Defence Science Journal, Vol. 67, No. 4, July 2017, pp. 460-464, DOI : 10.14429/dsj.67.11548

Planar Multi-body Dynamics of a Tracked Vehicle using Imaginary Wheel Model for Tracks

Ilango Mahalingam* and Chandramouli Padmanabhan

Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai - 600 036, India
**E-mail: me14d015@smail.iitm.ac.in*

ABSTRACT

Off-road vehicles achieve their mobility with the help of a track system. A track has large number of rigid bodies with pin joints leading to computational complexity in modelling the dynamic behaviour of the system. In this paper, a new idea is proposed, where the tracks are replaced by a set of imaginary wheels connected to the road wheels using mechanical links. A non-linear wheel terrain interaction model considering longitudinal slip is used to find out the normal and tangential contact forces. A linear trailing arm suspension, where a road arm connecting the road wheel and chassis with a rotational spring and damper system is considered. The differential algebraic equations (DAEs) from the multi-body model are derived in Cartesian coordinates and formulated using augmented formulation. The augmented equations are solved numerically using appropriate stabilisation techniques. The novel proposition is validated using experimental measurements done on a tracked vehicle.

Keywords: Tracked vehicle; Multi-body dynamics; Differential algebraic equations; Augmented formulation; Contact forces

1. INTRODUCTION

The major difficulty in the analysis of tracked vehicles using multi body dynamics is realistic modelling of tracks and their interaction with terrain. As the track involves large number of interconnected rigid bodies with complex constraints, analytical models of tracks ranging from single flexible band to complex rigid bodies interconnected with revolute joints are reported in literature.

Depending on the focus of the study the representation of tracks varies. An accurate analysis would require the model to have dynamics of each track link[1-3]. Such models are computationally expensive yet provide useful information in mechanical design of tracks. Focus of models discussed in[4,5] is to develop track models for ride dynamic analysis. These models take into account of the kinematic constraining effects of tracks on suspension system and predict the ride dynamic response. Models discussed in[6] are focused on studying the tractive performance of a tracked vehicle by considering the equilibrium of forces acting on the track system at steady state.

The proposed model comes under the scope of multi-body modelling approach. To overcome the drawbacks in having large number of bodies and complex joints for tracks, tracks are represented by a set of imaginary wheels and mechanical links. The purpose of this study is to look at the ride dynamic analysis of a tracked vehicle with this alternate model for tracks. In section 2 of this paper, the alternate model for tracks is introduced. In section 3, a nonlinear contact force model, used for finding normal and tangential forces is discussed. In section 4, the strategy adopted for solving the DAEs out of the alternate model is presented. In section 5, the results of the simulation of a tracked vehicle with alternate track model on a sinusoidal road profile are compared against the field measurements on a tracked vehicle.

2. ALTERNATE MODEL FOR TRACKS

2.1 Role of tracks in a tracked vehicle

The track performs several functions:

(a) It distributes the weight of the vehicle with its larger contact area and reduces sinkage resistance
(b) Generates tractive force to propel the vehicle
(c) Tracks envelop the road wheels and constraints the suspension system – track bridging effect.

The tracks varies in length and dimensions, with a long pitch for slow moving vehicles such as crawlers, whereas for a high speed tracked vehicle such as a battle tank have tracks with short pitch relative to their road wheels. Wong[6], *et al.* has done series of analytical and experimental studies to find the normal pressure distribution under the tracks. It can be understood from these studies that the pressure distribution is higher under the road wheels and lesser in between the road wheels; also the distribution varies with track tension and softness of terrain. For example, in case of a hard terrain with flat road profile the pressure distribution under the area between two road wheels is almost zero, which implies no traction generation. However, if the road profile is non-flat, the area between the road wheels will generate traction based on the tension of track.

Received : 06 March 2017, Revised : 17 May 2017
Accepted : 22 May 2017, Online published : 03 July 2017

2.2 Alternate Model

The planar multi-body model of a tracked vehicle used in this study is as shown in Fig. 1.

The model has twenty three rigid bodies with six road wheels, six road arms, five imaginary wheels, five imaginary road arms and a vehicle body. For simplicity the imaginary wheels are considered to be of same dimensions as the road wheels and are placed in between two road wheels. The imaginary wheels are attached to the road wheels using imaginary road arms with torsional spring and damper system at the pivot, this torsional spring and damper system used to mimic the track tension. The road wheels are attached to the chassis using road arm with torsional spring and damper system at the pivot, this torsional spring and damper system will mimic the vehicle suspension system.

Placing imaginary wheels in between the road wheels increases the contact area of vehicle running gear thus reducing the normal load and sinkage. To mimic the track driving characteristics all the wheels including the imaginary wheels will be given a proportion of driving torque based on the normal load acting on the wheel. Thus the role of tracksas mentioned earlier has been taken into account. The schematic diagram of the road wheel, imaginary wheel and suspension system is as shown in Fig. 2.

The suspension model is considered to be linear; however a nonlinear suspension model can be taken into account if needed. The suspension forces are formulated as

$$Q_i = k_s(\theta_i - \theta_j + \theta_s) + c_s(\dot{\theta}_i - \dot{\theta}_j) \tag{1}$$

$$Q_l = k_{sl}(\theta_l - \theta_j + \theta_s) + c_s(\dot{\theta}_l - \dot{\theta}_j) \tag{2}$$

$$Q_j = (Q_i - Q_l) \tag{3}$$

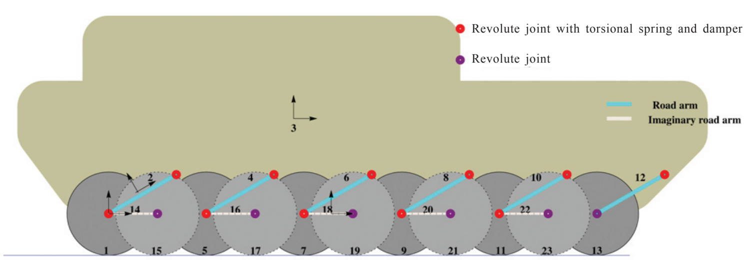

Figure 1. Tracked vehicle model with imaginary wheels.

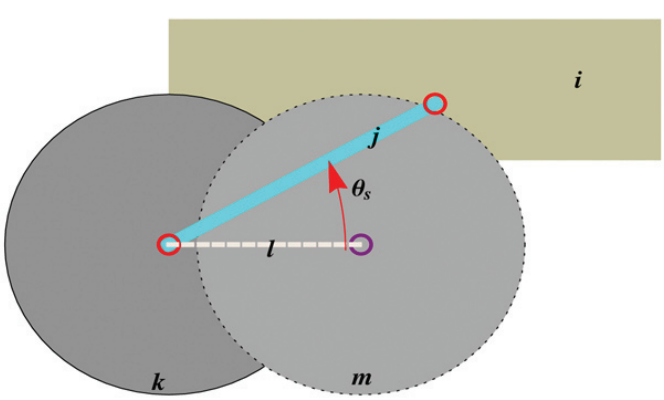

Figure 2. Road and imaginary wheel suspension.

where k_s and c_s are torsional stiffness and damping constants of the road wheel suspension system, k_{sI} and C_{sI} are torsional stiffness and damping constants of the imaginary wheel suspension system. The imaginary wheel stiffness is assumed to be quite lower than the road wheel stiffness.

3. WHEEL-TERRAIN CONTACT FORCE MODEL

Only the road wheels and imaginary road wheels are assumed to be in contact with the ground. The contact between a wheel and a ground can be represented in terms of constraint equations, but these constraint equations will prevent wheel-ground separation and wheel slip. A better way of representing wheel-ground contact will be by means of contact force models. Contact forces are generated when two bodies are interacting with each other. In general, two approaches are being used to study the interaction between bodies. If the interaction is for short interval of time impact mechanics is used, if the interaction is continuously occurring then elastic contact force models with dissipation are used. The ground is characterised with massless spring and damper system both in normal and tangential directions. A nonlinear normal force model and linear tangential force model are used.The schematic diagram for finding contact forces in wheel-ground contact is as shown in Fig. 3.

$$f_n = \begin{cases} k_n z^2 + c_n z^2 \dot{z} & if\ z \geq 0 \\ 0 & if\ z < 0 \end{cases} \tag{4}$$

$$f_{stick} = k_t x + c_t \dot{p} \tag{5}$$

$$f_t = \begin{cases} f_{stick} & if\ |f_{stick}| \leq \mu |f_n| \\ \mu |f_n| Sign[f_{stick}] & if\ |f_{stick}| > \mu |f_n| \end{cases} \tag{6}$$

$$\dot{x} = -\frac{k_t x + f_{stick}}{c_t} \tag{7}$$

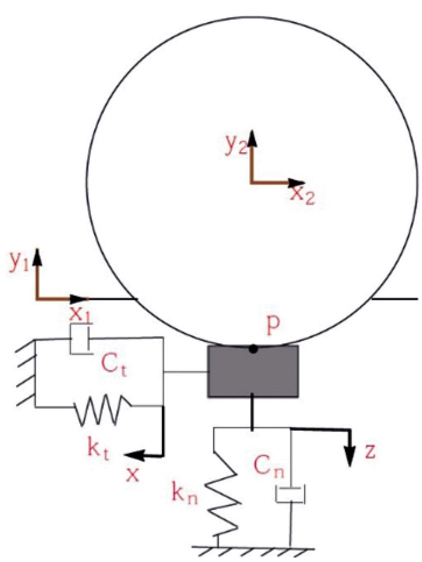

Figure 3. Normal and tangential contact force model.

where z is the penetration distance of a wheel on terrain, $\dot{z}$ is the rate of penetration – computed from state variables of the wheel in contact, k_n and c_n are normal stiffness and damping coefficients, k_t and c_t are tangential stiffness and damping coefficients, x is tangential displacement of the terrain at the contact point, $\dot{p}$ is the contact point velocity.

The contact point velocity is determined from the angular and longitudinal velocity of the wheel. To account for wheel slip, a Columb friction model is considered. The tangential contact forces are computed and compared with the limiting friction value and fed into the equations of motion. Along with the ordinary differential equations for rigid body motion, the differential equations arising out of contact model has to be solved simultaneously. To account for calculation of normal and tangential contact forces in a non-flat terrain profile, the slope of the terrain profile at the point of contact of the wheel with the terrain is computed. The force computed out of the contact force model is resolved into components in directions along and perpendicular to the slope to compute tangential and normal forces, respectively.

4 MATHEMATICAL MODEL

The configuration of each body in the system is represented using absolute Cartesian generalised coordinates that defines the position and orientation of a body fixed coordinate system. The position and orientation of the body fixed coordinate are measured with respect to a fixed global frame at the origin. As in a plane each body has three degrees of freedom, a unconstrained multi-body system with 23 bodies has 69 independent generalised coordinates represented as

$$q = \left\{ R_{x1} \; R_{y1} \; \theta_1 \; R_{x2} \; R_{y1} \cdots R_{x23} \; R_{y23} \; \theta_{23} \right\}^T \tag{8}$$

The multi-body system with alternate track model has 22 revolute joints introducing 44 constraint equations.

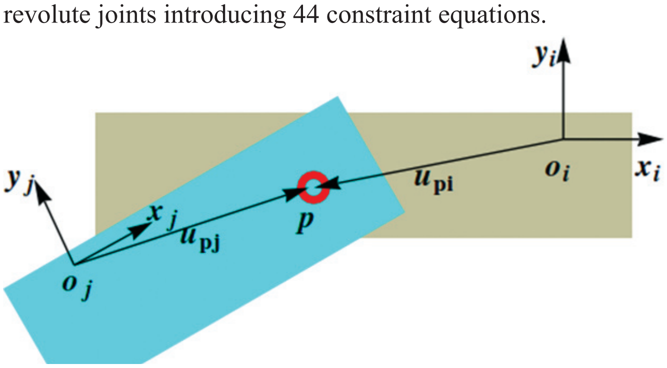

Figure 4. Revolute joint.

The kinematic constraint equation out of a revolute joint between two bodies i and j is represented in vector form as

$$\vec{r}_{pi} - \vec{r}_{pj} = \vec{0} \tag{9}$$

More explicitly in matrix form as

$$\begin{Bmatrix} R_{xi} \\ R_{yi} \end{Bmatrix} + \begin{bmatrix} Cos\,\theta_i & -Sin\,\theta_i \\ Sin\,\theta & Cos\,\theta_i \end{bmatrix} \begin{Bmatrix} u_{pxi} \\ u_{pyi} \end{Bmatrix} - \begin{Bmatrix} R_{xj} \\ R_{yj} \end{Bmatrix} - \begin{bmatrix} Cos\,\theta_j & -Sin\,\theta_j \\ Sin\,\theta_j & Cos\,\theta_j \end{bmatrix} \begin{Bmatrix} u_{pxj} \\ u_{pyj} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10}$$

The kinematic constraint equations with specified trajectories can be represented in constraint vector as

$$C(q,t) = 0 \tag{11}$$

The equations of motion of the multi-body system with alternate track model are obtained using Lagrangian framework as

$$M\,\ddot{q} + C_q^T\,\lambda = Q_e \tag{12}$$

where M is system mass matrix, C_q is the Jacobian matrix of the kinematic constraints, λ is the vector of Lagrangian multipliers and Q_e is the vector of external forces.

As the constraint equations are arising out of revolute joints, the constraints are holonomic and in terms of position coordinates of the interacting bodies. It can be seen that in equation the Lagrangian multiplier is in algebraic form, while coordinates are indifferential form with second order derivatives. Equations (11) and (12) form a set of DAEs which needs to be solved simultaneously. In general DAE solvers will repeatedly differentiate the equations and to get a set of ordinary differential Eqns (ODEs) called index reduction. However, index reduction technique is not preferred as the right hand side (RHS) of the Eqn (12) has discrete contact force model for wheel terrain contact. Instead, a stabilisation technique is used to solve the DAEs.

The constraint equations in acceleration form are augmented into the equations of motion in state space form as

$$C_q\,\dot{q} + C_t = 0 \tag{13}$$

$$C_q\,\ddot{q} + (C_q\,\dot{q})_q\,\dot{q} + 2C_{qt}\,\dot{q} + C_{tt} = 0 \tag{14}$$

$$\begin{bmatrix} I & 0 & 0 \\ 0 & M & C_q^T \\ 0 & C_q & 0 \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \dot{v} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} v \\ Q \\ -((C_q\,\dot{q})_q\,\dot{q} + 2C_{qt}\,\dot{q} + C_{tt}) \end{Bmatrix} \tag{15}$$

The suspension and wheel-terrain contact forces are computed ahead of rigid body simulation and fed into the equations of motion as external forces. With consistent initial conditions satisfying the constraints, these set of first order ODEs along with ODEs arising out of contact force models are solved using numerical integration technique. At every step of integration Baumgarte stabilisation technique[7] is used to satisfy the position and velocity constraints.

5. SIMULATION RESULTS

Simulation of the proposed model is carried out; the tracked vehicle with alternate model for tracks is made to run over a sinusoidal terrain profile of 100 mm amplitude and 7 m wavelength at a constant speed of 15 kmph. The dimensions and inertia properties of the tracked vehicle model used in the study are as shown in Table 1.

The simulation results are compared against the experimental measurements[8] made on a tracked vehicle with same dimensions and inertial properties. Figure 5. shows the time domain measurement of vertical acceleration of center of mass of the vehicle.

A fast Fourier transform (FFT) of the experimental and simulation vehicle center of mass acceleration measurements is shown in the Fig. 6. As seen from this figure, there is an excellent match of the response amplitudes at 0.6Hz, which

Table1. Dimensions and inertia properties of the tracked vehicle

Dimensions and Inertia properties	
Half sprung mass	5125 kg
Pitch moment of inertia	14630 kg m^2
Mass of road and imaginary wheels	75 kg
Stiffness of road wheeltorsion bar	75530 N/m
Damping coefficient for road wheel torsion bar	4732 Ns/m
Stiffness of imaginary wheel torsion bar	10000 N/m
Damping coefficient for imaginary wheel torsion bar	5000Ns/m
Road wheel radius	0.32 m
Wheel base	3.5 m
Distance between road wheels	0.583 m

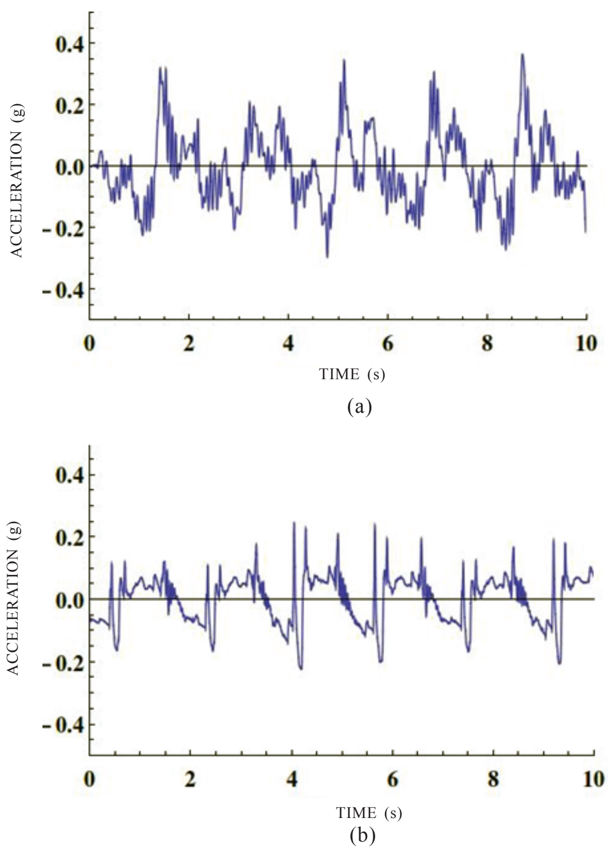

Figure 5. Vertical acceleration of center of mass of the vehicle: (a) Experiment and (b) Proposed model.

corresponds to the fundamental excitation frequency for 15 kmph speed on a sinusoidal track with 7 m wavelength.

Higher harmonics in the experimental data may be due to the fact that the constant speed of 15 kmph was not practically maintainable.

The normal load acting on the road and imaginary wheels while the vehicle passing over the sinusoidal track is as shown in the Fig. 7.

It is observed from the simulation that the normal load shared by the imaginary wheel can be varied by varying the torsion bar stiffness attached with the imaginary road arm. Also, the imaginary wheel stiffness has lesser significance in influencing the vertical acceleration of the vehicle.

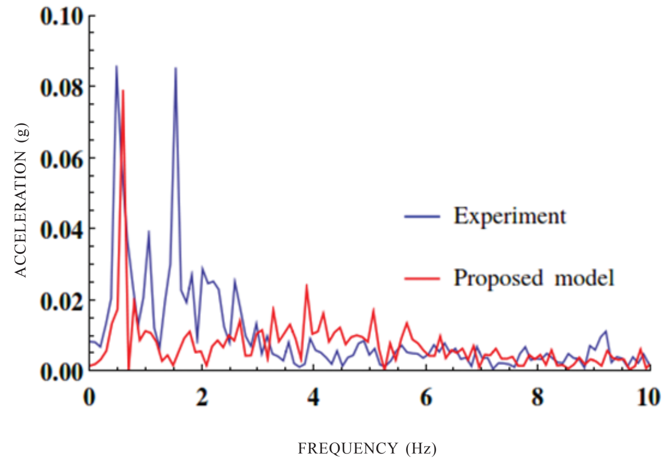

Figure 6. FFT of vertical acceleration of center of mass of the vehicle.

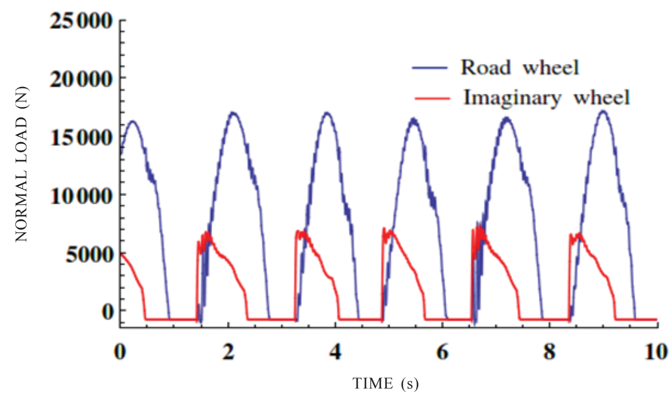

Figure 7. Normal loads.

6. CONCLUSIONS

It is been understood from the literature related to tracked vehicle dynamics that a multi-body model is necessary to study and analyse its performance on an unprepared terrain. The presence of large number of bodies and complexity of joints such as a sprocket track interaction pose computational difficulties. In this paper the tracks are replaced by novel imaginary wheels connected to the road wheels. Ride dynamic studies are simulated on the proposed model and compared with the experimental results. The ride dynamic study demonstrates the applicability of the alternate model for tracked vehicle performance studies. This proposed model can be easily integrated with other vehicle dynamic models such as engine, transmission and terrain models which can be used as a vehicle simulator.

REFERENCES

1. Choi, J.H.; Lee, H.C. & Shabana, A.A. Spatial dynamics of multibody tracked vehicles part I: spatial equations of motion. *Vehicle System Dynamics,* 1998, **29**(1), 27-49. doi: 10.1080/00423119808969365
2. Rubinstein, D. & Hitron, R. A detailed multi-body model for dynamic simulation of off-road tracked vehicles. *Journal Terramechanics,* 2004, **41**(2), 163-173. doi: 10.1016/j.jterra.2004.02.004

3. Nakanishi, T. & Shabana, A.A. On the numerical solution of tracked vehicle dynamic equations. *Nonlinear Dynamics,* 1994, **6**(4), 391-417.
4. Dhir, A. & Seshadri, S. Analytical track models for ride dynamic simulation of tracked vehicles. *Journal Terramechanics,* 1994, **31**(2), 107-138. doi: 10.1016/0022-4898(94)90010-8
5. Haug, E.J. & McCullough, M. K. Dynamics of high mobility track vehicles. *Trans. Am. Soc. Mehanical Engineers, J. Mechanisms, Transmissions, Automation Design* 1995, **108**(2), 189-196.
6. Wong, J.Y.; Garber, M. & Preston-Thomas, J. Theoretical prediction and experimental substantiation of the ground pressure distribution and tractive performance of tracked vehicles. *Proceedings Institution Mechanical Engineers, Pt D: J. Automobile Eng.,* 1984, **198**(4), 265-285. doi: 10.1243/pime_proc_1984_198_155_02
7. Baumgarte, J. Stabilization of constraints and integrals of motion in dynamical systems. *Comput. Methods Appl. Mech. Eng.,* 1972, **1**(1), 1-16. doi: 10.1016/0045-7825(72)90018-7
8. Rajkumar, S. Input force estimation from measured response data: An inverse problem. Indian Institute of Technology Madras, 2015. (MSc Thesis)

CONTRIBUTORS

Mr Ilango Mahalingam received his Bachelor of Engineering in Automobile Engineering from Madras Institute of Technology. He received his Master of Technology in Solid Mechanics from Indian Institute of Technology Madras, Chennai. Pursuing his Phd in the area of multi-body dynamic analysis of tracked vehicles at Indian Institute of Technology Madras, Chennai. His research interest includes Multi-body dynamics, Vehicle dynamics and Bio-mechanics.
Contribution in the current study, he has proposed the alternate model for tracks, formulated and carried out simulations.

Prof. Chandramouli Padmanabhan primary research interests are in the areas of Noise and Vibration as well as in Non-linear and Multi-body Dynamics. He has published nearly 150 international journal and conference papers and guided about 30 PhD and MS research scholars.
Contribution in the current study, he has suggested ideas, validation studies and edited the paper.