

# Mathematical Modelling of Indian Regional Navigation Satellite System Receiver

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## ABSTRACT

At present the armoured fighting vehicles are equipped with either global positioning system (GPS) receivers or integrated inertial navigation system (INS)/GPS navigation systems. During hostile situations, the denial/degradation of the GPS satellite signals may happen. This results in the requirement of an indigenous satellite based navigation system. Indian Space Research Organisation has developed an indigenous Indian regional navigation satellite system (IRNSS), with a seven satellite constellation to provide independent position, navigation and timing services over India and its neighbouring regions. In this paper, the development of IRNSS receiver using MATLAB as per IRNSS signal in space interface control document for standard positioning service is discussed. A method for faster IRNSS signal acquisition in frequency domain and delay locked loop code tracking for the acquired satellite signals are used. Models for navigation message decoding and pseudo range/user position calculations are developed using the algorithms provided in IRNSS ICD.

**Keywords:** IRNSS; Indian regional navigation satellite system. Acquisition; Tracking; Position Estimation

## 1. INTRODUCTION

At present, armoured fighting vehicles (AFVs) are equipped with either global positioning system (GPS) or integrated GPS/INS for navigation in which errors in the estimated navigation position increases with time at GPS outages. This affects the navigation of AFVs during critical missions. GPS satellite navigation system was developed and maintained by Department of Defense (DoD), United States of America's (USA). It is not reliable to depend on a foreign satellite navigation system for military applications during crucial operations since there is a possibility of blocking or degrading GPS signals by USA-DoD. This led to the development of an independent & indigenous satellite based navigation system by Indian Space Research Organisation (ISRO). Indian regional navigation satellite system (IRNSS) provides high accuracy position, velocity and timing (PVT) information to its users. It works in all weather conditions on 24/7 basis and provides two basic services such as standard positioning service (SPS) for civilians and Restricted Service (RS) for specific users for e.g. armed forces. IRNSS has seven satellite constellation in which three are geostationary orbit satellites (GEO) located at 32.5 °E, 83 °E and 131.5 °E latitude and four are geosynchronous orbit satellites inclined (IGSO) at an angle of 29°<sup>01</sup>. This constellation always makes, at least four satellites to be visible for the receiver to compute an uninterrupted user solution. Development of single frequency IRNSS receiver which is capable of receiving SPS signal at L5 or S band frequency is discussed in this paper. It shall be

noted that the errors associated with SPS are not considered. Essential blocks like satellite signal acquisition, tracking and receiver position calculation are mathematically modelled.

## 2. ACQUISITION

Acquisition is always performed first in any global navigation satellite system (GNSS) receiver to determine the visible satellites and the coarse values of carrier frequency. Acquisition method for satellite visibility is only presented in this paper since the SPS signal is considered here is a baseband signal. All the seven satellites of IRNSS are assigned with a unique Pseudo Random Number (PRN) code. PRN codes are nearly orthogonal and has the properties of auto correlation and cross correlation. Codephase, which is the beginning of each PRN code, is necessary to align the locally generated PRN code with the receive IRNSS satellite signal. The goal of the acquisition is to estimate the codephase of incoming signal by correlating it with the locally generated PRN code. The correlation process produces a peak value at particular sample if the codephase of received signal perfectly matches with (zero lag) the locally generated PRN code. This peak indicates that the particular satellite signal has been acquired. Acquisition is carried out on the samples of received signals for one period of PRN code i.e., 1 ms. Once the satellite signal has been acquired, the codephase is passed to the tracking stage of the receiver. Efficient acquisition method for a software receiver is discussed in this paper. Incoming signal is always delayed by some samples and it can be determined through acquisition. In this paper, the parallel codephase search acquisition method<sup>7</sup> is implemented. It is the most efficient acquisition method that uses circular cross correlation and its block diagram is shown

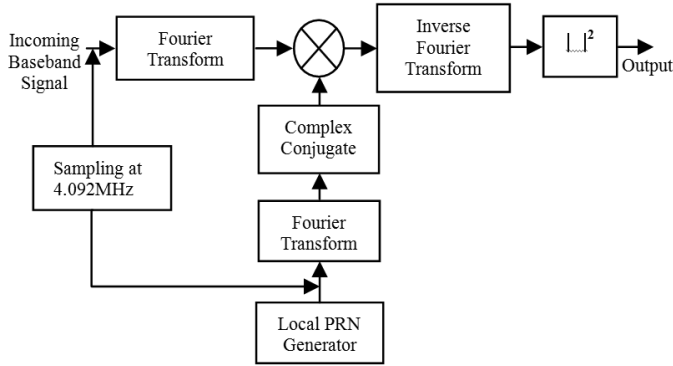


Figure 1. Parallel code phase search acquisition.

in Fig. 1. First one ms of incoming signal is sampled at the rate of 4.092 MHz and Fourier transform is performed on it. The result of the Fourier Transform is multiplied with the result of the lower branch of the block diagram as shown in Fig. 1. The lower branch signal is created as follows. The local PRN code generator generates a code with no code phase. The locally generated PRN code is sampled at 4.092 MHz and it performs a Fourier transform to the sampled PRN code. The result of the Fourier transform is complex conjugated and multiplied with the result of the before-mentioned multiplication and is given as input to an inverse Fourier transform block and the absolute value of the processed signal is computed<sup>2</sup>. Output of the parallel code phase search gives the highest correlation peak value at a particular sample. This sample is called the codephase of the signal and it has to be passed on to the tracking stage of the receiver. Figure 2 shows the output from the parallel code phase search method. The highest correlation value is obtained at the sample number 774 which is the codephase of the signal.

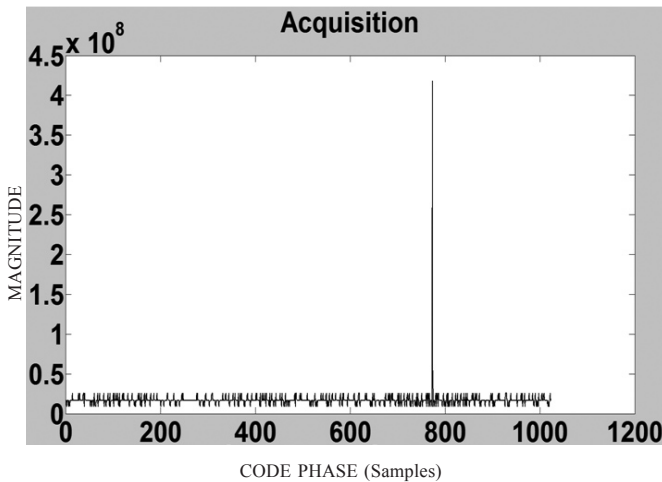


Figure 2. Output of the parallel code phase search method having the highest correlation peak at the codephase 774.

### 3. TRACKING

Tracking is performed to keep track of the codephase of a specific code in the signal<sup>2</sup>. Codephase from the acquisition stage is passed on to the tracking stage of the receiver. This codephase is fed to the local PRN generator of the tracking loop to generate a perfectly aligned PRN code. The tracking

method used in this paper is a delay lock loop (DLL) called an early-late tracking loop<sup>5</sup>. The DLL is used to correlate the incoming signal with the locally generated PRN codes, which is as shown in the Fig. 3. Local PRN code generator generates three different PRN code such as Early(E), Prompt(P), Late(L) code. These three PRN codes are separated by 0.5 chip length spacing. Early PRN code is advanced by 0.5 chip length, Late PRN code is delayed by 0.5 chip length and the Prompt PRN code is neither advanced nor delayed. Since all the PRN code used in tracking loops are of non return to zero (NRZ) representation, bit by bit multiplication is performed for all these three PRN codes with the incoming signal. After multiplication, the result is integrated and accumulated to produce  $I_E, I_P, I_L$  for each millisecond. The accumulated value after integration and dump blocks is a numerical value indicating how much the specific code replica correlates with the code in the incoming signal<sup>2</sup>. These values are fed to a Coherent DLL discriminator to identify whether the codephase has to be delayed or advanced. The output of code tracking loop is as shown in Fig. 4. It shows an incoming signal whose code is aligned perfectly with the prompt PRN code generated locally and hence the codephase is exactly matched.

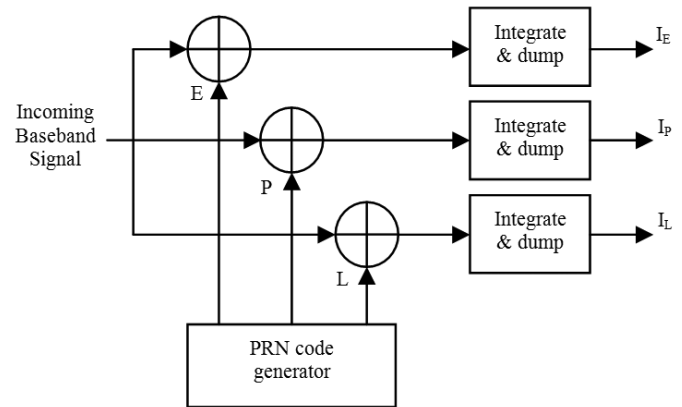


Figure 3. Code tracking loop.

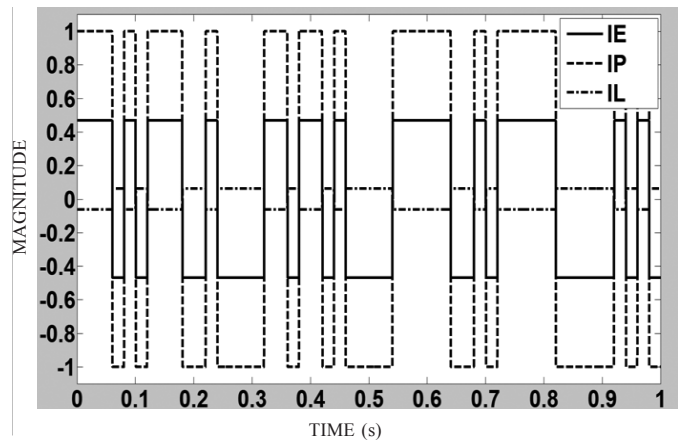


Figure 4. Output of the tracking loop showing 1 second of signal.

### 4. NAVIGATION MESSAGE DECODING

The output from the tracking loop is a binary signal truncated to the values of +1 and -1 with each bit having a

duration of one millisecond. Due to the noisy and weak signals, a mean value for 20 ms is computed and truncated to +1 or -1. The navigation data bit rate is 50 bps. The samples obtained from the tracking loop is at the rate of 1000 bps and it has to be converted into 50 bps. This can be done by averaging the 20 consecutive values into one value. This conversion procedure is called as bit synchronisation. The first task in the bit synchronisation is to detect the time at which the bit transitions occur i.e., where the output changes from +1 to -1, or vice versa. It is possible to find all bit transition times once the first bit transmission is identified. After the determination of bit transitions the 1000 bps signal is converted to a 50 bps signal by averaging twenty consecutive samples as mentioned earlier<sup>2</sup>. Next process is to identify all the four sub frames of a single IRNSS master frame. This is achieved by searching for a particular synchronisation pattern in the navigation message. Each sub frame consists of 600 bits out of which the first 16 bits represents the synchronisation pattern (which is not encoded). The synchronisation pattern for an IRNSS navigation message sub frame is  $(EB90)_{16}$  in hexadecimal form<sup>1</sup>. The remaining 584 bits are encoded and the information contained in it can be decoded using Viterbi decoding algorithm.

A sequence of 292 bits are obtained after decoding. The start of each sub frame is a TLM (Telemetry) word of 8 bits. Each sub frame ends with 24 bit cyclic redundancy check (CRC) followed by 6 tail bits. In sub frames 1 and 2 the navigation data is allotted 232 bits starting from bit number 31. In sub frames 3 and 4 the Navigation data is allotted 220 bits starting from bit number 37. The structure of a typical sub frame 1 and 2 is shown in Fig. 5. Figure 6 shows the structure of a typical sub frame 3 and 4<sup>1</sup>.

The 8 bits of TLM word are reserved for future use. Time of week count (TOWC) is of 17 bits in length and the value of TOWC is multiplied with 12 to obtain the time in seconds corresponding to the start of the next sub frame. Bit 26 is allotted to the alert flag. The alert flag signifies the users that

|        |         |       |         |              |       |          |         |        |
|--------|---------|-------|---------|--------------|-------|----------|---------|--------|
| 1      | 9       | 26    | 27      | 28           | 30    | 31       | 263     | 287    |
| TLM    | TOWC    | Alert | Autonav | Sub frame ID | Spare | Data     | CRC     | Tail   |
| 8 bits | 17 bits | 1 bit | 1 bit   | 2 bits       | 1 bit | 232 bits | 24 bits | 6 bits |

Figure 5. Structure of sub frame 1 and 2.

|        |         |       |         |              |       |            |          |        |         |        |
|--------|---------|-------|---------|--------------|-------|------------|----------|--------|---------|--------|
| 1      | 9       | 26    | 27      | 28           | 30    | 31         | 37       | 257    | 263     | 287    |
| TLM    | TOWC    | Alert | Autonav | Sub frame ID | Spare | Message ID | Data     | PRN ID | CRC     | Tail   |
| 8 bits | 17 bits | 1 bit | 1 bit   | 2 bits       | 1 bit | 6 bits     | 220 bits | 6 bits | 24 bits | 6 bits |

Figure 6. Structure of sub frame 3 and 4.

the utilisation of navigation data from the particular satellite shall be at the users’ own risk. Bit 27 is allotted to the AutoNav. Satellites store 7 days ephemeris and clock parameter sets as AutoNav data sets. Satellite can support broadcast of primary navigation parameters from AutoNav data sets with no uplink from ground for a maximum of 7 days. When the system is in AutoNav mode, the AutoNav flag is set to 1. Each sub frame in the master frame can be identified by the 2 bit sub frame ID allotted in bit numbers 28 and 29<sup>1</sup>. The mapping between 2-bit sub frame identifier and sub frame number is provided in Table 1.

Bit 30 is identified as spare bit for future use. Each message in the sub frame 3 and 4 has a 6 bit message identifier (bits 31-36) that uniquely identifies the message type in the sub frame. The lists of messages are defined in IRNSS ICD<sup>1</sup>. Each message in the sub frame 3 and 4 has a 6 bit (bits 257 – 262) PRN identifier (ID) that uniquely identifies the spacecraft transmitting the corresponding message. The PRN IDs for IRNSS spacecrafts are defined in IRNSS ICD. Broadcast ephemeris parameter for sub frame 1 and 2 is given in the Table 2. It gives the bit location of the ephemeris parameter and its size and scale factor. The parameters indicated as \* are in 2’s complement representation. IRNSS ICD<sup>1</sup> can be referred for the data format of sub frame 3 and 4.

Table 1. Sub frame ID to sub frame mapping

| Sub frame ID | Sub frame |
|--------------|-----------|
| 00           | 1         |
| 01           | 2         |
| 10           | 3         |
| 11           | 4         |

For determining a particular decoded navigation parameter, its corresponding bit location as specified in the Table 2 has to be extracted and converted into decimal values. The program flow for navigation message decoding is shown in the Fig. 7. MATLAB function *bin2dec*<sup>3</sup> is used to convert binary string to decimal values. This function can be reused to produce the decoded decimal values. The parameter extracted from navigation data is checked whether it is in the 2’s complement form before binary to decimal conversion. If it is in 2’s complement form then a 2’s complement operation is done before conversion else it is directly converted from binary to decimal and then multiplied with scale factors to get the decoded output. If the Most Significant Bit (MSB) of the binary navigation parameter is ‘1’ a negative sign is added with the decoded value else the decoded value is left as such. In similar way, all the decoded value for navigation parameter from different satellites are obtained and stored in the form of a matrix. These values are used to estimate the pseudo ranges and user position calculations.

## 5. PSEUDORANGE AND USER POSITION CALCULATION

In order to obtain the receiver position, it is necessary that the receiver should receive navigation message signal from at least four satellites. Calculations are done in three dimensional Cartesian coordinate system with geocentric origin. The range of the user from the four satellites can be

**Table 2. Ephemeris parameters in sub frame 1 and 2**

| Sub frame | Parameter  | Notation       | Location | Scale factor (LSB) | Size (bits) |
|-----------|--|----------------|----------|--------------------|-------------|
| 1         | Week number  | WN             | 31-40    | 1                  | 10          |
| 1         | Clock bias   | $a_{fo}$       | 41-62    | $2^{-31}$          | 22*         |
| 1         | Clock drift  | $a_{f1}$       | 63-78    | $2^{-43}$          | 16*         |
| 1         | Clock drift rate   | $a_{f2}$       | 79-86    | $2^{-55}$          | 8*          |
| 1         | SV accuracy  | URA            | 87-90    | 1                  | 4           |
| 1         | Time of clock  | $t_{oc}$       | 91-106   | 16                 | 16          |
| 1         | Time group delay   | $T_{GD}$       | 107-114  | $2^{-31}$          | 8*          |
| 1         | Mean motion difference   | $\Delta n$     | 115-136  | $2^{-41}$          | 22*         |
| 1         | Issue of data ephemeris & clock                                      | IODEC          | 137-144  | 1                  | 8           |
| 1         | Reserved   | -              | 145-154  | 1                  | 10          |
| 1         | L5 flag  |                | 155      | 1                  | 1           |
| 1         | S flag   |                | 156      | 1                  | 1           |
| 1         | Amplitude of cosine harmonic correction term to argument of latitude | $C_{uc}$       | 157-171  | $2^{-28}$          | 15*         |
| 1         | Amplitude of sine harmonic correction term to argument of latitude   | $C_{us}$       | 172-186  | $2^{-28}$          | 15*         |
| 1         | Amplitude of cosine harmonic correction term to angle of inclination | $C_{ic}$       | 187-201  | $2^{-28}$          | 15*         |
| 1         | Amplitude of sine harmonic correction term to angle of inclination   | $C_{is}$       | 202-216  | $2^{-28}$          | 15*         |
| 1         | Amplitude of cosine harmonic correction to orbit radius              | $C_{rc}$       | 217-231  | $2^{-4}$           | 15*         |
| 1         | Amplitude of sine harmonic correction to orbit radius                | $C_{rs}$       | 232-246  | $2^{-4}$           | 15*         |
| 1         | Rate of inclination angle  | IDOT           | 247-260  | $2^{-43}$          | 14*         |
| 1         | Spare  |                | 261-262  |                    | 2           |
| 2         | Mean anomaly   | $M_0$          | 31-62    | $2^{-31}$          | 32*         |
| 2         | Time of ephemeris  | $t_{oe}$       | 63-78    | 16                 | 16          |
| 2         | Eccentricity   | $e$            | 79-110   | $2^{-33}$          | 32          |
| 2         | Square root of semi major axis                                       | $\sqrt{A}$     | 111-142  | $2^{-19}$          | 32          |
| 2         | Long of ascending node   | $\Omega_0$     | 143-174  | $2^{-31}$          | 32*         |
| 2         | Argument of perigee  | $\omega$       | 175-206  | $2^{-31}$          | 32*         |
| 2         | Rate of RAAN   | $\dot{\Omega}$ | 207-228  | $2^{-41}$          | 22*         |
| 2         | Inclination  | $i_0$          | 229-260  | $2^{-31}$          | 32*         |
| 2         | Spare  |                | 261-262  |                    | 2           |

determined with the help of signal transit times between the satellites and the user. Due to the on board atomic clocks of the satellites, its signal transmission time is known very precisely. All satellite clocks are synchronised with each other and with the co-ordinated universal time (UTC). The receiver clock is not synchronised to UTC and the resultant time error causes inaccuracies in the measurement of signal transit time and an incorrect distance is measured that is known as pseudo range<sup>4</sup>. User position calculation program flow is shown in the Fig. 8 and its algorithm is given as follows.

- (1) Receiver position  $(X_{user}, Y_{user}, Z_{user})$  is initialised to be the previous estimated value. It is initialised to the origin of the ECEF frame if the previous value is not known.
- (2) Satellite position is computed from the algorithm described in the *Appendix B* of the IRNSS ICD<sup>1</sup>. Satellite position algorithm uses the decoded navigation data obtained from the section 4.
- (3) Satellite clock error is computed from the algorithm described in the *Appendix A* of the IRNSS ICD<sup>1</sup>.

- (4) Ionosphere delay computation using coefficients is computed from the algorithm presented in the *Appendix H* of the IRNSS ICD<sup>1</sup>.
- (5) The distance (pseudo range) between the satellite position  $(X_{sat_i}, Y_{sat_i}, Z_{sat_i})$  (for  $i = 1, \dots, 4$  number of satellites) and receiver position is calculated in cartesian coordinate system,  $(R_r)$ .  $R_i$  is calculated as given below<sup>6</sup>

$$R_i = \sqrt{(X_{sat_i} - X_{user})^2 + (Y_{sat_i} - Y_{user})^2 + (Z_{sat_i} - Z_{user})^2}$$

- (6) The signal transit time obtained from the signal is corrected for the satellite clock error, ionospheric delays computed from the steps 2 and 3 respectively. Corrected pseudorange,  $PSR_i$  can be calculated by multiplying the speed of the electromagnetic signal and the corrected signal transit time,  $\Delta t_i$  as given below<sup>6</sup>,  
 $PSR_i = c \cdot \Delta t_i$
- (7) Calculate the difference between  $R_i$  and  $PSR_i$ ,<sup>6</sup>  
 $dPR = PSR_i - R_i$

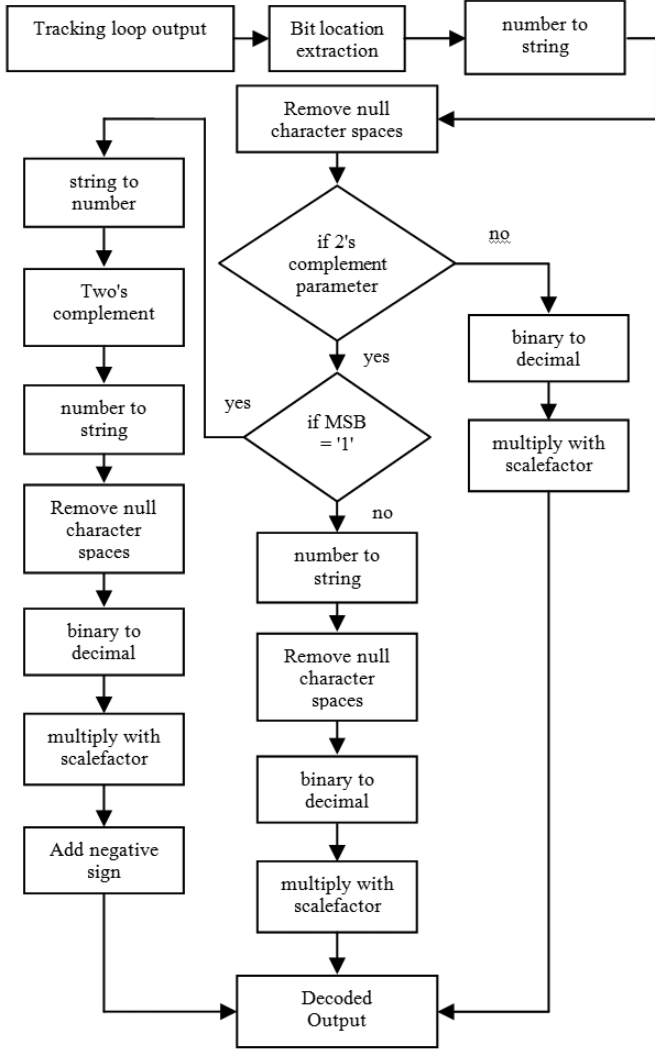


Figure 7. Program flow for the navigation message decoding.

$$dPR = \begin{bmatrix} PSR_1 - R_1 \\ PSR_2 - R_2 \\ PSR_3 - R_3 \\ PSR_4 - R_4 \end{bmatrix}$$

- (8) The error components in cartesian coordinate system is calculated as<sup>6</sup>

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta t_0 \end{bmatrix} = \begin{bmatrix} \frac{X_{user_1} - X_{sat_1}}{R_1} & \frac{Y_{user_1} - Y_{sat_1}}{R_1} & \frac{Z_{user_1} - Z_{sat_1}}{R_1} & c \\ \frac{X_{user_2} - X_{sat_2}}{R_2} & \frac{Y_{user_2} - Y_{sat_2}}{R_2} & \frac{Z_{user_2} - Z_{sat_2}}{R_2} & c \\ \frac{X_{user_3} - X_{sat_3}}{R_3} & \frac{Y_{user_3} - Y_{sat_3}}{R_3} & \frac{Z_{user_3} - Z_{sat_3}}{R_3} & c \\ \frac{X_{user_4} - X_{sat_4}}{R_4} & \frac{Y_{user_4} - Y_{sat_4}}{R_4} & \frac{Z_{user_4} - Z_{sat_4}}{R_4} & c \end{bmatrix}^{-1} * [dPR]$$

- (9) The error components  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  obtained from the step 8 is added to the receiver position  $(X_{user}, Y_{user}, Z_{user})$  to get the estimated user position.

$$X_{user\ est} = X_{user} + \Delta x$$

$$Y_{user\ est} = Y_{user} + \Delta y$$

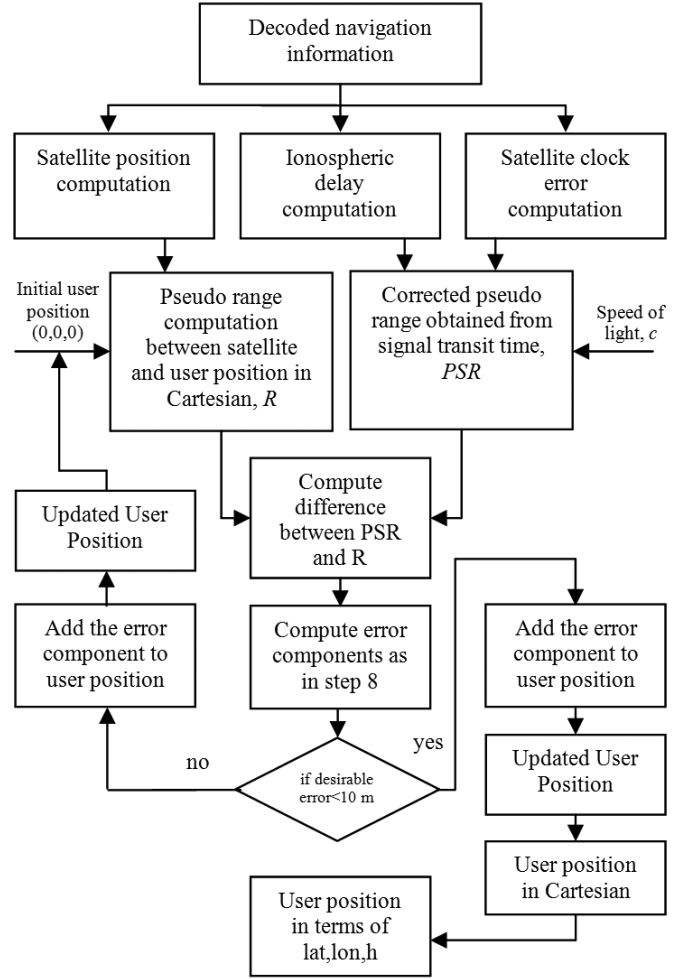


Figure 8. Program flow for the pseudo range and user position calculation.

$$Z_{user\ est} = Z_{user} + \Delta z$$

- (10) The position error obtained from the step 7 is checked whether it is in desirable limits. If it is not then the steps from 2 to 9 is repeated by updating the user position with the value obtained from step 9 till the desirable error limit is reached.
- (11) Finally, the user position is converted from cartesian coordinate system into geodetic coordinate system format i.e., latitude, longitude and altitude.

## 6. CONCLUSIONS

Demodulation model for acquiring and tracking of the INRSS baseband satellite signal is developed. Only coherent acquisition and tracking models were discussed in this paper. This can be extended to non coherent acquisition and tracking loops in future. The procedure for navigation message decoding from demodulated signal is discussed. Making use of the information obtained from the navigation message decoding, an algorithm for pseudo range and user position calculation is presented. These models and algorithms presented in this paper can be implemented in a suitable software defined radio (SDR) platform in future.



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