prominent among them are Global ionospheric maps provided by International GNSS analysis centre, Klobuchar model ${ }^{12}$, IRI 2007 model ${ }^{14}$, Nequick model ${ }^{16}$, etc.

## 4. APPLICATION TO RADAR TRACK DATA

 Following sections describe the application of the tropospheric and ionospheric refraction correction methodology to the tracking data of $C$-Band radars and the efficacy of various methodologies.
### 4.1 Tracking Network and Metric Accuracy

At Sriharikota range, the space port of India, long range tracking network consists of three C-Band precision radars and one conical scan radar. Measurements consist of timetagged slant range, azimuth and elevation. Tracking accuracy of precision radars is 10 m in slant range and 0.2 milli radian in angles. Conical scan radar accuracy is 10 m in slant range and a milli radian in angles ${ }^{20}$.

### 4.2 Tracking Scenario for Various Missions

From Sriharikota range, a number of sounding rockets and satellite launch vehicles are launched for meeting the atmospheric, remote sensing and communications needs of our country. Launch vehicles are instrumented with C-Band transponders for long range tracking. For each launch, radars track the launch vehicle in real time for range safety function as well as for trajectory monitoring. They are tracked up to a slant range of 3000 km with an angle accuracy of 0.2 milli radians. GPS receiver data onboard the vehicle is considered for aiding the inertial navigation system (INS) of the launch vehicle. It is known as GAINS system. It provides the position and velocity of the vehicle at 1 Hz . These data are acquired during real time for processing and for archival.

### 4.3 Radar Data Processing Methodology

Radar track data pertaining to a number of sounding rocket missions and satellite launch vehicle missions are analyzed. At the outset, radar track data are corrected for various errors random as well as systematic. Random noise in the data is
filtered using a non-recursive 41 point polynomial filter or a recursive Kalman filter. Systematic errors pertaining to $R F$ axis, pedestal, encoder and dynamics in the radar tracking data are modelled and corrected. These errors are well calibrated for each mission using IRS satellite tracking trials ${ }^{20}$.

Sriharikota range has a state of the art meteorology facility which acquires atmospheric data such as atmospheric pressure, temperature, humidity in real time to estimate tropospheric refractivity $N(0)^{11}$. Position and velocity data from the GAINS system is considered as the reference. Accuracy of the state vector from GAINS system is better than 50 m in position and $1 \mathrm{~m} / \mathrm{s}$ in velocity. This data are used to compute true target designations of the vehicle with respect to the station. Difference between the measurement data (filtered and corrected for all systematic errors) and reference values provided by GAINS system quantifies the refraction error.

### 4.4 Refraction Error Models

For the study a number of refraction correction models for Tropospheric refraction such as 1. Marini model 2. UNSW931 model 3. Davis CFA 2.2 model 4. Ifadis model and 5. Niell's NMF model respectively are considered ${ }^{3-8}$. All these models make use of differing and improved continued fraction forms for the mapping functions. Slant range delay and refraction error in elevation are estimated for the launch vehicle trajectory using different models. Radio refractivity estimated at Srihaikota range on the launch day is used. Figure 3 provides the slant range delays predicted by the models from $10^{\circ}$ of elevation to $3^{\circ}$ of elevation. At higher elevations, the range delay predicted by all these models converge to almost 1 m . As the elevation decreases to $10^{\circ}$, the delay gradually increases to 5 m . At $5^{\circ}$ of elevation, Marini model and UNSW931 predict a delay of 9 m , Niell and Ifadis predict around 10 m . CFA 2.2 predicts 11.5 m . At $3^{\circ}$ of elevation, Marini model and UNSW931 predict 13 m where as Niell and Ifadis predict around 15 m . CFA 2.2 only predicts 19.1 m .


Figure 3. Elevation vs tropospheric range delay using different mapping functions

Table 1 shows the refraction errors in elevation predicted by various models, the estimated error in the radar track data and the percentage of correction.

Also ionospheric range delays for VHF ( 160 MHz ), UHF ( 422 MHz ), $L$ band ( 1575 MHz ) and C-Band ( 5661 MHz ) frequencies are estimated. Range delays are estimated for these widely differing operational frequencies for a 100 units of total electron content. This value of 100 TEC units is considered to

Table 1. Predicted correction for refraction errors using different models

| Elevation <br> (degree) | Marini | UNSW931 | IFADIS | NIELL | CFA2.2 | Estimated <br> Error <br> (degree) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20.0 | 0.0499 | 0.0590 | 0.0591 | 0.059 | 0.064 |  |
|  | $[91.1 \%]$ | $[107.6 \%]$ | $[107.8 \%]$ | $[107.6 \%]$ | $[116.8 \%]$ | 0.0548 |
| 15.0 | 0.0677 | 0.0792 | 0.0795 | 0.0795 | 0.0825 | 0.0716 |
|  | $[94.6 \%]$ | $[110.6 \%]$ | $[111.0 \%]$ | $[111.0 \%]$ | $[115.2 \%]$ |  |
| 10.0 | 0.1028 | 0.1172 | 0.1184 | 0.1184 | 0.1243 | 0.1251 |
|  | $[82.2 \%]$ | $[93.7 \%]$ | $[94.6 \%]$ | $[94.6 \%]$ | $[99.4 \%]$ |  |
| 5.0 | 0.2057 | 0.2113 | 0.2179 | 0.2174 | 0.2487 | 0.2391 |
|  | $[86.0 \%]$ | $[88.4 \%]$ | $[91.1 \%]$ | $[90.9 \%]$ | $[104.0 \%]$ |  |
|  | 0.3470 | 0.2630 | 0.2745 | 0.2739 | 0.3379 | 0.3094 |
|  | $[112.2 \%]$ | $[85.0 \%]$ | $[88.7 \%]$ | $[88.5 \%]$ | $[109.2 \%]$ |  |

estimate the worst case ionospheric errors for a location like Sriharikota.

It is candidly clear from Fig. 4 that ionospheric delay for $C$-Band frequencies is 1000 times as small as the value for VHF frequencies. For operational tracking scenario of a $C$-Band radar ( 5.6 GHz ), the range delay is less than 10 m for elevations ranging between $2^{\circ}$ to $89^{\circ}$. The transversal displacement of the radio signal will also be of the same order and so it could be established that refraction error contribution by ionosphere for $C$-Band frequencies is less than 10 m in slant range and $0.001^{\circ}$ in elevation ${ }^{13}$.


Figure 4. Group delay at different elevations for different operational frequencies

## 5. DISCUSSIONS

This paper addresses the correction of tropospheric and ionospheric refraction observed in the radar tracking data of launch vehicles. Tropospheric refraction correction is carried out using mapping functions of astronomical refraction correction. Introduction of mapping functions with continued fraction expression is aimed at achieving higher accuracy at lower elevation coverage. It is observed that refraction effects begin to play significantly once elevation decreases below $20^{\circ}$ in the descending leg for longer ranges. Niells' NMF, UNSW391, Ifadis, CFA 2.2, Marini model show better convergence tendency as the elevation decreases. Marini
model could predict 80 per cent to 90 per cent of estimated error over the elevation range of $20^{\circ}$ to $5^{\circ}$ but overpredicts as elevation decreases to $3^{\circ}$. In a similar vein CFA2.2 also overpredicts the refraction error for elevation below $20^{\circ}$ to $3^{\circ}$ by almost 104 per cent to 116 per cent except at elevation of $10^{\circ}$. At $10^{\circ}$ of elevation, the error prediction is 99.4 per cent of the estimated error. In the same way UNSW391, Ifadis and Niells' NMF models overpredict the error by 110 per cent until $15^{\circ}$ of elevation. Later all these three models interestingly predict around 90 per cent of the estimated error until $10^{\circ}$ of elevation. UNSW931 predicts 88.4 per cent of the error at $5^{\circ}$ elevation and 85.0 per cent of error at $3^{\circ}$ of elevation. Niells’ NMF and Ifadis models correct around 91 per cent of the observed error at $5^{\circ}$ and 88 per cent at $3^{\circ}$ of elevation. From this it is candidly observed that these correction models i.e Marini, Niells' NMF, UNSW391, CFA 2.2, and Ifadis models substantially correct the observed refraction error.

Regarding tropospheric delay in slant range, Niells’ NMF, UNSW391, Ifadis, CFA 2.2 and Marini model show a convergence tendency as the elevation increases greater than $10^{\circ}$ of elevation. At $5^{\circ}$ of elevation all models predict a tropospheric delay of around 10 m . At low elevations such as $3^{\circ}$ of elevation, CFA 2.2 predicts the tropospheric delay of 19.1 m whereas Ifadis and Niell predict 15 m and Marini and UNSW931 12.5 m .

Since the refraction effects in ionosphere are significantly smaller for C-Band frequencies, a worst case TEC is considered for estimation of range and elevation errors. For existing operational tracking scenario at Sriharikota range, ionospheric range delay and elevation correction are less than 10 m and $0.001^{\circ}$. These are less than the tracking accuracies for precision C-Band radars.

After establishing the effects of troposphere and ionosphere on $C$-Band radar track data, correction methodologies are incorporated in trajectory estimation. At Sriharikota range, trajectory estimation of the launch vehicle is carried out using $C$-Band radar data from lift off to $5^{\circ}$ of elevation in the descending leg. It is observed in the study that tropospheric refraction errors, particularly refraction in elevation is the dominant error below $20^{\circ}$ of elevation. It
affects trajectory parameters such as altitude of the launch vehicle. For example, when the launch vehicle is at a slant range of 2300 km and at $5^{\circ}$ of elevation, refraction error of $0.24^{\circ}$ in elevation causes an error of 9.6 km in altitude. We have considered Niell's model for refraction correction in the trajectory estimation during real time. It is preferred to other models for its independence from local surface meteorological parameters and ease of implementation in the software. It has improved the computation of altitude of the vehicle by 90 per cent. Improvement for altitude computation for a typical PSLV mission ${ }^{21}$ is shown in Fig 5.


Figure 5. Efficacy of mapping function to correct refraction error in elevation thereby improving the computation of vehicle altitude

## 6. CONCLUSIONS

$C$-Band radar track data is mostly affected by troposphere during its transit. The effect is more clearly observed at elevations lower than $20^{\circ}$. Correction of elevation is imperative to estimate the trajectory of the space vehicle since it affects the computation of the altitude of the vehicle. This paper brings out some of the latest methods for correcting these errors and their efficacy of correction. Existing models with improved mapping functions could account for almost 90 per cent of the observed refraction error up to $5^{\circ}$ elevation of radar data. Niells's model is implemented in real time trajectory estimation methodology at Sriharikota range to correct refraction error and its application has significantly improved the computation of launch vehicle trajectory parameters.

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