

Geometric Scaling of Plates Subjected to Mine Blast

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ABSTRACT

In the design of AFVs, study of structures subjected to land mine blast is important. Generally, blast related experimental studies are very time consuming and costly. A simple first cut alternative is finite element modelling and analysis. Here, modelling of mine and simulating the blast effect involves large number of mesh elements, which makes the model computationally intensive and time consuming. Hence, instead of using full scale model for analysis, a suitable scaled down model would reduce analysis time and leads to a faster DOE studies. A proper scaling mechanism is to be evolved in order to get accurate results. Discusses about the scaling of plate subjected to mine blast using dimensional analysis approach. The out-of-plane surface deformation, including velocity fields during the blast loading are compared between the scaled and unscaled plate.

Keywords: Armoured fighting vehicles; Mines; Blast resistant structures; Explicit finite element analysis

1. INTRODUCTION

Buried mines are main threat in the modern war scenario. Apart from other effects, the land mines cause both physical damage and psychological damage to crew. On the other hand, modern war requires light armoured vehicles. Hence there is always a need for optimal design of vehicles and also ensure survivability of the crew.

Gives a brief introduction on material models, dimensional analysis based scaling and finite element modelling and analysis of mines. The present study is carried out using finite element explicit dynamics analysis. The type of analysis depends on the type of loading condition. Variable loads acting on the structures can be classified into three different types, namely, quasi static, dynamic and impulsive. The structural response behaviour greatly depends on these loading conditions. This is predominantly due to the material behaviour at these loading conditions. In case of blast loading, it is impulsive in nature¹. These types of problems are generally solved using explicit methods. The following section discusses the material behaviour beyond elastic region.

1.1 Structural Material Model

For a given ductile material, upto proportionality limit, the stress is proportional to strain and follows the Hooke's law. Beyond this region, the material behaves nonlinearly and greatly influenced by the type of loading. Blast loads typically produce very high strain rates in the range of $10^2 - 10^4 s^{-1}$. This high strain rate would alter the dynamic mechanical properties of material and, accordingly, the expected damage

mechanisms. Here, the total strain is the sum of the elastic and plastic strain. In case of nonlinear analysis involving plastic region, it is important to describe the material in the plastic region with much accuracy, for improved results. The obtained material data are used by the FE based software. Many FE software have the option of including the material model in the form of look up table. But, an empirical formula of true stress-strain curve is better choice in solving many of the dynamic analysis problems (e.g. blast, penetration, crash studies etc.). The type of approximation depends on the problems and the level of accuracy required. For most metals, the log-log plot of the true stress vs. strain curve is nearly linear¹. Hence, the simplest form of approximation is,

$$\sigma = K \epsilon^n \quad (1)$$

Several constitutive material models, which should adequately represent large strain behaviour over a wide range of strain rates and temperatures, have been proposed in literature. Examples of such models are Johnson-Cook, Zerilli-Armstrong etc. The Johnson-Cook material model is most popular model and particularly suitable to model high strain rate deformation of metals². The hardening is a particular type of isotropic hardening in which the yield stress is assumed to be of the form

$$\sigma_0 = \left(A + B(\epsilon^p)^n \right) \left(1 + C \log \left(\frac{\epsilon^p}{\epsilon_0} \right) \right) (1 - T^m) \quad (2)$$

where,

$$T^m = \begin{cases} 0, T < T_r \\ \frac{(T - T_r)}{T_m - T_r}, T_r \leq T \leq T_m \\ 1, T > T_m \end{cases}$$

In the above expression ϵ^p is the equivalent plastic strain, $\dot{\epsilon}_0$ is the strain rate at reference temperature, $\dot{\epsilon}^p$ is the strain rate, T_m is melting temperature and T_r is the reference temperature. The parameters A , B , C , n and n are obtained from the experimental data³. Here, is the normalised yield stress σ_y at temperature T_r , is the hardening parameter, n the hardening exponent, C is the strain rate constant and is the temperature exponent. In this paper, the above Johnson-Cook material model for Steel 4340 as shwon in Table 1 is used for the analysis.

Similarly, the following Johnson-Cook failure criteria is used for modelling the failure and erosion of failed material. Generally, the structural components are meshed using Lagrangian formulation.

Table 1. Johnson-Cook material parameters

A	792 MPa
B	510 MPa
C	0.014
n	0.260
m	1.030
$\dot{\epsilon}_0$	0.001

1.2 Blast and Sand Materials Model

Modelling the mine is one of the critical elements in the mine blast studies. The blast effects are categorised into two, viz. near field and far field effects. The classification is based on the most commonly used scaled distance formulation, namely Hopkinson-Cranz scaling law which states the following

$$\lambda = \frac{R_1}{R_2} = \frac{d_1}{d_2} = \left(\frac{W_1}{W_2} \right)^{1/3} \tag{3}$$

where, λ is the scale factor, W_1 and W_2 are the masses of the charges of diameter d_1 and d_2 , respectively. Further, the scale factor is also same as the ratio of the distances R at which the same overpressure occurs. The following Fig. 1 demonstrates⁴ the relation.

Blast effects are classified as near field effects and far field effects. As a general thumb rule, scaled distances, below

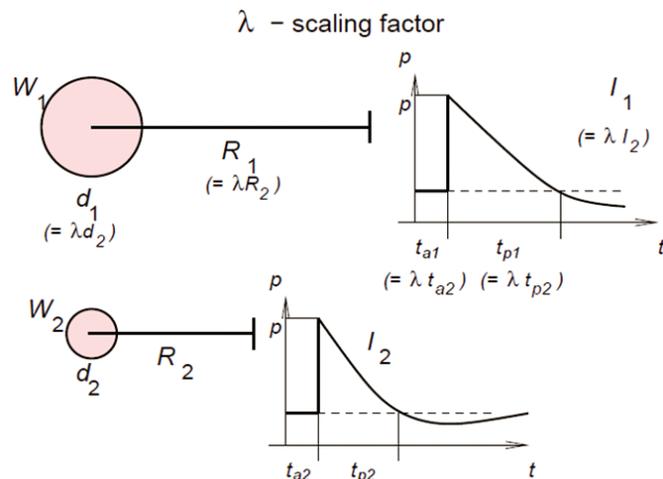


Figure 1. Comparison of pressure curves between different scaled distances.

are considered as near field and far field, otherwise. The pressure calculation for far field effects can be approximated by means of empirical formulas in terms of radius and time, like CONWEP⁵. It is observed that CONWEP is unable to provide accurate results for near field effects, but, for far field, the model performs close to experiments. The near field effect of blast mine is formulated using the JWL material model⁴. Generally, in this type of analysis, the finite element model of land mine along with air and sand are made using Eulerian formulation. In this present analysis, the Eulerian sand, air and mine interacts with Lagrangian plate elements. Such interactions are dealt by Combined Eulerian Lagrangian (CEL) analysis. Proper interactions are to be provided for propagating the blast pressure onto the structure. Based on this, the deformation and stresses of the structure are analysed.

The following are the JWL parameters considered for explosive materials in the analysis.

Table 2, A , B , R_1 , R_2 and W are JWL material parameters. In case of air, the ideal gas formulation was used with air density 1.22 kg/m³. For sand, granular sand material model available in ANSYS Autodyn is applied.

Table 2. JWL parameters for TNT and PETN

	TNT	PETN
A	373.77 GPa	617.05 GPa
B	3.747 GPa	16.92 GPa
R_1	4.15	4.40
R_2	0.90	1.20
W	0.35	0.25

2. SCALING BASED ON DIMENSIONAL ANALYSIS

Mine blast simulation involves complex experimental set up. A scaled down model gives a good control over the setup and better understanding of the complex phenomenon. Moreover, scaling is done to reduce the cost involved creating the facility. Also, it is useful for DOE studies. Finite element based analysis of this scaling gives a better understanding of the scaling method. This paper describes a method for scaling based on FE analysis. This can be used as a design tool for designing the experimental model. Further, FE analysis of scaled model reduces the computational time compared to the full model analysis. Hence scaling can be used as a simplified analysis method also.

Dimensional analysis is one of the classical method for scaling. Identification of appropriate set of variables and application of Buckingham Pi theorem produces scaling method. As discussed in⁶, the non-unique set of input and output – terms as mentioned in the Table 3 are considered

From the above relations, displacement vector can be expressed in expressions of π – terms as follows

$$u = \sqrt[3]{\frac{U}{E}} f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$$

Considering the two, viz., scaled (A) and unscaled (B), when $E_A = E_B$ (Modules of elasticity), $(\rho_p)A = (\rho_p)B$ (density of plate) and

Table 3. Scaling terms as per Buckingham Pi theorem

$$\pi_1 = \frac{X}{\sqrt[3]{U}} \quad \pi_2 = \frac{\sigma_0}{E}$$

$$\pi_3 = \frac{t}{\sqrt[6]{\rho^3 U^2}} \quad \pi_4 = \frac{\dot{u}}{\sqrt[2]{\frac{E}{\rho}}}$$

$$\pi_5 = \frac{u}{\sqrt[3]{\frac{E^4}{\rho^3 U}}} \quad \pi_6 = \frac{\rho}{\rho_p}$$

Here, $U =$ Explosive energy, $E =$ Young's modulus, $\sigma_0 =$ Yield stress, $t =$ Time, $\dot{u} =$ Plate velocity vector,

$\ddot{u} =$ Plate acceleration vector, $\rho_p =$ Density of plate, $\rho =$ Density of sand.

From the above equations, it is clear that the scaling is based on the energy factors which are directly proportional to cube root mass of the explosive. Indeed, considering a typical spherical explosive, it is clear that mass is proportional to the cube root of radius.

In literature, there are a number of studies, both theoretical and experimental, related to deformation of simple circular or rectangular plates subjected to mine blasts^{7,8}. Such studies have helped in better understanding of blast phenomenon as well as behaviour of structures subjected to such loads.

In the present paper, the aim is to study the effect of PETN explosive of 0.5 gm. and 1.9 gm. on circular Steel 4340 plate with dimension as in Table 4. Very small amount of explosive and plate dimensions are considered so that experiments can be conducted in future for validation. Based on the ratio of masses and based on the above relation between the energies, $\beta \approx 1.56$. As a result of this scaling, the scaled dimensions are obtained as per Table 4 below.

Here, SOD refers to standoff distance of plate from surface of the soil and DOB refers to depth of burial. In line with the above scaling relation, same material is used for both the plate models.

3. ANALYSIS, RESULTS AND DISCUSSIONS

3.1 FE Model and BCs

Two different analysis were performed for large and small plate. Figure 2 shows the finite element model of the large and small plate. Here, element size is in order of 2 mm, for both large and small plate models. Both the analysis was modelled as 2D, axisymmetric model, as the explosion at the center is symmetric about the central axis. The plate is modelled using Lagrangian element formulation. The air, sand and TNT are modelled using Eulerian element formulation.

Observe that, due to the present scaling ratio, for a typical 3D finite element model, the total number of mesh elements is reduced by 3.79 times and total time reduced by 1.56 times. But,

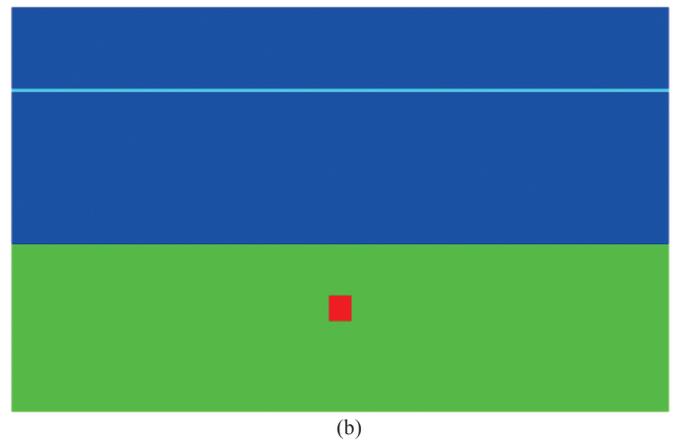
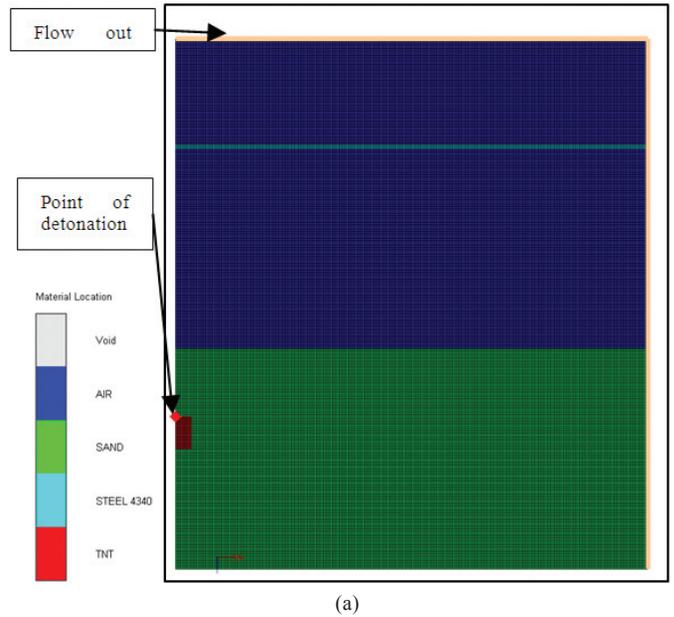


Figure 2. (a) Large plate FE model (b) Small plate FE model.

in the present study, 2D axisymmetric model is considered, for which, the number of elements is reduced by 1.33 times, due to the mesh approximation. In fact, the number of elements in the case of large plate model is 405714, whereas that of small plate model is 304458. Here, it is should be observed that the elements size is 2 mm, in case of both the analysis. As the blast problem can be solved through explicit solvers and explicit solvers are conditionally convergent, element sizes are critical in obtaining better results.

Hence, finite element based scaled down approach is also computationally efficient apart from being a viable testing tool to handle large blast problems.

Flow out boundary condition is applied to the air and sand volume in order to avoid reflection issues. For the plate, fixed boundary condition is applied. The following Fig. 2 shows the fixed BC for the plate and Eulerian domain, marked in cream colour. The red dot in the Fig. 2 shows the point of detonation of the land mine.

In the present analysis, two different simulations are carried out for large plate model, for the purpose of verification of explosive materials and solver. Since, TNT equivalent is considered as a STANAG standard for mine blast effects, the

Table 4. Dimensions for small and large plate simulation model

	Small plate	Large plate
Plate thickness (mm)	1.0	1.59
Plate diameter (mm)	227.6	355.6
SOD (mm)	48.8	76.2
DOB (mm)	16.3	25.4
PETN(gram)	0.5	1.9

present PETN explosive is converted to TNT equivalent, based on the mass specific energy conversion factor⁹. As per⁹, mine blast effect due to 1 gm. of PETN explosive is equivalent to 1.282 gm. of PETN explosive.

3.2 Results and Discussions

The JWL material model for both PETN and TNT available in ANSYS Autodynare used for this comparative study. Based on the above mentioned conversion factor, mine is modelled for both PETN explosive of 1.9 gm and TNT equivalent explosive of 2.43 gm, respectively. The following Fig. 3 shows the simulation at the end of 1ms for both explosives.

As the shock wave emanating from the point of detonation travels spherically and the first point to come in contact with it is the plate centre, maximum deformation and damage occurs in the centre of the plate. Hence, the out of plane deformation of the plate centre is studied in the present analysis.

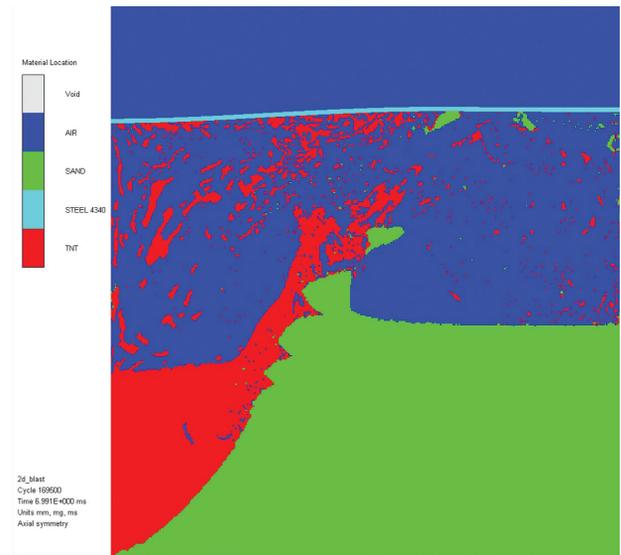
Figure 4 shows comparison of the out of plane displacement of center of plate, subjected to PETN explosive of 1.9 gm and TNT equivalent explosive of 2.43 gm, respectively. From the Fig. 4, it is observed that both the PETN and TNT explosive models are in good agreement.

Subsequent analysis was carried out using TNT equivalent masses so as to cater for future requirements. The 2D axisymmetric analysis was repeated for both large plate and small plate. The simulation end time was taken as 15 ms, even though the peak deformation is around 2 ms. This is in order to capture the secondary effects after the blast event. In fact, this 2 ms is the time taken for the shock front to reach the plate structure, after the initiation of the blast.

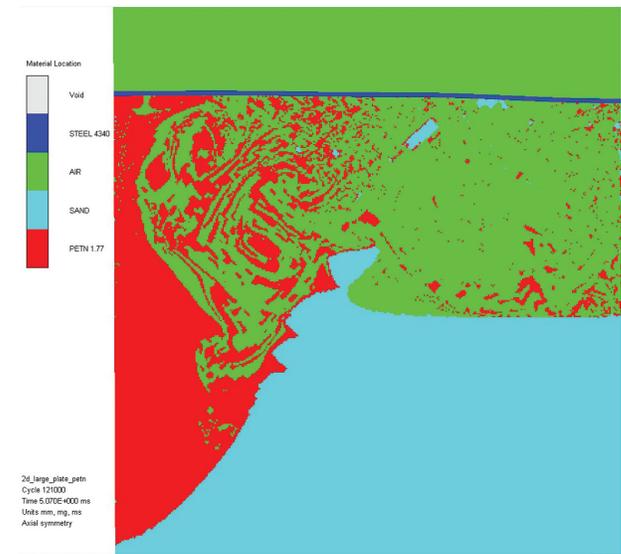
Figure 5 shows the unscaled time history output of out of plane displacement of plate centre in case of both large plate and unscaled small plate both large plate and small plate. Figure 6 shows the comparison between scaled measurement of small plate and original large plate. Here observe that time is also scaled as per the dimensional analysis.

From the above, a good agreement in displacement values, between the scaled small plate and large plate can be observed. Here, it is to be noted that time is also scaled as per the scaling rule obtained from the above mentioned dimensional analysis.

Figure 6, it can be observed that even though there is an excellent agreement between the two deformations till 5 ms, there is a slight shift between the two values beyond 5 ms. This is due to the fact that the scaling factor is approximated to 1.56 which based on the cube root of ratios of the two masses as per the scaling rule discussed in the previous section. Further, the model is made as 2D axisymmetric model which has further approximation in terms of volume calculation. This



(a)



(b)

Figure 3. (a) FE model using TNT explosive and (b) FE model using PETN explosive.

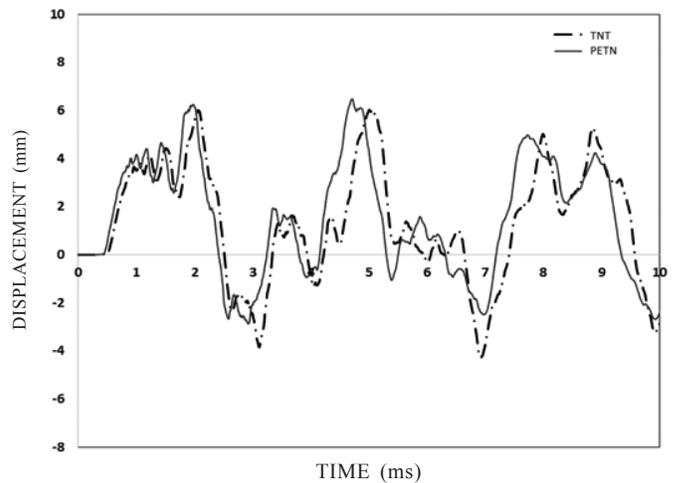


Figure 4. Comparison of Displacement of plate centre between PETN and TNT explosive.

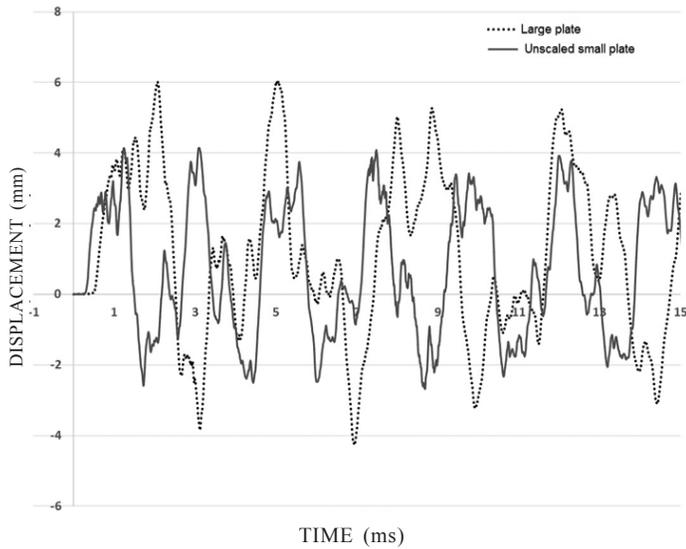


Figure 5. Comparison of displacement of plate centre between unscaled small plate and original large plate.

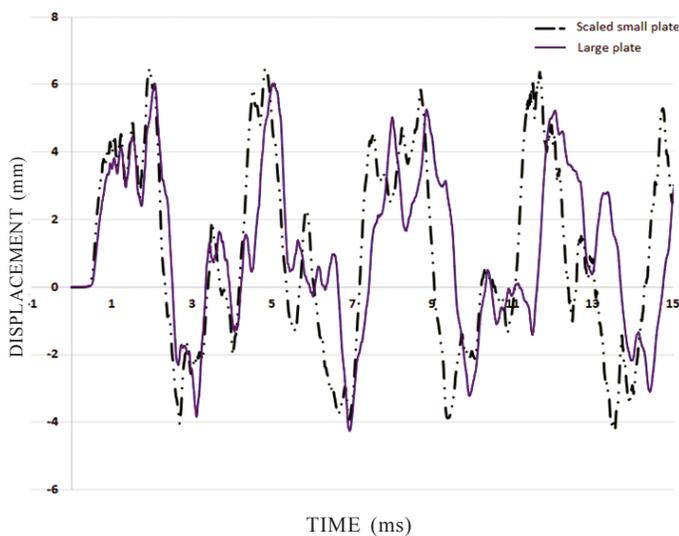


Figure 6. Comparison of displacement of plate centre between scaled small plate and original large plate.

approximation error increases with the time step, as the time variable is a scaled variable. In spite of the shift in the values, a good correlation can be observed between the peak values of the two outputs.

Figure 7 shows the comparison of velocities between large plate and scaled small plate, at the plate centre.

From the above graph, a good agreement of velocity profiles is seen between the two cases.

Figure 8 shows the 3D rendering of absolute velocity contour of large plate along with sand and TNT at 1.574 ms. Air volume is hidden in the Fig. 8 for better visualisation. It is to be noted that the model is simulated as 2D axisymmetric and rendered in 3D mode.

Even though the model has captured the phenomenon accurately upto 5 ms, there are certain limitations. Beyond 5 ms, there is an observable shift / offset in the values with respect to time scale. This is in fact due to the approximation in case of 2D axisymmetric model. This can be overcome by

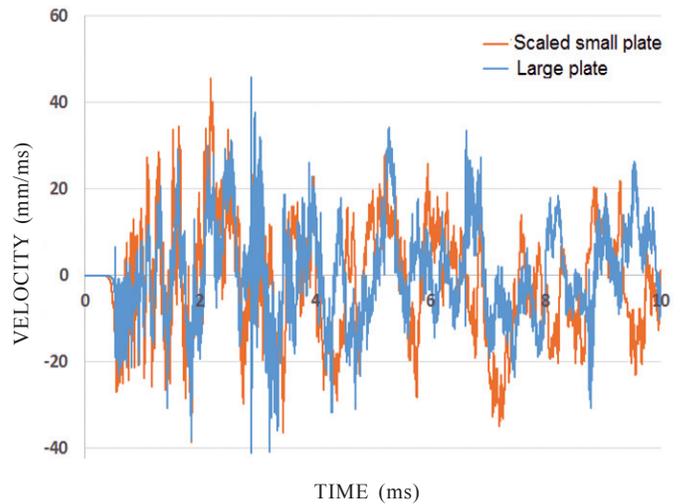


Figure 7. Comparison of velocities of plate centre between scaled small plate and original large plate.

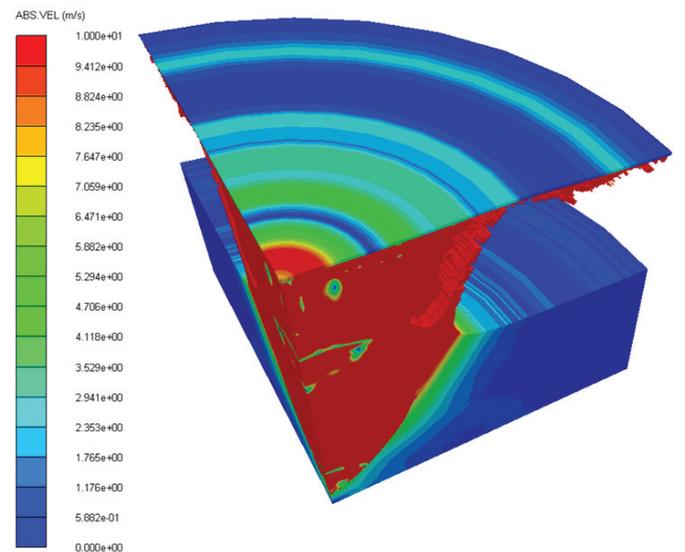


Figure 8. Absolute velocity contour for large plate simulation.

performing a 3D analysis. But a 2D counterpart gives a first cut analysis for subsequent simulations.

4. CONCLUSIONS

This present paper discusses about the dimensional analysis application for scaling of blast event. In general, FE analysis of structure subjected to blast load is very large computationally intensive problem. Hence better methods are required to reduce the computational cost and time. FE based scaling method is one such approach which is explored in the present paper. From the analysis, a good agreement was observed between the scaled and unscaled models FE results. This approach would provide a viable solution to simulate blast effect on large structures like MBT hull etc. at a comparatively lesser run time and hardware resources. Moreover, this method can be used as tool to understand the scaled model, well before the physical test setup. Thereby, the number of field experiments can be optimised and effective output can also be obtained.

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