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Parameter Estimation of Unstable Aircraft using Extreme Learning Machine

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ABSTRACT

The parameter estimation of unstable aircraft using extreme learning machine method is presented. In the past, conventional methods such as output error method, filter error method, equation error method and non-conventional method such as artificial neural-network based methods have been used for aircraft's aerodynamic parameter estimation. Nowadays, a trend of finding an accurate nonlinear function approximation is required to represent the aircraft's equations-of-motion. Such type of nonlinear function approximation is usually achieved using artificial neural-network which is trained with the aircraft input-output flight data using a training algorithm. The accuracy of estimated parameters, which is achieved using the trained network, is highly dependent on the generalisation capability of the network which can be improved using extreme learning machine based network in contrast to artificial neural-network. To estimate the unstable aircraft parameters from the simulated flight data, Gauss-Newton based optimisation method has been used with a predefined aerodynamic model using the trained network. Further, the confidence of the estimated parameters has been shown in comparison to that of the standard parameter estimation methods in terms of the Cramer-Rao bounds.

Keywords: Unstable aircraft; Extreme learning machine; Gauss-Newton method; Parameter estimation

NOMENCLATURE

β	Weight matrix connecting hidden layer and output layer
δ_e	Elevator deflection (rad)
I_{I_i}	Input at ith neuron of input layer
I_{O_i}	Output at ith neuron of input layer
H_{I_j}	Input at jth neuron of hidden layer
H_{O_j}	Output at jth neuron of hidden layer
O_{I_k}	Input at kth neuron of output layer
O_{O_k}	Output at kth neuron of output layer
M_w, M_q, M_{δ_e}	First-order dimensional derivatives of pitch acceleration per unit moment of inertia
M_{w^2}, M_{q^2}	Second-order dimensional derivatives of pitch acceleration per unit moment of inertia
N_Z	Normal acceleration (m/s^2)
q	Pitch rate (rad/s).
V	Weight matrix connecting input layer and hidden layer
w	Normal velocity (m/s)
X	Input vector
Z	Target vector
Z_w, Z_q, Z_{δ_e}	First-order dimensional derivatives of normal acceleration per unit mass
Z_{w^2}, Z_{q^2}	Second-order dimensional derivatives of normal acceleration per unit mass

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1. INTRODUCTION

Parameter estimation is an important phase of system identification process which is useful for designing control law, flight simulator and expansion of flight envelope. It uses the measured flight motion and control variables gathered while exciting the mode of the aircraft in finding the input-output relationship either in the form of parametric approach or nonparametric approach[1,2]. The parametric approach involves a prior model, which requires an understanding of physical phenomenon occurring between inputs and outputs, either in the linear or nonlinear form such as transfer function or state-space form whereas the nonparametric approach requires the concept of a network to be defined a prior without any physical knowledge of the system. The parametric approaches for estimating parameters are such as output error method (OEM), filter error method (FEM), equation error method (EEM), and some Kalman filter based methods[1]. OEM is mostly used for estimating stability and control derivatives of stable aircrafts. However, it produces a numerical divergence problem in the integration of the unstable aircraft dynamics which occurs due to improper and round-off of the initial estimates. Many variants of OEM have been presented to overcome the numerical divergence problem with some increased complexity in the existing method[3]. FEM has a filtering property which takes care of process and measurement noises by estimating stabilising parameters of dynamic system's state equations. Therefore, FEM can be applied to estimate the parameters of unstable aircraft due to its inherent stability in the state propagation[3]. The methods discussed above use initial guess values of the parameters for integration of states which are obtained either analytically or computed from wind tunnel or fluid dynamics software results. EEM overcomes the above issues and employs a single-shot solution based on least-square principle which reduces the computational burden. Therefore, it is used as a standard method for estimating parameters of stable and unstable aircrafts[4].

To overcome the shortcomings of the above-mentioned methods, artificial neural-network (ANN) was used to estimate the aircraft parameters. The trained network can generate a nonlinear function approximation based on the input-output data set[5,6]. It is a multi-layer network which comprises a number of neurons with their activation functions and biases in each intermediate layer, and these layers are interconnected in forward direction with some weights. Generally, the weights and biases are updated in an iterative way using the steepest-descent algorithm, namely the back-propagation (BP) error algorithm, until the mean-square-error (MSE) between the measured and predicted outputs reduces to a predefined lower value. Thus, many researchers around the world have used ANN to have a nonlinear input-output relationship and shown its capability in the system identification and estimation of aerodynamic parameters. For estimation of parameters, forces and moments have been mapped with respect to the corresponding variations in the motion and control variables using the ANN[7-11].

Except the earlier methods of estimating stability and control derivatives using ANN, the two methods namely, delta and zero have been reported in the literature using the concept of numerical finite difference approach[12-14]. An extension of these methods, the modified delta and Neural Gauss Newton (NGN) method have also been reported which yield the estimates with lesser standard deviations[15-18]. A radial basis function neural network can also be used for estimation of parameters as discussed in earlier methods[19]. The only difference is the use of radial basis function instead of the sigmoid function at the hidden neurons. However, this method requires a large number of hidden neurons and is slower than the earlier methods. A similar approach has been reported where a physical insight in the form of partial differentiation approach has been used for estimating the parameters, and has shown satisfactory results[20,21].

However, ANN based methods are dependent on the architecture and training algorithm. There is a high chance of BP error algorithm to trap in a local minima which causes a poor generalisation of the network. As, the training method is slower, hence it consumes a lot of time in terms of more no of iterations to conclude the relationship which may be a case of over-fitting. Therefore, an extreme learning machine (ELM) has been suggested which overcomes the limitations of the conventional ANN[22]. ELM is a three-layer network in which the weights and biases of the first two layers are randomly chosen, while the weights between the last two layers are computed analytically using the Moore-Penrose method. By choosing an optimum number of hidden neurons on trial and error basis, ELM can produce a nonlinear input-output relationship from the given measured data set of the dynamic system in a non-iterative way. The generalisation ability of the trained ELM network is more accurate and robust in comparison to ANN. Some of its applications is found in the field of forecasting[23,24].

To estimate the stability and control derivatives, the first step is to train the ELM network using flight measured motion and control variables. In the next step, the trained network is used for parameter estimation. An aerodynamic model, whose unknown parameters have to be estimated, is computed analytically and propagated through the trained network. Further, this process is followed by Gauss-Newton based optimisation method in an iterative way to update the parameters which converges in a few iterations. The statistical analysis of the estimated parameters is given in terms of Cramer-Rao bound. The simulated flight data of unstable aircraft in the linear and nonlinear forms has been dealt. The proposed approach has been applied to both cases and has shown satisfactory results in comparison to the standard methods of estimation.

2. EXTREME LEARNING MACHINE

Extreme learning machine is a single hidden layer feed forward neural network with a least-square based learning approach. It has been developed by Huang[22], *et. al.*

Its network architecture is as shown in Fig. 1 which is like a conventional ANN architecture. It has three layers: input, hidden, and output. In the input layer, there are *m* number of neurons which have a linear transfer function. The mathematical expression to represent the input layer is as follows:

$$I_{O_i} = I_{I_i} \tag{1}$$

where $i = 1, 2, ..., m$

The weights and biases to connect n number of hidden neurons with *m* number of input neurons can be represented in the matrix form as follows:

$$[V] = \begin{bmatrix} v_{11} & \cdots & v_{1j} & \cdots & v_{1n} \\ \vdots & & \vdots & & \vdots \\ v_{i1} & \cdots & v_{ij} & \cdots & v_{in} \\ \vdots & & \vdots & & \vdots \\ v_{(m+1)1} & \cdots & v_{(m+1)j} & \cdots & v_{(m+1)n} \end{bmatrix} \tag{2}$$

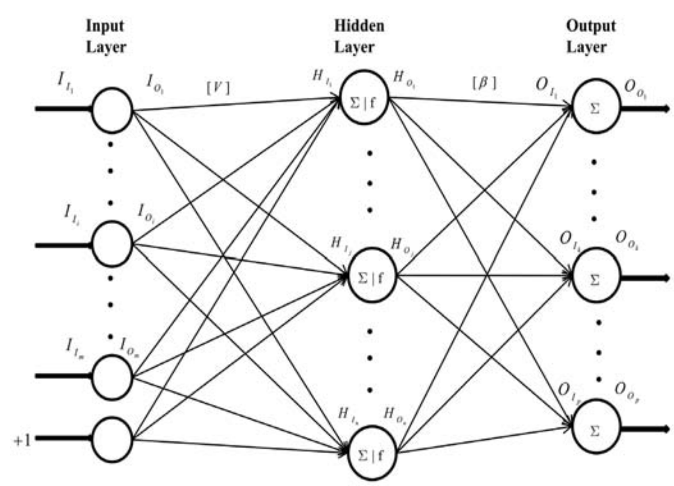

Figure 1. Structure of ELM network.

It is noted that V is a $(m+1)\times n$ matrix where the last row corresponds to the biases of the hidden neurons. The input at the j[th] hidden neuron can be given as follows:

$$H_{I_j} = \sum_{i=1}^{m} I_{O_i} v_{ij} + v_{(m+1)j} \tag{3}$$

where $j = 1, 2, ..., n$

The output of the hidden neuron using sigmoid activation function is as follows:

$$H_{O_j} = \frac{1}{1+e^{-H_{I_j}}} \tag{4}$$

Let's assume that β is the weight matrix connecting the hidden layer neurons to the output layer neurons. Thus, the estimated input to the k[th] output neuron can be given as follows:

$$O_{I_k} = \sum_{j=1}^{n} H_{O_j} \beta_{jk} \tag{5}$$

where $k = 1, 2, ..., p$

As the output layer neurons use the linear activation functions therefore, their outputs are same as the inputs as follows:

$$O_{O_k} = O_{I_k} \tag{6}$$

For multi-input multi-output case, the output can be represented in the matrix form as follows:

$$O_O = H_O \beta \tag{7}$$

where $H_O = \begin{bmatrix} H_{O_1} & \cdots & H_{O_j} & \cdots & H_{O_n} \end{bmatrix}_{1X\,n}$,

$$\beta = \begin{bmatrix} \beta_{11} & \cdots & \beta_{12} & \cdots & \beta_{1p} \\ \vdots & & \vdots & & \vdots \\ \beta_{j1} & \cdots & \beta_{j2} & \cdots & \beta_{jp} \\ \vdots & & \vdots & & \vdots \\ \beta_{n1} & & \beta_{n2} & & \beta_{np} \end{bmatrix}_{nX\,p}$$ and O_O is a $1\times p$ row vector.

Here the objective is to find the matrix β with the following condition:

$$\left\| H_O\hat{\beta} - Z \right\| = \min_{\beta} \left\| H_O\beta - Z \right\| \tag{8}$$

This a norm based entity. The above equation can be solved using least square method. Therefore,

$$\hat{\beta} = H_O^{*} Z \tag{9}$$

where $H_O^{*} = \left(H_O^T H_O\right)^{-1} H_O^T$ is a Moore-Penrose inverse generalised matrix. Thus,

$$\hat{\beta} = \left(H_O^T H_O\right)^{-1} H_O^T Z \tag{10}$$

For N number of input-output data samples, the above procedure can be applied, and similar expressions are obtained.

The performance of the network is given in terms of root-mean-square error (RMSE) as follows:

$$E = \sqrt{\frac{\sum_{i=1}^{N}\left(Z_i - O_{O_i}\right)^2}{N}} \tag{11}$$

where, E is the error of the model, Z_i is i[th] target output, O_{O_i} is i[th] estimated output of the model, and N is the total number of data samples.

3. NONLINEAR SYSTEM MODELLING AND PARAMETER ESTIMATION

3.1 Nonlinear System Modelling

For modelling of the nonlinear system, a network is chosen a prior based on the input and output variables. The network seems to be same as shown in Fig. 1. As it is a supervised learning so, the input and output dataset are defined to have a nonlinear relationship between them. To fulfil our purpose, the input variables are considered at the i[th] instant which are written in the vector form as follows:

$$X(i) = \left[x_1(i),\ x_2(i),\ ...x_m(i) \right] \tag{12}$$

And the target is chosen at (i+1)[th] instant which is represented using the output variables in the vector form as follows:

$$Z(i+1) = [z_1(i+1),\ z_2(i+1), ..., z_p(i+1)] \tag{13}$$

To have a dynamic model, either some or all of the output variables are feedback to the input side with some time delay, thus applying a feedback dependency in the network. Further, an optimal number of hidden neurons are determined on the basis of low value of RMSE using trial and error method. After choosing the optimal values of random weights and biases, the analytical approach of determining the weights between the hidden and the output layer is used as per Eqn. (10). Thus, the process of training is completed in a single shot by following the above procedure.

3.2 Parameter Estimation

The training procedure discussed in last subsection is followed for optimising an aerodynamic model. As we know that the motion of the aircraft is influenced by the generated forces and moments on the body hence, their coefficient forms are chosen as some of the input variables for training of the network which can be further represented as a linear or nonlinear function of motion and control variables of aircraft. This function also consists of some unknown constants represented in vector form as Θ which is determined through some optimisation method. The popularly known Gauss-Newton method has been selected for the said purpose. With some initial guess values of Θ, the analytically determined inputs are applied to the trained network and the predicted output Y is computed. Further, the residual error E is defined as the difference between the target output Z and the predicted output, Y which is given at the i^{th} instant as follows[1]:

$$E(i) = [Z(i) - Y(i)] \tag{14}$$

And the covariance matrix, R is defined as follows:

$$R = \frac{1}{N}\sum_{k=1}^{N}[Z(i)-Y(i)][Z(i)-Y(i)]^T \tag{15}$$

The cost function, J has to be minimised and given as follows:

$$J(R) = \frac{1}{2}\sum_{k=1}^{N}[Z(i)-Y(i)]^T R^{-1}[Z(i)-Y(i)] \tag{16}$$

The updating of the parameter vector, Θ using GN method is as follows:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \quad (17)$$

$$F\Delta\Theta = -G \quad (18)$$

where the Hessian Matrix, F is computed as follows:

$$F = \sum_{i=1}^{N} \left[\frac{\partial Y(i)}{\partial \Theta}\right]^T R^{-1} \left[\frac{\partial Y(i)}{\partial \Theta}\right], \quad (19)$$

And the gradient vector, G is computed as follows:

$$G = -\sum_{i=1}^{N} \left[\frac{\partial Y(i)}{\partial \Theta}\right]^T R^{-1} \left[Z(i) - Y(i)\right] \quad (20)$$

As we know that Gauss-Newton method is an iterative way of optimisation so, the Eqns. (14) - (20) are repeated until the cost function reaches to the desired convergence value to find the optimal value of unknown vector Θ .

4. RESULTS AND DISCUSSION

The extreme learning machine based estimation method has been applied to estimate the parameters of de Havilland DHC-2 Beaver aircraft's linear and non-linear models.

4.1 Linear Unstable Aircraft

The short-period motion of de Havilland DHC-2 Beaver aircraft has been considered as a first case whose data is generated through simulation of a simplified linear model[1].

The state equations of the linear model are:

$$\begin{aligned} \dot{q} &= M_w w + M_q q + M_{\delta_e} \delta_e \\ \dot{w} &= Z_w w + (u_0 + Z_q) q + Z_{\delta_e} \delta_e \end{aligned} \quad (21)$$

And the observation variables are $w,\ q,\ \dot{w},\ \dot{q}$, and N_z. The normal acceleration N_z is defined as

$$N_z = Z_w w + Z_q q + Z_{\delta_e} \delta_e \quad (22)$$

The nominal values of the aerodynamic derivatives and an optimised Mehra-Input signal δ_e are considered from Jategaonkar[1,3], *et. al.* The state equations are integrated using fourth-order Runge-Kutta method to generate the response for a time span of 12.5 s with a sampling time of 0.05 s by adjusting the static stability parameter M_w at a nominal speed of 44.57 m/s. The simulated data is as shown in Fig. 2 which has been used to estimate parameters $M_w, M_q, M_{\delta_e}, Z_w, Z_q$, and Z_{δ_e} (considered as Θ) by the methods namely: least-squares (LS), stabilised output error method (SOEM), and ELM.

The classical methods such as OEM, FEM require initial guess values to estimate the parameters which may not be accurate while LS method does not require any guess values. SOEM was unable to estimate parameters with stabilisation matrix due to intermediate divergence of the algorithm. Therefore, Z_q has been fixed to estimate the parameters and have shown satisfactory results.

ELM based network has inputs $w, q, \dot{w}, \dot{q}, \delta_e$ at the ith instant and $w, q, \dot{w}, \dot{q}, N_z$ at (i+1)th instant for its training. By following the procedure discussed in section 3, a non-linear input-output relationship is generated with 100 numbers of hidden neurons, and the unknown parameters of equation are optimised by propagating the states through the network. It is seen that in a few iterations, the algorithm converges closer to the actual/nominal values. A comparative analysis is presented amongst the values from ELM, conventional methods and nominal as shown in Table 1. The parameters are estimated satisfactorily while their standard deviations are a little higher in contrast to SOEM. It is observed that the parameters converge from different initial guess values. A closer guess value causes the optimisation method to converge early while it takes more number of iterations in case of others. Thus, it can be concluded that ELM based network neither depends on the initial guess values nor fixing of the parameters like SOEM method. The validation of estimated parameters has been carried out with the values of the other methods as shown in Fig. 3. The estimated parameters from the conventional methods have shown a fairly well matching with the nominal values. In case of ELM, a satisfactory matching is found with the simulated data up to 8s whereas a deviation may have occurred due to the sensitive Z_q value.

4.2 Nonlinear Unstable Aircraft

The short-period motion of the same aircraft as described

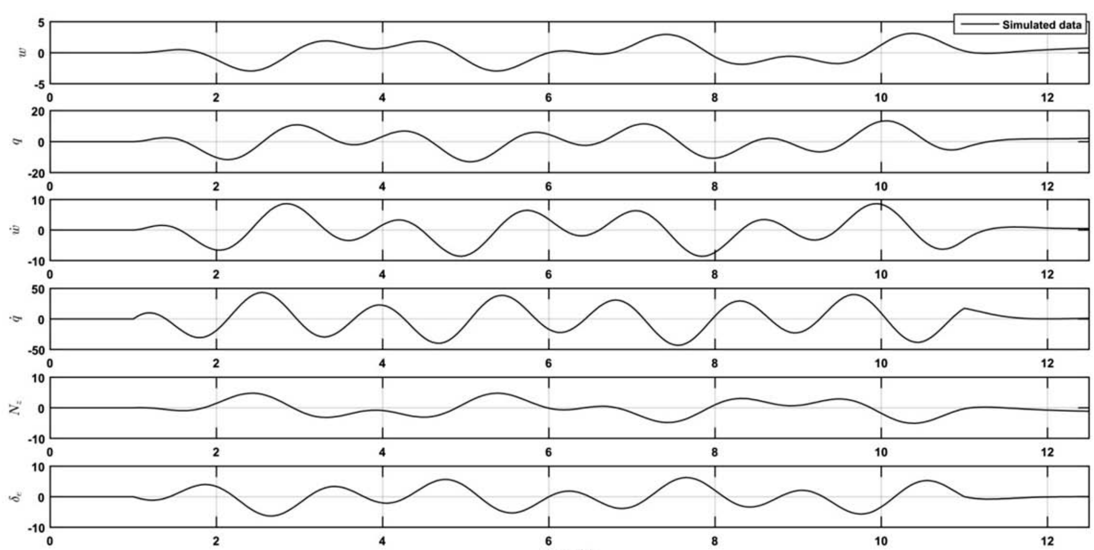

Figure 2. Simulated data of linear model of beaver aircraft.

Table 1. Parameter estimates of linear model using LS, SOEM, and ELM methods

Parameter	Nominal value[1]	LS[1]	SOEM[1]	ELM
Z_w	-1.4249	-1.4249 (0.0)[#]	-1.4274 (0.0001)	-1.4206 (0.0009)
Z_q	-1.4768	-1.4768 (0.0)	-1.4768*	-1.7959 (0.0110)
Z_{δ_e}	-6.2632	-6.2632 (0.0)	-6.1619 (0.0009)	-6.4738 (0.0237)
M_w	0.2163	0.2161 (0.0003)	0.2172 (0.0)	0.2132 (0.0002)
M_q	-3.7067	-3.7042 (0.0033)	-3.7238 (0.0002)	-3.6649 (0.0019)
M_{δ_e}	-12.784	-12.7653 (0.0071)	-12.8205 (0.0011)	-12.6175 (0.0043)

Note: (i) $^*Z_q$ is kept fixed at the nominal value for estimating parameters.
(ii)[#] Values in parentheses indicate standard deviations.

in the last subsection has been generated through simulation of a nonlinear model as presented[21]. The state equation of the nonlinear model is:

$$\begin{aligned}\dot{q} &= M_w w + M_q q + M_{\delta_e}\delta_e + M_{w^2}w^2 + M_{q^2}q^2 \\ \dot{w} &= Z_w w + (u_0 + Z_q)q + Z_{\delta_e}\delta_e + Z_{w^2}w^2 + Z_{q^2}q^2\end{aligned} \quad (23)$$

The equations of the nonlinear model simply add two higher order terms of states which make it different from the linear model. A similar approach is applied for observation variable N_z. The normal acceleration, N_z is defined as

$$N_z = Z_w w + Z_q q + Z_{\delta_e}\delta_e + Z_{w^2}w^2 + Z_{q^2}q^2 \quad (24)$$

Four more observations w, q, $\dot{w}$, and $\dot{q}$ are considered along with N_z. The aerodynamic coefficients are same as the linear case except the second degree coefficients which are taken 10 per cent of the values of the corresponding linear coefficients to avoid the divergence in short duration of the simulation. Further, the integration is done using fourth-order Runge-Kutta method to generate the response with the same Mehra-Input δ_e for a time span of 5 s with a sampling time of 0.05 s. The simulated flight data is shown in Fig. 4.

A similar approach as discussed earlier, has been carried out in the present subsection. Due to the divergence of classical parameter estimation methods, SOEM has been used to estimate the parameters with a proper stabilisation matrix by fixing Z_q. For the training of ELM network, the inputs $w, q, \dot{w}, \dot{q}, N_z, \delta_e$ at the i[th] instant and the outputs $w, q, \dot{w}, \dot{q}, N_z$ at the (i+1)[th] instant are considered.

By following the procedure discussed in section 3, a non-linear input-output relationship is produced and the unknown parameters of the Eqn. (23) are optimised by propagating the states through the network. It is seen that the parameters are estimated in a few iterations after converging from their initial random chosen guess values as shown in Fig. 5 which has similar remarks on the convergence of the parameter from two different initial guess values as the linear case. The estimated parameters using ELM are compared with nominal, LS, and SOEM values as shown in Table 2. It is found that the values are satisfactorily matching with the other methods whereas standard deviations of the parameters are a little higher than that of the SOEM. The validation of estimated parameters has been presented with the other methods in Fig. 6 which has shown a perfect matching.

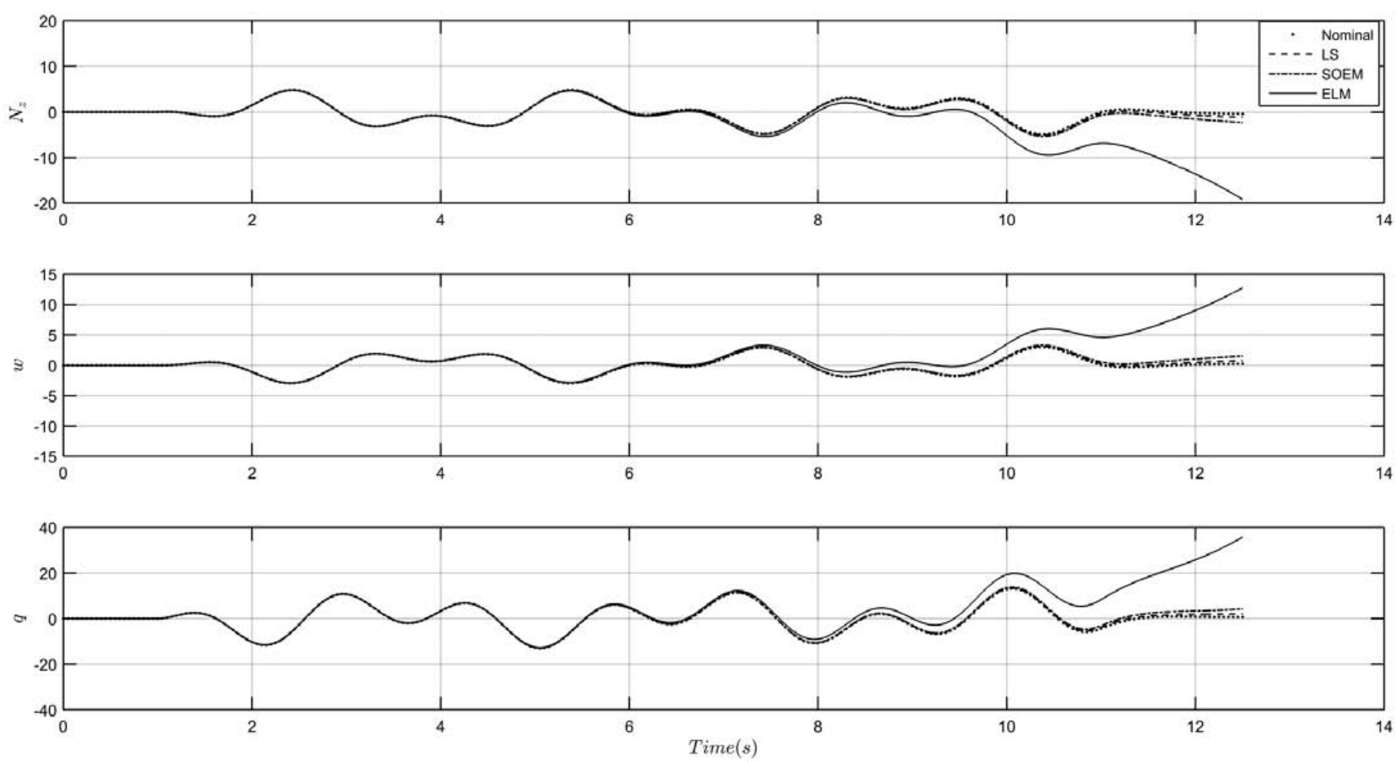

Figure 3. Data validation of estimated parameters with the simulated data of linear model.

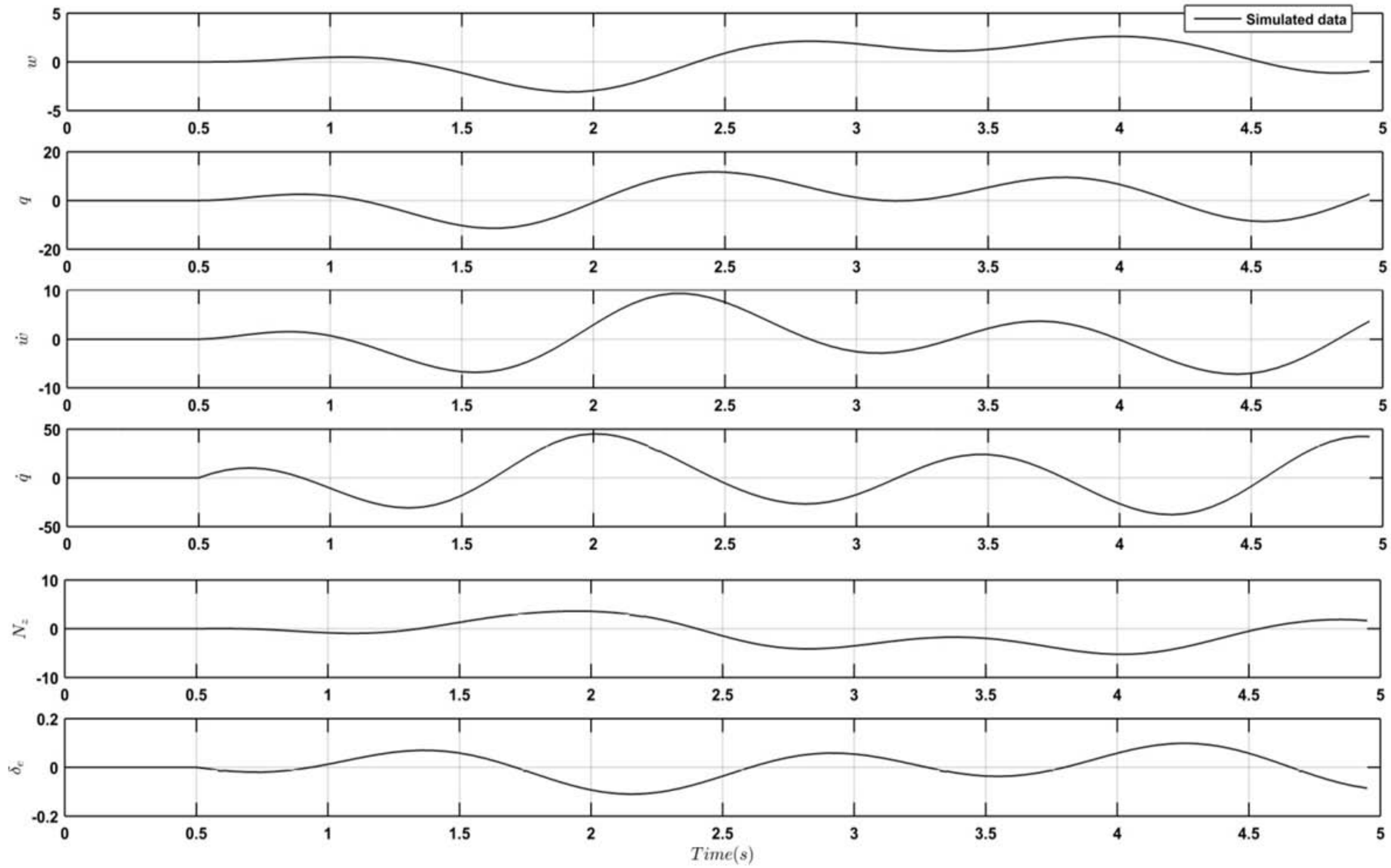

Figure 4. Simulated data of non-linear model of Beaver Aircraft.

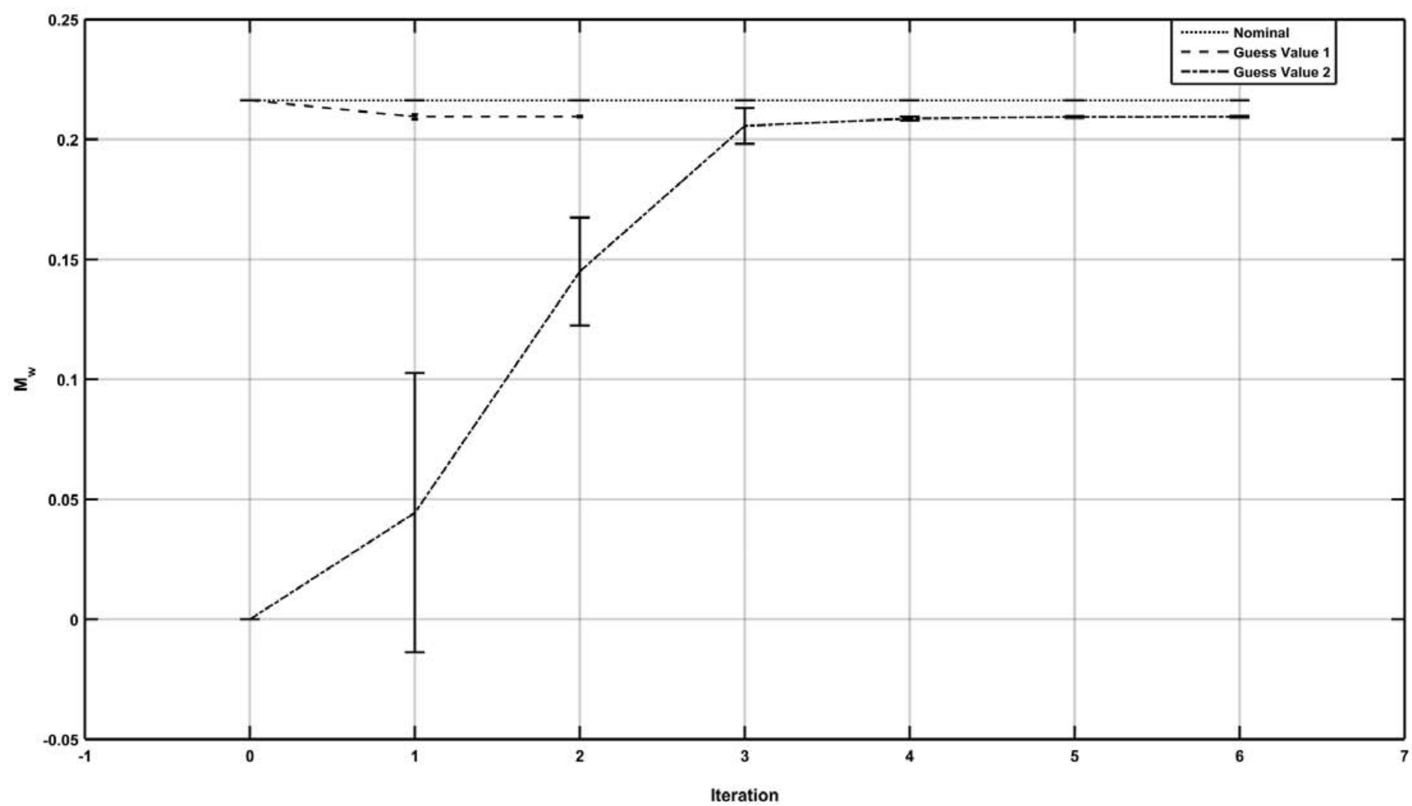

Figure 5. Parameter *vs* iteration.

Table 2. Parameter estimates of non-linear model using LS, SOEM, and ELM methods

Parameter	Nominal value[21]	LS[21]	SOEM	ELM
Z_w	-1.4249	-1.4249 (0.0)[#]	-1.4302 (0.0001)	-1.4097 (0.0023)
Z_q	-1.4768	-1.4768 (0.0)	-1.4768[*]	-1.8429 (0.0288)
Z_{δ_e}	-6.2632	-6.2632 (0.0)	-6.1279 (0.0037)	-6.5219 (0.0618)
Z_{w^2}	-0.1425	-0.1425 (0.0)	-0.1427 (0.0)	-0.14 (0.0003)
Z_{q^2}	-0.1477	-0.1477 (0.0)	-0.1795 (0.0079)	-0.1469 (0.0193)
M_w	0.2163	0.2163 (0.0)	0.2177 (0.0)	0.2137 (0.0003)
M_q	-3.7067	-3.7067 (0.0)	-3.7271 (0.0003)	-3.6777 (0.0038)
M_{δ_e}	-12.784	-12.784 (0.0)	-12.8220 (0.0009)	-12.6203 (0.0083)
M_{w^2}	0.0216	0.0216 (0.0)	0.0217 (0.0)	0.0213 (0.0)
M_{q^2}	-0.3707	-0.3707 (0.0)	-0.3646 (0.0020)	-0.3205 (0.0073)

Note: (i) [*]Z_q is kept fixed at the nominal value for estimating parameters.
(ii)[#] Values in parentheses indicate standard deviations.

5. CONCLUSIONS

Extreme learning machine in combination with Gauss-Newton method has been proposed for estimating the aerodynamic parameters of an unstable aircraft. Linear and non-linear cases of the unstable aircraft's longitudinal dynamics have been discussed with their corresponding simulated flight data, and their estimated parameter results are compared with the classical methods such as LS and SOEM. It is proved that LS based parameters are same as the nominal values chosen for generating the flight data to ensure the data consistency in both the cases whereas SOEM results are seen with dependency on stabilising and fixing of the parameters. In case of ELM, the estimation process is fully dependent on the chosen network. Therefore, a careful attention is made on the network parameters such as number of hidden neurons, weights, and biases. It is seen that a large number of hidden neurons in the network corresponds to a lower value of root-mean-square error (RMSE) which may be over-fitting the network, and further it makes difficult to apply any iterative optimisation method for estimating the aerodynamic parameters. Here, the input-output states of the dynamic systems are chosen such that ELM network can be constituted with moderate value of RMSE in both cases, and GN method can be applied for estimating parameters from random initial guess values. Further, the validation of the estimated parameters has been presented for both the cases. In a short duration of simulation, responses are found fairly well whereas deviations are seen in a longer duration of time. Thus, the results obtained using

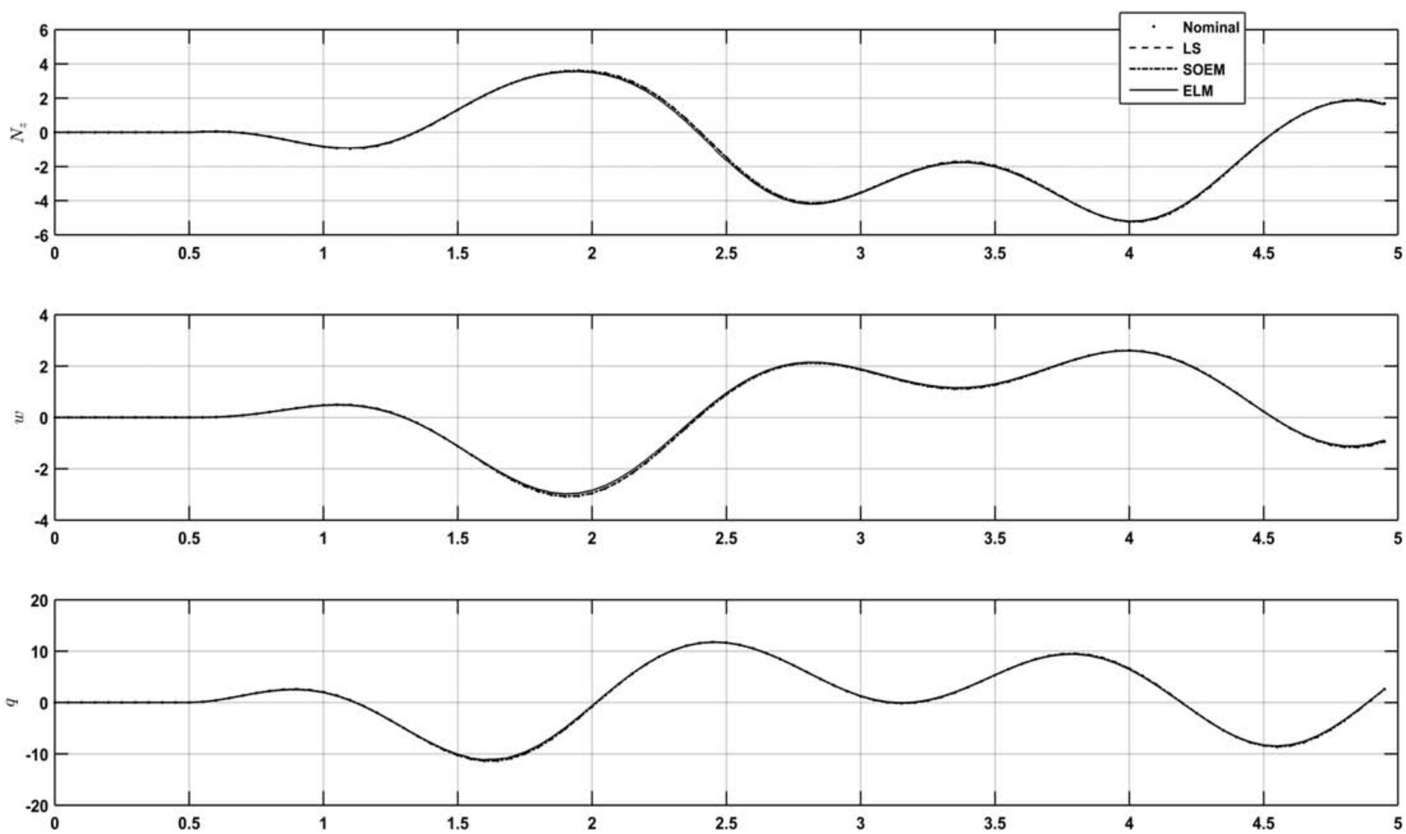

Figure 6. Data validation of estimated parameters with the simulated data of non-linear model.

the ELM network suggests its applicability in the estimation of aerodynamic parameters provided a sufficient number of variables in the dataset.

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