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Sensor/Control Surface Fault Detection and Reconfiguration using Fuzzy Logic

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ABSTRACT

In the aircraft flight control systems, a quick detection of the faults, that occur in actuators, control surfaces or sensors, is necessary. In this paper, sensor fault detection and reconfiguration is performed using Kalman filter by estimating the states of the plant and comparing them with respective measured values from the sensors. Sensor fault detection and reconfiguration is carried out using non-model-based fuzzy logic technique. Control surface fault detection and reconfiguration is carried out by identifying the elements of control distribution matrix using extended Kalman filter and fuzzy logic. In estimating the factor of effectiveness of the control surface using fuzzy logic, different implication methods such as Mamadani's minimum, Larsen's product, bounded product and drastic product have been used and a comparison is made.

Keywords: Fuzzy logic, SFDIR, sensor fault detection isolation and reconfiguration, fault reconfiguration, sensor fault detection, control surface fault detection, Kalman filter.

1. INTRODUCTION

Aircraft is a complex vehicle as its motion is three dimensional and the environment in which it flies generates disturbance forces. In practice, faults may occur in an aircraft in sensors, actuators, and control surfaces. The purpose of fault tolerant flight control systems is to detect, identify, and accommodate or reconfigure for the fault that may occur during a flight. Two critical faults are generally classified as actuator (or/and control surface) and sensor faults. To maximise survivability of the aircraft, any type of control surface fault or sensor fault should be immediately detected, isolated and accommodated. This needs basically fault detection and isolation, and reconfiguration. The motion of the aircraft is measured by sensors, such as rate gyroscopes and accelerometers. When sensor faults occur, automatic flight control system (AFCS) is expected to maintain its normal performance. This needs sensor fault detection, isolation, and reconfiguration (SFDIR). Many approaches have been proposed for SFDIR which involve the concept of hardware or analytical redundancy. Analytical redundancy techniques are basically signal-processing techniques, which involve state estimation, parameter identification, statistical decision theory, etc. A novel approach based on a Kalman filter (KF) innovation sequence is used to detect and locate the aircraft sensor faults[1,2]. Faults that change the system dynamics by abnormal measurements, sudden shifts, and other difficulties such as the decrease of instrument accuracy, etc affect the characteristics of the normalised innovation sequence by changing its white noise nature, displacing its zero mean and varying unit covariance matrix. Hence, the objective of the problem is to detect as quickly as possible, any change of these parameters from their nominal values and provide the necessary remedies.

In studying control systems, one must be able to model dynamic systems and analyse dynamic characteristics. It is generally difficult to represent a complex process accurately by a mathematical model. Fuzzy logic is a non-model-based technique which deals with knowledge of process behaviour and experience of people working with the process and it can handle non-crisp and incomplete information. Since the first successful application of the idea of fuzzy sets of Zadeh to the control of a dynamic plant by Mamdani and Assilian 'Fuzzy control Systems Engineering' has gained worldwide interest[3,4]. It is possible to control many complex systems effectively by experienced human operators who have no knowledge of their underlying dynamics, while it is difficult to achieve the same with conventional controllers.

In this paper, a model-based scheme using KF is used for SFDIR. If the system operates normally, the normalised innovation sequence in KF is a Gaussian white noise with a zero mean and with a unit covariance matrix. When a KF is used, the decision statistics change under fault condition and its effect is more significant for the faulty sensor channel, hence the faulty sensor is isolated. Subsequently, the KF is reconfigured by ignoring the measurement from faulty sensor. It is assumed that all the states of the system are observable and can be measured. Fuzzy logic is used for detection, and reconfiguration of sensor fault in an aircraft. It basically involves fuzzification, rule base, and

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inference engine and defuzzification. In this case, perturbation elevator deflection is the input to the fuzzy module of the plant and true states are estimated as outputs. Then, measured states are compared with true estimated states and if their difference exceeds the threshold value, then the particular sensor measurement is ignored and replaced by the true estimated state. In this paper it is shown that, fuzzy logic algorithm can also be extended for multiple sensor faults. The aircraft becomes unstable due to fault in actuator or if there is a loss of control surface effectiveness due to damaged or blown surfaces. One of the popular methods to detect and reconfigure the surface fault is the model-based approach e.g. extended Kalman filter (EKF)[5]. In this paper, the parameters of control distribution matrix are estimated as augmented states of the system using EKF which are subsequently used to compute feedback gain to reconfigure the impaired system using pseudo-inverse technique. Detection and reconfiguration of surface fault in elevator of an aircraft is demonstrated using non-model-based fuzzy logic in terms of determining the factor of effectiveness of control surface and in turn new control gain for reconfiguration. A comparison has been carried out using two T-norm operations, namely intersection and algebraic product and different implication methods, such as Mamadani's minimum, Larsen's product, bounded product and drastic product in estimating the factor of effectiveness or correction factor and the results obtained are compared.

2. SENSOR FAULT DETECTION, ISOLATION, AND RECONFIGURATION

Sensor fault detection and identification(SFDI) is very important, particularly when the measurements from a faulty sensor are used in feedback control loop. Since the aircraft control laws use sensor feedback to establish the current dynamic state of the airplane, even slight sensor inaccuracies, can lead to closed-loop instability, if not attended, and may lead to unrecoverable flight conditions.

2.1 Sensor Fault Detection, Isolation, and Reconfiguration–using Kalman filter

One of the techniques of sensor fault detection involves the generation of residuals that carry information about the failures. The most common method used for generating residuals is using state estimators such as KF. If the system operates normally, the normalised innovation sequence in KF is a Gaussian white noise with a zero mean and with a unit covariance matrix. When a fault occurs, the decision statistics change and its effect is more significant for the faulty sensor channel, hence the faulty sensor is identified.

2.1.1 Sensor Fault Detection

In this study, SFDIR is carried out considering the longitudinal dynamics of an aircraft in the simulation. The state space equations of the longitudinal motion of an aircraft in continuous domain are given by

$$\dot{x} = Ax + Bu_c + \Gamma w_n \tag{1}$$

$$z = Hx + v \tag{2}$$

where

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ M_{\delta_E} \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

and $u_c = \delta_E$ is perturbation elevator deflection = control input, w_n is a white Gaussian process noise (random) with zero mean and covariance Q. Γ is a perturbation noise transition matrix. z is measurement vector and v is a white Gaussian measurement noise with zero mean and covariance R and is uncorrelated with process noise w_n. It is assumed that all the states are measurable. The simulation of the KF is as follows:

State and covariance propagation

$$\dot{\tilde{x}} = A\hat{x} + Bu_c \tag{3}$$

$$P(k/k-1) = F(k/k-1)P(k-1/k-1)F^T(k/k-1) + \Gamma(k/k-1)Q(k-1)\Gamma^T(k/k-1) \tag{4}$$

where, $F = e^{AT}$ is the state transition matrix and T is sampling time interval. Measurement update

$$\hat{x}(k/k) = \tilde{x}(k/k-1) + K(k)\gamma(k) \tag{5}$$

where, $\gamma(k)$ is innovation sequence, given by

$$\gamma(k) = z(k) - H(k)\tilde{x}(k/k-1) \tag{6}$$

$$K(k) = P(k/k-1)H^T(k)\,S(k)^{-1} \tag{7}$$

where, S is covariance matrix of residuals or innovations and is given by

$$S(k) = H(k)P(k/k-1)H^T(k) + R(k) \tag{8}$$

$$P(k/k) = [I - K(k)H(k)]P(k/k-1) \tag{9}$$

where, $P(k-1/k-1)$ is a covariance matrix of estimate errors at the preceding step, K (k) is the gain matrix of the KF, and I is an identity matrix. To detect the faults changing the mean of the innovation sequence, the following statistical function is used:

$$\beta(k) = \sum_{j=k-M+1}^{k} \tilde{\gamma}^T(j)\tilde{\gamma}(j) \tag{10}$$

where, M is the number of the samples (window length). The two hypothesis tests used to detect the faults are:

$$\beta(k) \le \chi^2_{\alpha,Ms} \quad \text{then} \quad H_0 \text{ (no fault)}$$

$$\beta(k) > \chi^2_{\alpha,Ms} \quad \text{then} \quad H_1 \text{ (fault)} \tag{11}$$

$\chi^2_{\alpha,Ms}$ is a threshold taken from chi-square table, α is probability of confidence level, and Ms is degree of freedom (DOF) which is equal to M, multiplied by s (no. of sensors). If the mean of the innovation sequence exceeds statistical function value, then fault is detected.

2.1.2 Sensor Fault Isolation Algorithm

For the isolation of sensor fault, the approach presented[1] is used. For isolation of sensor fault, s-dimensional innovation sequence is transformed into s-one-dimensional sequences. The statistics of the faulty sensor is assumed to be affected much more than those of the other sensors.

The statistics, which is a rate of sample and theoretical variances, $\frac{\hat{\sigma}_i^2}{\sigma_i^2}$ is used to verify the variances of one-dimensional innovation sequences $\tilde{\gamma}_i(k), i = 1,2,...,s$ where

$$\hat{\sigma}_i^2(k) = \frac{1}{M-1}\sum_{j=k-M+1}^{k}[\tilde{\gamma}_i(j) - \bar{\tilde{\gamma}}(k)]^2$$
$$\bar{\tilde{\gamma}}_i(k) = \frac{1}{M}\sum_{j=k-M+1}^{k}\tilde{\gamma}_i(j) \qquad (12)$$

When, $\tilde{\gamma}_i = N(0, \sigma_i)$ it is known that

$$\frac{v_i}{\sigma_i^2} \sim \chi^2_{\alpha,M-1};\quad \forall i,\ i = 1,2,...,s$$

where $v_i = (M-1)\hat{\sigma}_i^2 \quad ;\forall i, \quad i = 1,2,...,s$

As $\sigma_i^2 = 1$ for normalised innovation sequence, it follows that

$$v_i \sim \chi^2_{\alpha,M-1}, \qquad \forall_i,\ i = 1,2,...,s \qquad (13)$$

Using Eqn (13), any change in the mean of the normalised innovation sequence can be detected, and hence, the i^{th} sensor, in which γ_i exceeds the threshold value can be identified as the faulty sensor.

2.1.3 Sensor Fault Reconfiguration

When a sensor fault occurs, the effect of the sensor fault to its channel is more significant which needs reconfiguration. Once the fault is detected in a particular channel, KF is reconfigured by ignoring the feedback from the faulty sensor and using measurements from healthy sensors only. Therefore, there will be no more faulty measurements, and KF estimates the states with reduced healthy measurements, thus providing the necessary reconfiguration.

2.2 Sensor Fault Detection and Reconfiguration using Fuzzy Logic

In this case, perturbation elevator deflection is the input to the fuzzy module of the plant and true states are estimated as outputs. The measured states are compared with true estimated states, and if their difference exceeds the threshold value, then the fault is detected. The fault detection automatically triggers the sensor isolation algorithm to locate the faulty channel and once the fault is located, then accommodation algorithm bypasses the faulty measurements, i.e., the particular sensor measurement is ignored and replaced by the true estimated state. Thus, the reconfiguration is provided for sensor fault.

Figure 1 shows the block diagram for sensor fault detection and reconfiguration-based on fuzzy logic. Input to the plant is perturbation elevator deflection, i.e., $u_c = \delta_E$ and universe of discourse (UOD) for the input is defined as UOD $u_c = [u_c^1\ u_c^2\ u_c^3\ u_c^4\ u_c^5]$, triangular membership functions are constructed for input u_c on its UOD as shown in Fig. 2. The outputs are the four states of the system u, w, q and θ.

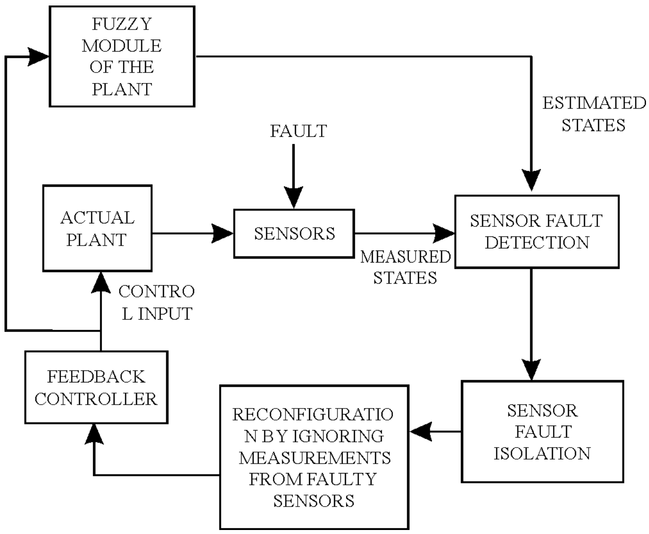

Figure 1. Fault detection, isolation and reconfiguration schematic using fuzzy logic.

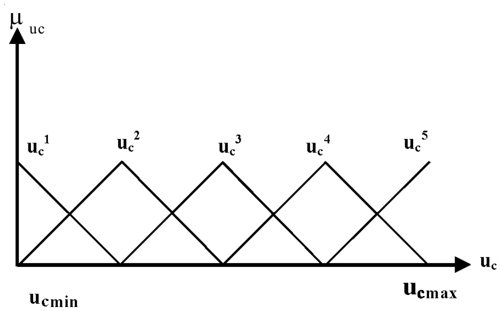

Figure 2. Membership functions for input variable u_c.

Figure 3 shows the general shape of the membership functions for the states of the system (e.g. perturbation velocity along x-axis, u) which are unsymmetrical and triangular. To partition the outputs, triangular membership functions for each output are constructed on their respective UODs.

For an output state x (representing w, q and θ) it is chosen as UOD $x=[x^1\ x^2\ x^3\ x^4\ x^5]$. Table 1 shows if-then form of fuzzy rules described for output state u, Table 2 shows Fuzzy Associative Memory (FAM) table for output states w, q, and θ.

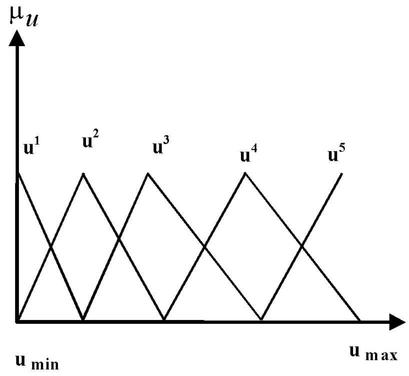

Figure 3. Membership functions for *u*.

Table 1. Inference rules for horizontal velocity *u*

Control input u_c	Control output u
u_c^1	u^1
u_c^2	u^2
u_c^3	u^3
u_c^4	u^4
u_c^5	u^5

Table 2. Inference rules for *x* (*w, q* and θ)

Control input u_c	Control output x
u_c^1	x^5
u_c^2	x^5
u_c^3	x^4
u_c^4	x^2
u_c^5	x^2

Using the inference rules expressed as in Tables 1 and 2, a simulation of the developed fuzzy system is carried out. Each simulation cycle results in membership functions for the four outputs. The membership functions for the outputs are defuzzified-based on inferred rule and intersection T-norm to get their crisp values. Thus, true states are estimated. For detection and reconfiguration of sensor fault, the estimated states are compared with the respective measured states of the plant and if the difference (or residual) exceeds the pre-computed threshold of the measurement noise (for respective channel), then fault is detected in that sensor. For the detection purpose, thresholds are computed using Monte-Carlo simulation of random noises (for all four sensors with standard deviation specified in Section 4) for 1000 runs. The minimum and maximum values of these noises are computed for each run and then average of all run are considered as thresholds for detection purpose. If the fault is detected, then output of that faulty sensor is ignored and it is replaced by the respective estimated state. Hence, the reconfiguration for sensor fault is carried out.

This method is also capable of handling multiple sensor faults, i.e., if two sensors become faulty, then fault is detected in the faulty sensors and reconfiguration is provided by replacing measurements of faulty sensors with estimated values.

3. CONTROL SURFACE FAULT DETECTION AND RECONFIGURATION

3.1 Extended Kalman Filter Implementation

Here an actuator surface fault detection algorithm-based on EKF has been used for estimation of elements of control matrix[6].

3.1.1 Identification of Control Distribution Matrix

Figure 4 shows the block diagram structure of the identification algorithm using EKF.

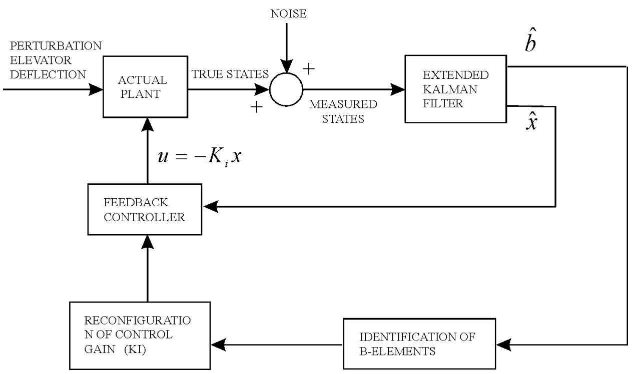

Figure 4. Schematic for control surface fault detection and reconfiguration using EKF.

The $b_{i,j}$ $(i=1,n\ ;\ j=1,m)$ elements of the control distribution matrix *B* are identified using EKF to detect the actuator surface faults.

For this purpose, the state vector *x* is augmented as follows:

$$x_a = [\, x_1, x_2, ..., x_n, b_{11}, b_{12}, ..., b_{ij}, ..., b_{nm}\,] \tag{14}$$

and the augmented dynamic system can be represented by

$$x_a(k+1) = \tilde{F}(k+1,k)\, x_a(k) + \tilde{\Gamma}(k+1,k)\, w(k) \tag{15}$$

and the measurement Eqn turns out to be

$$\tilde{z}(k) = \tilde{H}(k)\, x_a(k) + v(k) \tag{16}$$

where, x_a is an (*n*+ *nm*) dimensional augmented system state vector, $\tilde{F}(K, K+1)$ is a (n + nm) by (*n* +*nm*) augmented system matrix, $\tilde{\Gamma}(K, K+1)$ is an *(n + nm)* by *(n + nm)* augmented perturbation noise transition matrix, $\tilde{z}(k)$ is an *s* by *nm* dimensional system measurement matrix. The matrix $\tilde{F}$ is the discrete form of the system matrix $\tilde{A}$ where,

$$\tilde{A} = \begin{bmatrix} A_{nxn} & \begin{bmatrix} u_1 ....u_m\ 0 .... 0 \\ 0 ......0\ \ u_1 ....u_m\ 0 .... 0 \\ .... \\ 0 .... 0\ \ .... 0\ u_1 ......u_m \end{bmatrix} \\ 0_{nm\,x\,n} & I_{nm\,x\,nm} \end{bmatrix}$$

The discrete form of $\widetilde{A}$ is computed as

$$\widetilde{F} = e^{\widetilde{A}T} \quad (17)$$

The EKF estimation algorithm is as follows:
State and covariance propagation:

$$\widetilde{x}_a(k, k-1) = \int \widetilde{A}\hat{x}_a(k-1, k-1) \quad (18)$$

$$P(k,k-1) = \widetilde{F}(k,k-1)P(k-1,k-1)\widetilde{F}^T(k,k-1) + \widetilde{\Gamma}(k,k-1)Q(k-1)\widetilde{\Gamma}^T(k,k-1) \quad (19)$$

Measurement update:

$$\widetilde{x}_a(k, k-1) = \int \widetilde{A}\hat{x}_a(k-1, k-1)$$

$$\hat{x}_a(k,k) = \widetilde{x}_a(k,k-1) + K(k)\gamma(k) \quad (20)$$

where, $\gamma(k) = \widetilde{z}(k) - \widetilde{H}(k)\widetilde{x}_a(k,k-1)$ is innovation sequence

$$K(k) = P(k,k-1)\widetilde{H}(k)P(k,k-1)\widetilde{H}^T(k) + R(k)]^{-1} \quad (21)$$

$$P(k/k) = [I - K(k)\widetilde{H}(k)]P(k/k-1) \quad (22)$$

3.1.2 Control Reconfiguration Algorithm

State feedback discussed Franklin[7], *et al.* can be used to improve the stability properties of the control system as follows:

Consider the state equation:

$$\dot{x} = Ax + Bu \quad (23)$$

Then the state feedback control law is given by

$$u = -Kx \quad (24)$$

where, $K = [k_1, k_2\ k_3\ k_4\ k_5]$ is a constant state feedback gain matrix. If this state feedback control law is connected to the Eqn (23), the close-loop system is described by the state equation:

$$\dot{x} = (A - BK)x \quad (25)$$

Once the feedback control law is designed for fault-free system, then under surface fault condition, control reconfiguration is realised using pseudo-inverse technique. Pseudo-inverse technique can be stated as follows:

Let the dynamics of the closed system be

$$\dot{x}_0 = (A - B_0 K_0)x_0 \quad (26)$$

After an actuator surface fault occurs, the dynamics may be represented as:

$$\dot{x}_i = (A - B_i K_i)x_i \quad (27)$$

To ensure the closed-loop dynamics is the same as before, the following condition must be satisfied:

$$B_0 K_0 = B_i K_i \quad (28)$$

where, B_0 is unimpaired control distribution matrix, K_0 is gain matrix for unimpaired system and B_i is estimated (EKF) impaired control distribution matrix and K_i is gain matrix for impaired system.

The gain matrix for impaired system is obtained as

$$K_i = B_i^{\#} B_0 K_0 \quad (29)$$

where, the matrix $B_i^{\#}$ is the pseudo inverse of the matrix B_i. New gain matrix for surface fault is then computed from Eqn (29). The state feedback is then provided for reconfiguration by computing the new control input according to Eqn (24)

3.2 Control Surface Fault Detection and Reconfiguration using Fuzzy Logic

Figure 5 shows the schematic of actuator fault detection and reconfiguration. Perturbation elevator deflection is the input to the actual plant (plant with actuator fault). The errors or the difference in output states of nominal plant and those of actual faulty plant are used as the inputs to the fuzzy module. Correction factor for loss of effectiveness of control surface is used as the desired output. Seven triangular membership functions (with two overlapping membership functions) are constructed for both inputs. To partition the control output, i.e., correction factor, seven triangular membership functions (two membership functions overlapping) are constructed on its universe of discourse (UOD) in the range of 0 to 1.

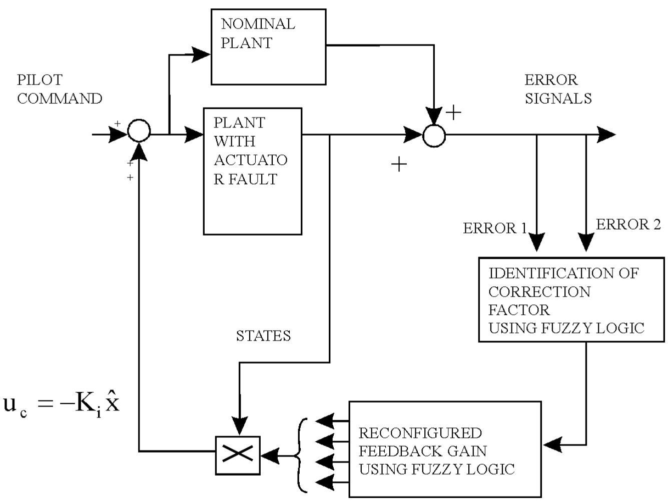

Figure 5. System identification and reconfiguration using state feedback.

To determine the factor of effectiveness of control surface, in case of loss of effectiveness of control surface or surface fault, errors are computed for different factors of effectiveness ranging from 0 to 1.

Membership functions used for error inputs e_1 and e_2 are as shown in Figs 6 and 7.

Then the errors in the first and second channels, i.e., errors in *u* and *w* states denoted as e_1 and e_2 respectively are used as inputs to fuzzy model in determining the factor of effectiveness. Since it is found that the errors in *u* and *w* are comparatively much larger and sufficient to estimate the factor of effectiveness of the control surface; only these two errors are used in computation.

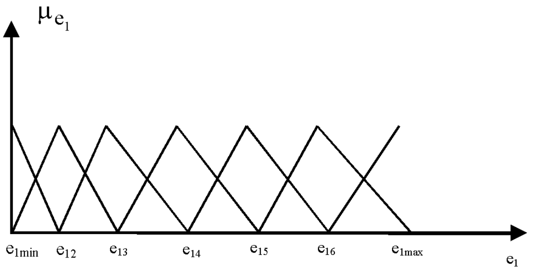

Figure 6. Membership functions for error e_2 (error for w).

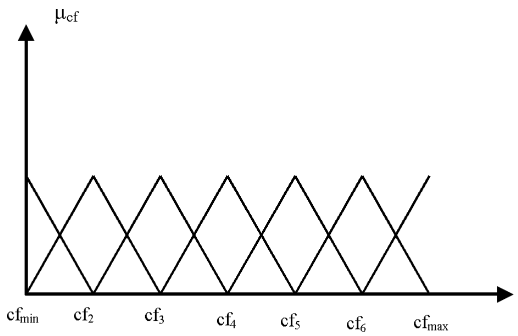

Figure 7. Membership functions for output correction factor (cf).

Inference rules are constructed in 7x7 FAM table, shown in table 3. The entries in the table 3 are the control actions or reference points of membership functions of the output corresponding to particular rule. Using the rules as expressed in FAM table, simulation of the problem is conducted for fuzzy operations. Each simulation cycle will result in membership function for the two input variables. From the FAM table, output contributed from each rule is inferred. The basic function of the inference engine is to compute the overall value of the output variable-based on the individual contributions of each rule in the rule base.

Table 3. FAM Table for correction factor (cf)

	e_2^1	e_2^2	e_2^3	e_2^4	e_2^5	e_2^6	e_2^7
e_1^1	cf^4	cf^4	cf^4	cf^4	cf^4	cf^2	cf^1
e_1^2	cf^5	cf^5	cf^4	cf^4	cf^4	cf^2	cf^2
e_1^3	cf^6	cf^6	cf^4	cf^4	cf^3	cf^3	cf^2
e_1^4	cf^6	cf^4	cf^4	cf^4	cf^4	cf^3	cf^2
e_1^5	cf^6	cf^5	cf^4	cf^4	cf^4	cf^3	cf^2
e_1^6	cf^6	cf^4	cf^4	cf^4	cf^4	cf^3	cf^2
e_1^7	cf^5	cf^4	cf^4	cf^4	cf^4	cf^1	cf^1

Ross[9] has explained each such individual contribution represents the value of the output variable as computed by a single rule. In this, first the degree of match between the crisp inputs and fuzzy sets describing the meaning of the rule antecedent is computed for each rule using the triangular norm (*T*-norm): intersection or algebraic product as follows:

$$\mu_A(u) = \mu_{ant}(e_1, e_2)$$

$$\mu_{ant}(e_1, e_2) = \min(\mu(e_1), \mu(e_2)) \text{ intersection } T\text{-norm} \quad (30)$$

$$\mu_{ant}(e_1, e_2) = \mu(e_1).\mu(e_2) \text{ algebraic product } T\text{-norm} \quad (31)$$

Then, based on this degree of match, the clipped fuzzy set representing the value of the output variable is determined via one of the inference methods. As a sensitivity study, different implication methods are used in estimating the factor of effectiveness or correction factor and the results obtained are compared.

3.3 Comparison with Different Implication Methods

The different implication methods used for comparison are:

- Mamdani's minimum implication
- Larsen's product implication
- Bounded difference and
- Drastic product or intersection implication[4,10]

1. Mamdani's minimum implication

With respect to fuzzy control, this is the most important implication. Its definition is based on the intersection operation. The relation from conjunction is defined as

Rule: If A then B = $A \rightarrow B \equiv AXB$ where AXB is Cartesian product which is given by:

$$\begin{aligned} A \rightarrow B &= AXB \\ &= \sum_{UXV} (\mu_A(u) \cap \mu_B(v))/(u,v) \\ &= \sum_{UXV} min(\mu_A(u), \mu_B(v))/(u,v) \end{aligned} \quad (32)$$

where, $u \in U$ & $v \in V$

2. Larsen's product or algebraic product implication

This is another commonly used implication method. It is given by

$$\begin{aligned} A \rightarrow B &= AXB \\ &= \sum_{UXV} (\mu_A(u) \bullet \mu_B(v))/(u,v) \end{aligned} \quad (33)$$

3. Bounded difference or Bounded product implication:

This is defined by the relation:

$$\begin{aligned} A \rightarrow B &= AXB \\ &= \max(0, (\mu_A(u) + \mu_B(v) - 1)) \end{aligned} \quad (34)$$

4. Drastic intersection implication**:**

This relation is given by

$$A \rightarrow B = AXB$$
$$= \begin{cases} \mu_A(u)/(u,v) & if \ \mu_B(v) = 1 \\ \mu_B(v)/(u,v) & if \ \mu_A(u) = 1 \\ 0/(u,v) & otherwise \end{cases} \quad (35)$$

The fuzzy outputs recommended by each rule are aggregated. Defuzzification is then applied to get crisp value for output or correction factor. Centroid method is used in determining the defuzzified value of output. This is given by the expression

Table 4 gives the outcome of T-norm operator for rule antecedent, defined in Eqn (31) and Eqn (32) for i^{th} rule,

$$crisp\,value\,of\,output = \frac{\sum value\ of\ output\,member \times \inf erred\ area\,of\ output\ \ member}{\sum \inf erred\,area\,of\ output\,member} \tag{36}$$

control decision due to different implication methods and area of the clipped output fuzzy set[3,4]. Once the correction factor (output) is determined then actual B-matrix elements under surface fault condition are determined and subsequently new control gain is determined by another fuzzy module. For this module, the input is the estimated correction factor and output is the feedback control gain matrix, K_i. The elements of matrix K_i where $K_i = [K_1\, K_2\, K_3\, K_4]$ are defined in their respective universe of discourses and partitioned into seven fuzzy partitions. e.g. UOD $K_1 = [K_1^1\ \ K_1^2\ \ K_1^3\ K_1^4\ K_1^5\ K_1^6\ K_1^7]$. The inference rules for determining new control gain matrix elements are as given in Tables 5 and 6.

From the new computed control gain matrix, reconfiguration is provided using state feedback, i.e., $u = -K_i\, x_i$ where , x_i = states under fault conditions. Hence, the control law is reconfigured for actuator (surface) fault. Using the new control law, the reconfiguration is carried out.

4. RESULTS AND DISCUSSIONS

In the simulation, the longitudinal dynamics of a Delta-4 aircraft is considered[11]. The state space matrices for the given model are

$$A = \begin{bmatrix} -0.033 & 0.0001 & 0.0 & -9.81 \\ 0.168 & -0.367 & 260 & 0.0 \\ 0.005 & -0.0064 & -0.55 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.45 \\ -5.18 \\ -0.91 \\ 0.00 \end{bmatrix},$$

C = I (4x4), Γ = I(4x4)

In this case noise measurement vector, is considered as:v = [0.36*randn; 0.3*randn; 0.15*randn; 0.1*randn]

4.1 For Sensor Fault Detection and Reconfiguration

In the simulations, sensor faults are modelled by adding the appropriate sensor measurement fixed bias shifts. To introduce sensor fault, a bias of 4 is added at iteration 300, changing the mean value of the innovation sequence in the first measurement channel as follows:

v = [1+0.36*randn; 0.3*randn; 0.15*randn; 0.1*randn]

For multiple sensor faults, fixed bias is added in first and third measurement channels as

v = [1+0.36*randn; 0.3*randn; 1+ 0.15*randn; 0.1*randn]

For the simulation, the sampling time is chosen as T=0.01 s and the number of iterations is chosen as N=1000. Figure 8 shows true (non-faulty), measured (with fault) and estimated reconfigured states. It can be seen that fault is detected in the first measurement channel at 301st iteration (3 s) when the difference between measured and true values exceeds pre-computed threshold bounds. The fault detection automatically locates the faulty channel and reconfiguration is provided by replacing the faulty measurement with estimated value. Figure 9 shows the residuals in all channels and the respective threshold bounds when the fault is in a single sensor, i.e., in 1st channel with and without reconfiguration using fuzzy logic.

Table 4. Fuzzy implications[3,4]

Fuzzy reasoning method	**Control decision due to i^{th} rule** $\mu_{Ai}(u) = \mu_{ant_i}(e_1, e_2)$	**Area of the clipped output fuzzy set**
Mamdani's minimum(MM)	$\mu_{A_i}(u) \wedge \mu_{B_i}(v)$	$H\mu_{A_i}(2-\mu_{A_i})$
Larsen' product (LP)	$\mu_{A_i}(u)\mu_{B_i}(v)$	$H\mu_{A_i}$
Bounded product(BP)	$\max(0, \mu_{A_i}(u) + \mu_{B_i}(v) - 1)$	$H\mu_{A_i}\wedge 2$
Drastic product (DP)	$\mu_{A_i}(u)$ *if* $\mu_{B_i}(v) = 1$ $\mu_{B_i}(v)$ *if* $\mu_{A_i}(u) = 1$ 0 *otherwise*	H

Table 5. Inference rules for k_1 (K_1 and K_2).

input cf	output k_1
cf^1	k_1^7
cf^2	k_1^6
cf^3	k_1^5
cf^4	k_1^4
cf^5	k_1^3
cf^6	k_1^2
cf^7	k_1^1

Table 6. Inference rules for k_2 (K_3 and K_4)

input cf	output k_2
cf^1	k_2^1
cf^2	k_2^2
cf^3	k_2^3
cf^4	k_2^4
cf^5	k_2^5
cf^6	k_2^6
cf^7	k_2^7

Figure 10 shows true (non-faulty), measured (with fault) and estimated reconfigured states for faults. Figure 11 shows the residuals in all channels and threshold bounds when the fault is in multi sensors, i.e., in 1st and 3rd channels with and without reconfiguration using fuzzy logic. It can be seen that fault is detected in the first and third measurement channels at 301st iteration (fault is introduced at the same time in both the channels) when the difference between measured and true values exceeds pre-computed threshold bounds for respective measurement channels. From the plots, it is seen that detection and reconfiguration of sensor fault are effectively provided using fuzzy logic. It is also seen from Fig. 11 that detection and reconfiguration can be effectively provided for multi-sensor faults using fuzzy logic without additional computational burden. Figure 12 shows the errors between the true and measured states with and without reconfiguration using both fuzzy logic and KF schemes for sensor fault in the first measurement channel. It can be seen that the residual with reconfiguration lies within the threshold bounds, here the states with closed-loop state feedback are shown.

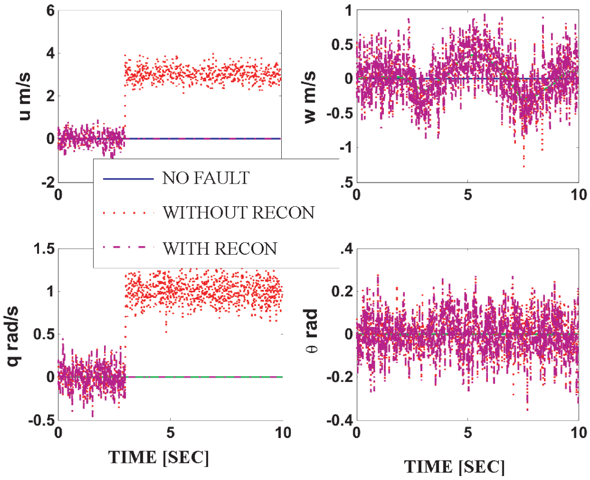

Figure 10. Comparison of true, measured, reconfigured states using fuzzy logic for faults in 1st and 3rd channels.

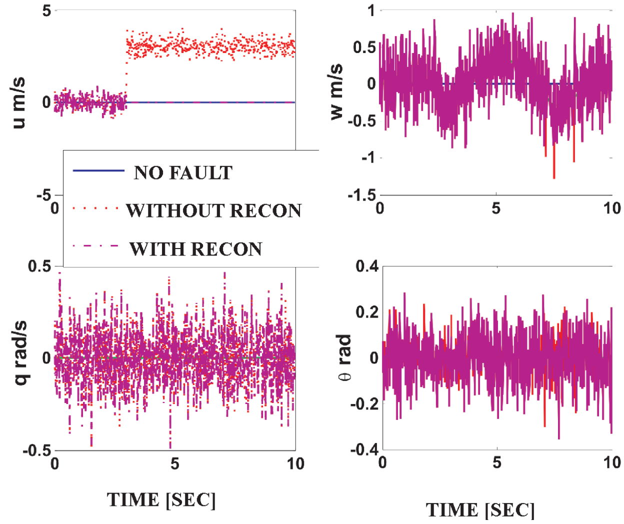

Figure 8. Comparison of true, measured, reconfigured states using fuzzy logic for fault in 1st channel.

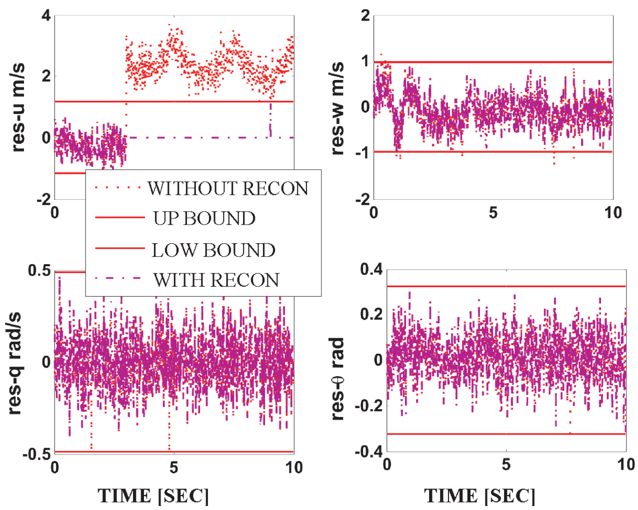

Figure 9. Residuals with/without reconfiguration using fuzzy logic for fault in 1st channel.

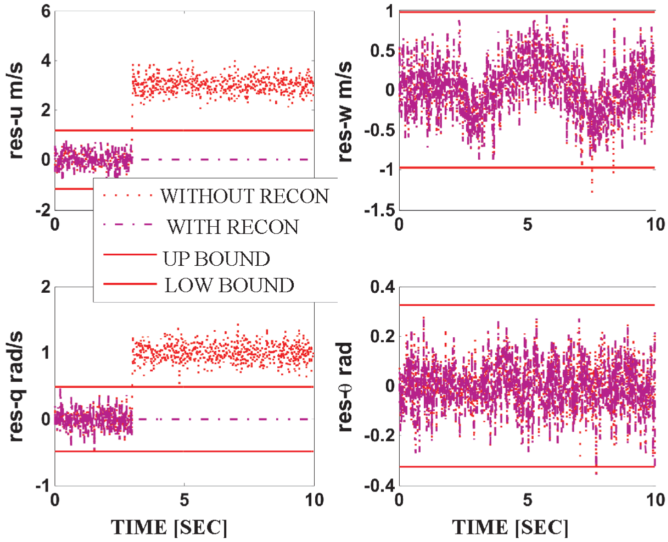

Figure 11. Residuals with/without reconfiguration using fuzzy logic for faults in 1st and 3rd channels.

4.2 For Control Surface Fault Detection and Reconfiguration

In the simulation, the longitudinal dynamics of a Delta-4 aircraft is considered. To simulate the fault, factor of effectiveness is changed to 50 per cent of the normal value (100 per cent). Hence, B-matrix elements are multiplied by 0.5. In EKF method, for the reconfiguration, feedback control gain K_0 for fault free plant is determined using LQR technique. For this, the values of LQR are chosen as: QQ (i.e., Q in LQR) = zeros(4,4) ; QQ(1) =0.01; Q(2,2) =0.00001; QQ(3,3) =0.00001; QQ(4,4) =0.00001. With this, control gain for fault free aircraft K_0 value is obtained as

K_0= [0.0 0.0887 0.0046 -0.8968 -2.1067] which is used in reconfiguration.

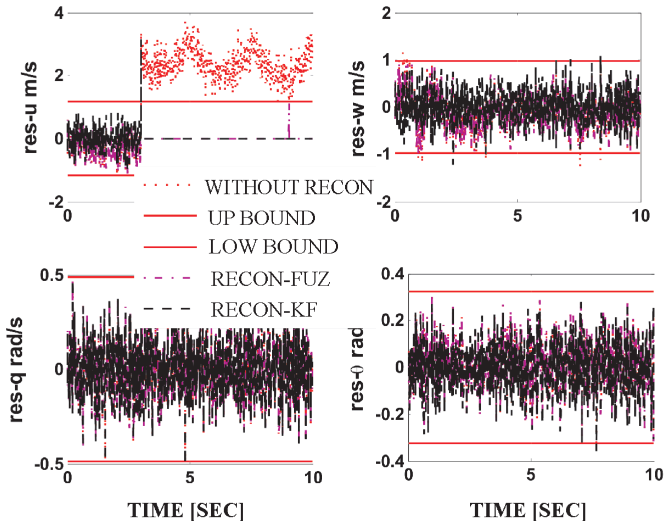

Figure 12. Residuals with/without reconfiguration using fuzzy logic and KF for faults in 1st channel.

In fuzzy logic control, the correction factor is estimated under fault condition using the errors in states for a factor of effectiveness of 50 per cent and then this is used in determining the control gain. Figures 13 to 16 show the comparison of the closed loop responses of unimpaired, impaired without reconfiguration and reconfigured impaired aircraft using fuzzy logic and KF schemes. It is observed from the plots that the reconfigured states converge to those of the unimpaired one in both the schemes.

Figure 17. shows the estimated values of control distribution matrix using model-based EKF and non-model-based fuzzy logic schemes. It is seen that the estimated parameters are close to the true values for both the schemes and the delay in estimation is noticed in fuzzy logic scheme. The delay in fuzzy logic scheme indicates that some fine tuning of membership functions and inference rules is required.

Figure 18 shows the error in estimated states for the cases with and without reconfiguration using both fuzzy logic and KF schemes. From the plots it can be interpreted that reconfigured aircraft states converge to the true states in both cases, and hence, the error between the true (unimpaired) and reconfigured states is reduced. For perturbation velocity along x-axis u, the error is less using KF; otherwise error is less in all other states using fuzzy logic.

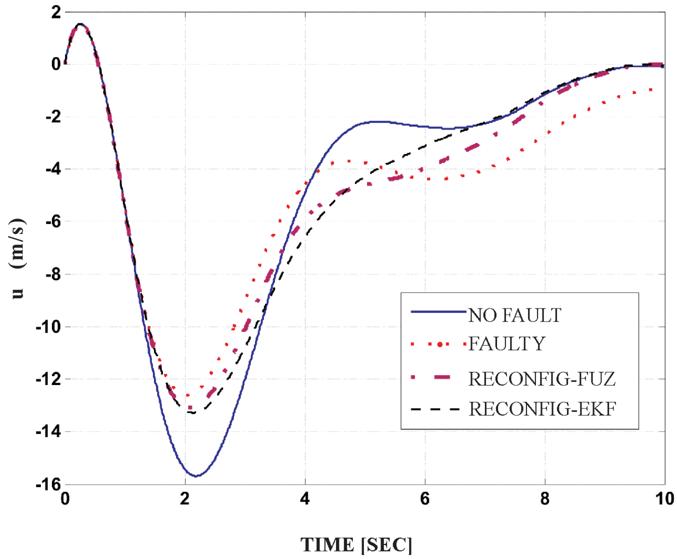

Figure 14. Perturbation velocity along Z-axis for impaired, unimpaired and reconfigured aircraft using EKF and fuzzy logic.

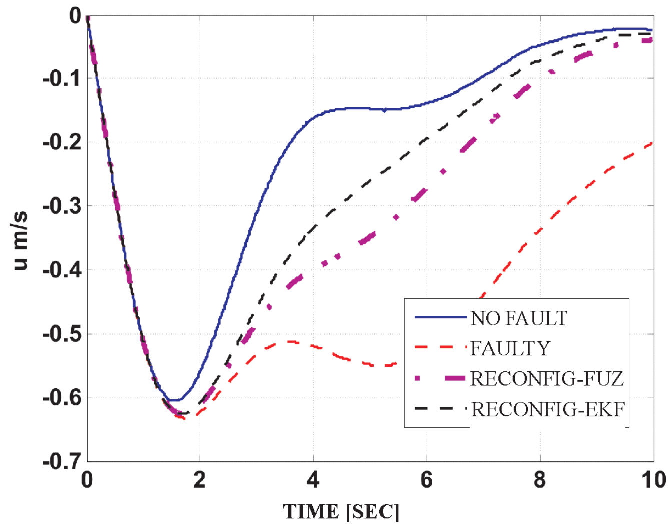

Figure 13. Perturbation velocity along X-axis for impaired, unimpaired and reconfigured aircraft using fuzzy logic and EKF.

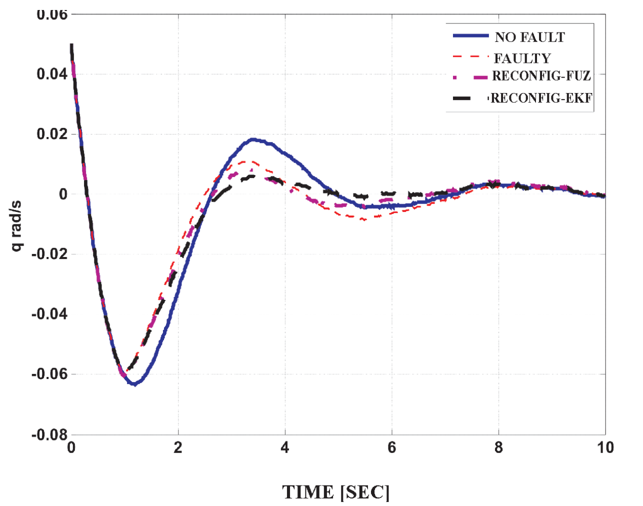

Figure 15. Perturbation pitch rate for impaired, unimpaired and reconfigured aircraft using EKF and fuzzy logic.

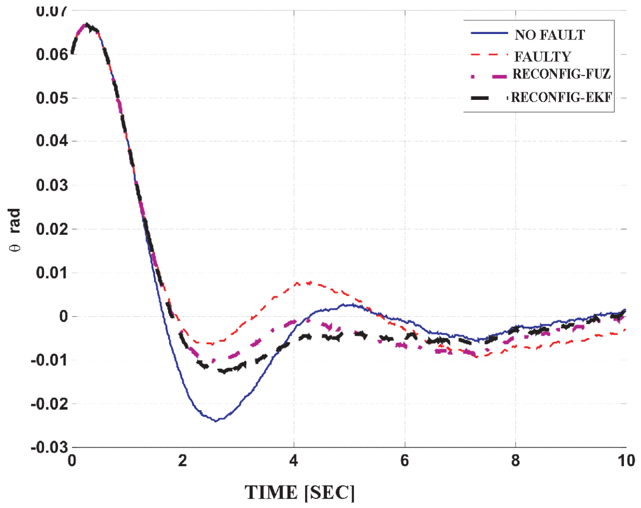

Figure 16. Perturbation pitch angle for impaired, unimpaired and reconfigured aircraft using EKF and fuzzy logic.

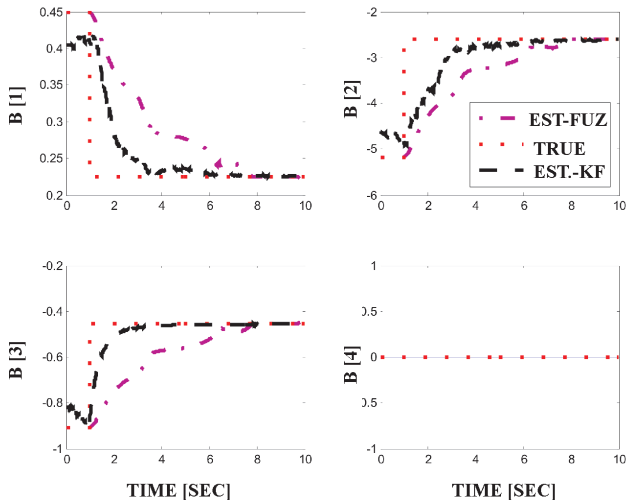

Figure 17. Actual and estimated values of B parameters.

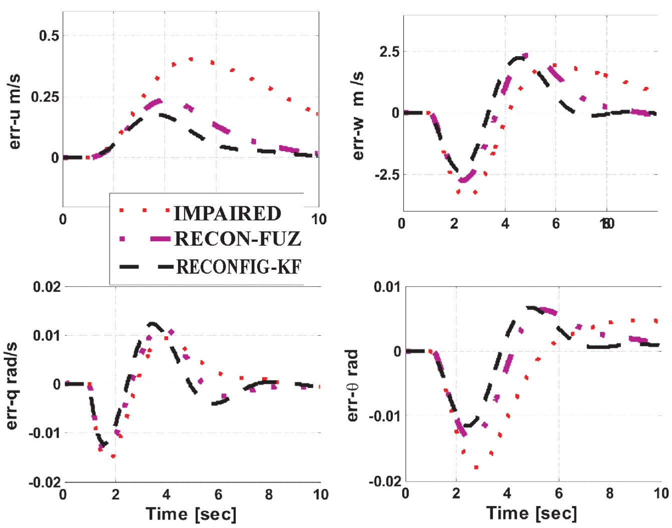

Figure 18. Errors in states with state feedback control – with and without reconfiguration using fuzzy logic and EKF

4.3 Sensitivity Study using Intersection and Algebraic Product *T*-Norms and Different Implication Methods

The results using intersection *T*-norm and different implication methods for estimation of factor of effectiveness under control surface fault are shown in Fig. 19. From the figure it can be seen that the estimation is satisfactory using all the methods. Using the estimated value of factor of effectiveness, the reconfiguration is carried out. Figure 20 shows the results using algebraic product *T*-norm and different implication methods for estimation of factor of effectiveness under control surface fault. From the figure it can be seen that the estimation is satisfactory for different implication methods using the algebraic product *T*-norm

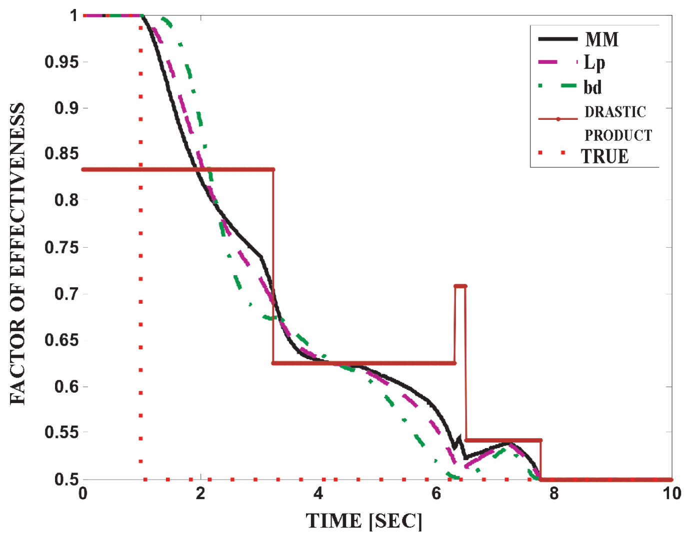

Figure 19. Comparison of estimated values of factor of effectiveness using different implication methods and using intersection *T*-norm.

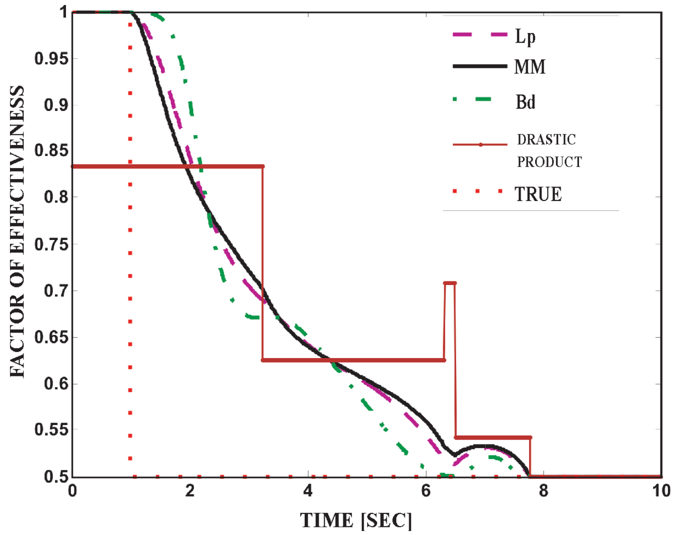

Figure 20. Comparison of estimated values of factor effectiveness using different implication methods and using of algebraic product *T*-norm.

also. Mamadani's minimum, Larsen's product, drastic product methods of fuzzy implications are in general suitable for control applications, but in the determination of correction factor under control surface fault of an aircraft, bounded difference or bounded product implication method has yielded satisfactory results along with Mamadani's minimum and Larsen's product methods whereas Drastic product method of implication shows stepwise results.

5. CONCLUSIONS

In this study, fuzzy logic is used in sensor fault detection and reconfiguration and the results are compared with those using KF. It is shown that using fuzzy logic, multi-sensor faults can also be detected and reconfiguration can be effectively provided. Model-based EKF is used in the parameter estimation under control surface fault. Fuzzy logic is used to detect the factor of effectiveness under control surface fault, and thereby, new control gain is determined. In both the schemes, state feedback is adopted to reconfigure for surface fault. In fuzzy logic, different implication methods are used for parameter estimation of control distribution matrix and a sensitivity study of performance is carried out. The effects of (sensitivity study) using intersection and algebraic *T*-norms and different implication methods are demonstrated through computer simulation. Future research work may be taken up to derive the analytical structure of the overall implementation to prove the efficacy of using any *T*-norm and any implication methods.

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