# Ergonomic Level Improving of Armoured Fighting Vehicle Crew 

N.V. Ramamurthy ${ }^{\#, *}$, B.K. Vinayagam ${ }^{@}$ and J. Roopchand ${ }^{\#}$<br>"DRDO-Combat Vehicles Research and Development Establishment, Chennai - 600 054, India<br>${ }^{\text {@ Department of Mechatronics, SRM University, Kattankulathur - } 603 \text { 203, India }}$<br>*E-mail: nvram2001@gmail.com


#### Abstract

The armoured fighting vehicle (AFV) - occupant composite system is modelled as a lumped parameter system, in this paper, wherein the 4 degrees of freedom (dof) biodynamic occupant model is integrated with 10 dof inplane AFV model including the crew seat, thus leading to the 14 dof vehicle-occupant composite model and the governing equations of motion are obtained. The composite model is subjected to idealised road input simulating the ground reaction forces. Natural frequencies and the frequency domain vibration responses of various masses of model are obtained. The natural frequency of chassis thus obtained is compared with the result established by an earlier research work, to validate the model. The study is focused on crew seat location. A 2 dof occupantseat suspension model is formulated and validated through case study. The optimised values of seat suspension parameters for ride comfort are obtained using the said model, through two methods of Invariant points theory and genetic algorithm toolbox of Matlab 2014a software. Acceleration responses of body for the current and optimised parameter values obtained illustrate that comfort of crew is improved with optimised values through minimization in the acceleration responses.


Keywords: Vibration responses; Vehicle-occupant system; Seat suspension system; Mathematical model, Optimisation algorithms; GA

## NOMENCLATURE

| M | Ch |
| :---: | :---: |
| $Y$ | Vertical displacement of c.g (m) |
| $m p$ | Passenger/driver seat mass (kg) |
| $m w 1, \ldots . . m w 7$ | Mass of road wheel + track pad distributed at 7 stations (kg) |
| Iy | Mass M.I of half/ in-plane chassis mass about pitch axis (kg.m ${ }^{2}$ ) |
| K1, ...K7 | Stiffness/ spring const. of suspension unit at stations 1-7 ( $\mathrm{N} / \mathrm{m}$ ) |
| Кр | Stiffness/ spring const. of Passenger/ driver seat ( $\mathrm{N} / \mathrm{m}$ ) |
| $C 1, \ldots C 7$ | Damping coeff. of suspension unit at stations 17 (N.s/m) |
| $C p$ | Damping coeff. of Passenger / driver seat (N.s/m) |
| $\theta$ | Pitch displacement of chassis (sprung) mass about vehicle c.g (rad.) |
| Kw1,...Kw7 | Stiffness/ spring const. of road wheel rubber + track pad at stations 1-7 (N/m) |
| $C w 1, \ldots C w 7$ | Damping coeff. of road wheel rubber + track pad at stations 1-7 ( $\mathrm{N} . \mathrm{s} / \mathrm{m}$ ) |
| Yh, Yut, Ylt \& Yth | Vertical displacement of human body parts viz. head, upper torso, lower torso \& thorax, thigh with pelvis, respectively (m) |
| $Y 1, \ldots Y 7$ | Vertical displacement of chassis mass directly above the road wheels at stations 1-7 (m) |

Received: 31 May 2017, Revised: 03 October 2017
Accepted: 09 October 2017, Online published: 18 December 2017

| $Y p$ | Vertical displacement of Passenger/ driver seat mass (m) |
| :--- | :--- |
| $Y w 1, . . Y w 7$ | Vertical displacement of wheels at road wheel stations <br> $1-7(\mathrm{~m})$ |
| $l 1, \ldots l 7$ | Distance from vertical C.L of wheels 1-7 to the c.g of <br> chassis (sprung) mass (m) |
| $a 1, \ldots a 7$ | Road input - vertical displacement of wheels 1-7 (m) <br> $X p$ |
| Horizontal distance of seat centre position from c.g of <br> chassis (sprung) mass $(\mathrm{m})$ |  |
| $g$ | Acceleration due to gravity $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| $f n$ | Natural frequency $(\mathrm{Hz})$ |
| $\omega$ | Circular frequency $(\mathrm{rad} / \mathrm{s})$ |

## 1. INTRODUCTION

Armoured fighting vehicle (AFV) is a typical tracked military vehicle, capable of running at far higher speeds. High speed tracked vehicle like AFV are generally provided with passive suspension systems using torsion bar and shock absorbers, which contribute in the attenuation of terrain vibrations. Although such tracked vehicles vary widely in size, shape and physical appearance, they share many common characteristics in track and suspension assembly. For analytical modelling purpose, components of such vehicle may be divided into two groups viz. track with suspension components and hull components. Track, hull/ bogie wheels, road wheel assemblies and suspension components fall under first group. All remaining components of vehicle are covered under the name 'Hull'. Typical AFV additionally possess a
rotating top part known as 'Turret'. Ride dynamics of off-road vehicles has drawn extensive research and development efforts to improve suspension system to achieve comfort of vehicle operator, considering the adverse effects of terrain induced vibrations ${ }^{1}$. However in this work, a cost effective method of locally modifying the seat suspension system by optimising its parameter values, to achieve the same goal of attaining comfort of vehicle operator is discussed.

Fatigue and safety of the crew of an off-road heavy vehicle like AFV is very well dominated by the vibration environment during the riding of such vehicle over cross-country terrains. Earlier studies revealed that both the wheeled and tracked offroad heavy vehicles were subjected to low frequency and high amplitude vibration environment during the riding. It was also found that the continuous exposure to such vibrations was the cause for the bodily discomfort, insufficient performance and physiological damage of the operators of such heavy vehicles. Ride characteristics of such ground vehicles are evaluated using their analytical models through computer simulation. This avoids the need to resort to repeated testing involving expense and time consumption. However it should be kept in view that computer simulations are beneficial, if and only if the vehicles analytical model accurately reflect the vehicles behaviour during its running ${ }^{1}$. Accordingly an attempt is made here to include the detailed model of AFV.

It should be kept in mind that vehicle vibration not only causes just one part of the body but it causes whole body of the operator to vibrate ${ }^{2}$. When the exposure time of the operator to vibration exceeds the standard recommended and set by ISO (ISO 2631-1, 1997) ${ }^{3}$, harmful effect of whole - body vibration will be experienced. Now-a-days, people are more concerned about the vibrations to seek better and comfortable environment. Mostly the interaction between the road and vehicle only leads to vibrations which the vehicle crew is exposed to, with the increase in his travelling activity. Discomfort, fatigue and even injury are the possible effects of vibration. Specifically in the seated posture, vehicle occupant is more sensitive to whole body vibrations. This is the reason for the bio-dynamics of seated vehicle occupant to become the topic of interest since long. Also varieties of mathematical models were derived for analytical purposes. Experimental method for estimating the discomfort and fatigue of vehicle occupant is a time consuming process involving lot of efforts. Alternatively bio-mechanical models are much useful to predict the vibration response of bodily parts of vehicle occupant in a vibration environment. Thus the dynamic characteristics of vehicle occupant can be estimated through simulation of human mechanical models. In common, mathematical models are found to be suitable for bio-dynamic response studies, as they describe the vehicle occupant in the best way under vertical whole body vibration conditions. Lumped parameter modelling (LPM), finite element modelling (FEM) and multi-body dynamic modelling (MBDM) are few groups of modelling techniques. Considering human body as several concentrated masses interconnected by springs and dashpots is the approach followed under LPM method. Analysis being simple and the experimental validation task being easier are the common advantages of this method. But their application is restricted to one-directional analysis
and the low frequency problems of vibration ${ }^{4}$.
However, LPM still remains as one of the most popular methods used by researchers as its validity is found to be much suitable for the analysis in the low frequency range i.e. below $100 \mathrm{~Hz}^{5}$. Also LPM is found to be one of the most popular analytical methods in the study of biodynamic responses of seated human subjects, though it is limited to one directional analysis. However as the vertical vibration exposure of driver is our main concern ${ }^{6}$, LPM is suited for the current problem.

Also reports reveal that an off-road vehicle is subjected to a ride vibration environment of low frequency and large amplitude ${ }^{1}$. Summarising the above facts, it may be justified that LPM method is best suited for the current problem, wherein we focus our study on low frequency vertical vibration of vehicle occupant.

Scientists and engineers pay more attention now-adays, on the study of comfortness of driver and occupants of vehicle, because of the wide spread development of variety of kinds of vehicles. Both the security and comfort level of passengers are much dominated by the suspension system, which is an important device in any type of vehicle. While taking a comparative look on the conventional passive and the two modern suspension systems viz. active and semiactive suspensions, it may be understood that the later i.e nonconventional system suffer from few major short comings like consumption of large energy, bad reliability, etc. Hence these two modern systems are not used widely. On the other side, conventional passive suspension system is comparatively more reliable and economic. This makes the control performance improving study of passive suspension as meaningful, still. Vehicle body acceleration is still the most important indicator, among the various comfort level evaluation methods. An important functional device for reducing the acceleration of vehicle body is the vehicle suspension. Thus, comfort level of driver and the other occupants of the vehicle would be enhanced by selecting reasonably good suspension parameters thus achieving good isolation effects ${ }^{7}$. Thus the necessity of optimising the suspension parameters either for the whole vehicle suspension or locally for the seat suspension alone, as done in the current work is very well justified, for enhancing the comfort level of vehicle occupants.

Analysing the comfort level of occupant of typical heavy vehicle like tractor, it is apparent that the occupant of such vehicle is not equipped to tolerate the vibration levels in the 0.5 Hz to 11 Hz discomfort range for extended periods of time, which he is exposed to during the operation of such vehicle. It was observed that the earlier design works done for tractor suspension systems to isolate the occupants from the pitch and vertical vibrations were not very effective, due to the simple reason that such designs were based only on seat level measurement of vibration transmissibilities, as the vibration measurement on the suspended seat alone does not truly reflect the vibration level, which the human body parts are exposed to. Thus it is obvious that the seat suspension designs done without taking into account the combined effect of the vehicle and occupant do not yield satisfactory results. This is also supported by the work at the National Institute of Agricultural Engineering, United Kingdom, who stressed that a mechanical
simulation of human body characteristics together with the seat is necessary ${ }^{8}$.

The task of modelling the occupant and the tractor together in the form of lumped mass system interconnected by springs and dashpots was carried out by Patil and Palanichamy ${ }^{8}$, based on the above concept. Said composite model was subjected to sinusoidal i.e idealised field or road profile vibration at the tire contact points. It is pertinent to mention here that for vibration elimination, seating system designs of on-road as well as off-road vehicles are optimised using mathematical models of vehicle occupants through the estimation and prediction of vibration response of vehicle occupant ${ }^{4}$. Accordingly said researchers found out the resulted responses of various body parts through computer simulation which were further studied for the selection of new seat suspension parameters i.e optimised parameter values so that the intensity of vehicle occupant vibration was reduced in the $0.5 \mathrm{~Hz}-11 \mathrm{~Hz}$ harmful frequency range to a minimum level ${ }^{8}$.

The methodology followed by the above researchers in their work for the wheeled off-road heavy vehicle viz. tractor is adopted in the present work for the tracked off-road heavy vehicle viz. Armoured fighting vehicle (AFV), to arrive at the ideal parameter values for the crew seat suspension system using the 14 dof vehicle - occupant composite model. However, the suspension parameters giving minimum vibration response i.e the optimised values were obtained by the above researchers ${ }^{8}$ in their work, by varying the parameters involved in their study i.e. by the conventional trial and error method. In the said conventional method, values of design variables are iteratively changed and the system is reanalysed until acceptable design criteria are achieved. This is both time-consuming and tedious. But in the present work, modern optimisation techniques are used to arrive at the ideal suspension parameter values of vehicle.

Limited works are reported in the literature considering off-road vehicle-occupant composite model, that too, not relating to AFV applications. Ramamurthy and Patil ${ }^{9}$ developed a seat suspension system for the use of AFV and assessed the improvement achieved in crew comfort, through case study. However the parameter values used in the said seat design were adopted from the results of earlier research works carried out for optimising the parameter values of crew seat suspension of other heavy vehicle viz. tractor.

Rakheja ${ }^{10}$, et al. made an analytical experimental investigation of the Driver-seat suspension system, for the ride vibration environment of off-road vehicles, but the work was not specific to the AFV environment. Dhir and Sankar ${ }^{1}$ carried out a simulation study with field validation on the ride dynamics of high speed tracked vehicles, but the study was exclusively done for a conventional military armoured personnel carrier (APC) and not exactly for the AFV like military tank, as done in the present work.

Shirahatt ${ }^{11}$, et al. made a study on optimal design of passenger car suspension for ride and road holding, but the mathematical model covered the chassis with the passenger seat only and not the body segments of vehicle occupant. Singh $^{4}$, et al. have done the modal analysis of human body vibration model for Indian subjects under sitting posture,
wherein the vehicle model has not been included in the study and also un-damped bio-dynamic model only has been considered, thus ignoring the damping characteristics of connective tissues between various body segments.

Abbas ${ }^{6}$, et al. arrived at an optimal seat design using genetic algorithm, wherein 4 dof biodynamic model interfaced with an on-road vehicle model is considered for study. Thus it does not discuss about off-road vehicle. Said 4 dof biodynamic model ${ }^{6}$ is adopted in the present work. Also the widely accepted in-plane model of AFV, proposed in AMC Pamphlet ${ }^{12}$ has been adopted by modifying the number of wheel stations and both models were integrated for use in the present study. The natural frequencies and the frequency domain vibration responses of various masses of the vehicle-occupant composite model are obtained by subjecting the model to idealised road inputs simulating the ground reaction forces. Matlab 2014 a software package has been used in these simulations. Natural frequency of major mass segment viz. the chassis thus obtained is compared with the value established by an earlier research work, to validate the vehicle-occupant model.
$\mathrm{Liu}^{7}$, et al., presented the Invariant Points Theory, wherein the authors have arrived at the invariant equation of suspension from the differential equations of a 2 dof quarter vehicle suspension model and illustrated that there are no suspension parameters in the said invariant equation demonstrating that the vehicle suspensions are all under the control of the invariant equation. Same concept is adopted in the present work for optimising the seat suspension parameters.

In the current work, first a vehicle-occupant composite model is formulated and output values for specified road input forces applied to the model, are obtained through simulation and analysis study. The results of such study are compared with those obtained in an earlier research, to validate the formulated model. Then the study is focused on driver seat location and the optimal values of seat suspension parameters are obtained by visualising a 2 dof occupant-seat suspension model. The formulated model is validated using driver seat level vibration data acquired through case study, before using the model to obtain optimal seat suspension parameter values. Then said optimal values for ride comfort are obtained, using the two methods of Invariant points theory proposed by $\mathrm{Liu}^{7}$, et al. involving direct mathematical formula and also G.A optimisation algorithm. Driver seat level acceleration graphs obtained for old and optimal parameter values are given to demonstrate the improvement achieved in ride comfort, with the new optimal seat suspension parameter values. Thus, one purpose of the current work is to derive, first, a detailed i.e 14 dof vehicle - occupant (man - machine) composite model for the use of future researchers in this area and validate the said model. The other purpose of the current work is to obtain optimal parameter values for the driver seat suspension system using a simplified model duly validated, to improve the ride comfort i.e Ergonomic level of AFV crew/ driver in a cost effective manner.

## 2. MATHEMATICAL MODEL FORMULATIONS

### 2.1 Proposed Model

This section is devoted to the mathematical modelling
of proposed system i.e. the biodynamic lumped human linear model coupled with the in-plane model of AFV. As mentioned earlier, the 4 dof biodynamic model proposed by Abbas ${ }^{6}$, et al. wherein the human body was constructed with four separate mass segments interconnected by four sets of springs \& dampers with the arms and legs combined with the upper torso \& thigh respectively, finally amounting to a total human mass of 60.67 kg ., is adopted here for use as occupant model but with slight changes in the notations. With the modified notations, the four masses represent following body segments: head and neck ( $m h$ ), upper torso ( $m u t$ ), lower torso ( $m l t$ ) and thigh with and pelvis ( $m t h$ ).The stiffness and damping properties of thigh with pelvis are $K t h \& C t h$, those of lower torso are Klt \& Clt, upper torso are Kut,Ktth \& Cut, Ctth and those of head are $K h \& C h . Y h, Y u t, Y l t \& Y t h$ are the vertical displacements of body parts viz. head, upper torso, lower torso, and thorax, thigh with pelvis about the c.g of chassis (sprung) mass. The biomechanical parameters of the model are listed in Table 1.

In-plane model of AFV of 10 dof duly integrated with the seat and biodynamic model is illustrated in Fig. 1, thus making
Table 1. The biomechanical parameters of occupant model, as adopted from Abbas model ${ }^{6}$

| Mass (kg) | Damping coeff. (N.s/m) | Spring constant (N/m) |
| :---: | :---: | :---: |
| $m h=4.17$ | $C h=310$ | $K h=166990$ |
| $m u t=15$ | $C u t=200$ | $K u t=10000$ |
| $m l t=5.5$ | $C l t=330$ | $K l t=20000$ |
| $m t h=36$ | $C t t h=909.1$ | $K t t h=144000$ |
|  | $C t h=2475$ | $K t h=49340$ |

the composite model of 14 dof. In the Fig. 1, a1, ..a7 refer to the road input vertical displacement of wheels 1-7 (m), which are defined such that:

$$
\begin{aligned}
& a 1=\mathrm{A}^{\prime} \cdot \sin (\omega \mathrm{t}) ; a 2=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 1)) \\
& a 3=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 2)) ; a 4=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 3)) \\
& a 5=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 4)) ; a 6=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 5)) \\
& a 7=\mathrm{A}^{\prime} \cdot \sin (\omega(\mathrm{t}-\alpha 6))
\end{aligned}
$$

$\alpha 1, . . \alpha 7$ refer to the time lag between adjacent wheels to cross the same obstacle as explained in section 2.2.

Said composite model is subjected to sinusoidal vibrations simulating the ground reaction forces, that the AFV model would be subjected to, at its speed range, while negotiating the cross country. While deriving the governing equations of motion, pitch motion of AFV chassis, in addition to its vertical motion is included. The equation of motion of each mass consists of the inertia terms and forces exerted on the mass by the springs and dashpots due to the relative motion between the connected masses. Description of variables with respect to the in-plane vehicle model is given in Table 2 and the parameters of the AFV model obtained from literature and reports ${ }^{12-14}$ are listed in Table 3.

### 2.2 Governing Equations of Motion

The governing second order coupled ordinary differential equations of various masses of the composite model of Fig. 1 are put down as:
Head

$$
\begin{equation*}
(m h . \ddot{Y} h)+[K h .(Y h-Y u t)]+[C h .(\dot{Y} h-\dot{Y} u t)]=0 \tag{1}
\end{equation*}
$$



Figure 1. Vehicle-occupant composite model.

Table 2. Description of variables of in-plane AFV model

| Symbol | Description |
| :---: | :---: |
| M | Chassis mass for half/ in-plane model (kg) |
| Y | Vertical displacement of centre of gravity (c.g) (m) |
| $m p$ | Passenger/ driver Seat mass (kg) |
| $m w 1, \ldots . m w 7$ | Mass of road wheel + track pad distributed at 7 stations (kg) |
| Iy | Mass M.I of half/ in-plane Chassis mass about pitch axis (kg.m²) |
| K1, ...K7 | Stiffness/ spring const. of suspension unit at stations 1-7 ( $\mathrm{N} / \mathrm{m}$ ) |
| Kp | Stiffness/ spring const. of Passenger/ driver seat ( $\mathrm{N} / \mathrm{m}$ ) |
| C1, ...C7 | Damping coeff. of suspension unit at stations 1-7 $(\mathrm{N} . \mathrm{s} / \mathrm{m})$ |
| Cp | Damping coeff. of Passenger/ driver seat (N.s/m) |
| $\theta$ | Pitch displacement of chassis (sprung) mass about vehicle c.g (rad.) |
| $K w 1, \ldots K w 7$ | Stiffness/ spring const. of road wheel rubber + track pad at stations 1-7 (N/m) |
| $C w 1, \ldots C w 7$ | Damping coeff. of road wheel rubber + track pad at stations 1-7 (N.s/m) |
| Yh, Yut Ylt \& Yth | Vertical displacement of human body parts viz. head, upper torso, lower torso \& thorax thigh with pelvis, respectively (m) |
| Y1, ...Y7 | Vertical displacement of chassis mass directly above the road wheels at stations 1-7 (m) |
| $Y p$ | Vertical displacement of Passenger/driver seat mass (m) |
| Yw1,..Yw7 | Vertical displacement of wheels at road wheel stations 1-7 (m) |
| $l 1, \ldots l 7$ | Distance from vertical C.L of wheels 1-7 to the c.g of chassis (sprung) mass (m) |
| $a 1, \ldots a 7$ | Road input - vertical displacement of wheels 1-7 (m) |
| Xp | Horizontal distance of seat centre position from c.g of chassis (sprung) mass (m) |
| $g$ | Acceleration due to gravity ( $\mathrm{m} / \mathrm{s}^{2}$ ) |

Table 3. Parameters of in-plane AFV model as adopted from AMCP $^{12}$, Banerjee ${ }^{13}$, et al. \& Chandramouli ${ }^{14}$

| Parameter | Value | Parameter | Value |
| :--- | :--- | :--- | :--- |
| $M$ | 10000 kg | $C w 1, \ldots C w 7$ | $5793 \mathrm{~N} . \mathrm{s} / \mathrm{m}$ |
| $m p$ | 10 kg | $l 1$ | 2.5 m |
| $m w 1, \ldots m w 7$ | 575 kg (each) | $l 2$ | 1.7 m |
| $I y$ | $1.94 \times 10^{5}\left(\mathrm{~kg} . \mathrm{m}^{2}\right)$ | $l 3$ | 0.8 m |
| $K 1, \ldots K 7$ | $14483 \mathrm{~N} / \mathrm{m}$ | $l 4$ | 0 |
| $K p$ | $90000 \mathrm{~N} / \mathrm{m}$ | $l 5$ | 0.8 m |
| $C 1, \ldots C 7$ | $2,98,188 \mathrm{~N} . \mathrm{s} / \mathrm{m}$ | $l 6$ | 1.6 m |
| $C p$ | $400 \mathrm{~N} . \mathrm{s} / \mathrm{m}$ | $l 7$ | 2.7 m |
| $X p$ | 0 | $g$ | $9.81 \mathrm{~m} / \mathrm{s}^{2}$ |
| $K w 1, \ldots K w 7$ | $45,00,000 \mathrm{~N} / \mathrm{m}$ |  |  |

Upper Torso

$$
\begin{align*}
& (\text { mut . ̈̈ut })+[\text { Kh .(Yut-Yh) }]+[\text { Ch . }(\dot{Y} u t-\dot{Y} h)] \\
& +[K u t(Y u t-Y l t)+[\text { Cut } .(\dot{Y} u t-\dot{Y} l t)]  \tag{2}\\
& +[K t t h .(Y u t-Y t h)]+[\text { Ctth } \cdot(\dot{Y} u t-\dot{Y} t h)]=0
\end{align*}
$$

Lower Torso

$$
\begin{align*}
& (m l t . . \ddot{Y} l t)+[\text { Kut . }(Y l t-Y u t)]+[\text { Cut } .(\dot{Y} l t-\dot{Y} u t)]  \tag{3}\\
& +[K l t .(Y l t-Y t h)]+[C l t .(\dot{Y} l t-\dot{Y} t h)]=0
\end{align*}
$$

Thigh (Pelvic):

$$
\begin{align*}
& \text { (mth .Ÿth)+[Klt .(Yth-Ylt)]+} \\
& [\text { Clt .(́th- } \dot{Y} t)]+[\text { Ktth . }(Y t h-Y u t)]+  \tag{4}\\
& [\text { Ctth . }(\dot{Y} t h-\dot{Y} u t)]+\text { Kth } .(Y t h-Y p)+\text { Cth . }(\dot{Y} t h-\dot{Y} p)]=0
\end{align*}
$$

Driver Seat

$$
\begin{align*}
& (m p \cdot \ddot{Y} p)+\{K p \cdot[Y p-Y-(X p \cdot \theta)]\} \\
& +\{C p \cdot[\dot{Y} p-\dot{Y}-(X p \cdot \dot{\theta})]\}  \tag{5}\\
& +[K t h \cdot(Y p-Y t h)]+[C t h \cdot(\dot{Y} p-\dot{Y} t h)]\}=0
\end{align*}
$$

Chassis (Bounce)

$$
\begin{align*}
& M . \ddot{Y}+[K 1 .(Y+l 1 . \theta-Y w 1)]+[C 1 .(\dot{Y}+l 1 . \dot{\theta}-\dot{Y} w 1)] \\
& +[K 2 .(Y+l 2 . \theta-Y w 2)]+[C 2 .(\dot{Y}+l 2 . \dot{\theta}-\dot{Y} w 2)] \\
& +[K 3 \cdot(Y+l 3 . \theta-Y w 3)]+[C 3 .(\dot{Y}+l 3 . \dot{\theta}-\dot{Y} w 3)] \\
& +[K 4 .(Y+l 4 . \theta-Y w 4)]+[C 4 .(\dot{Y}+l 4 . \dot{\theta}-\dot{Y} w 4)] \\
& +[K 5 .(Y-l 5 . \theta-Y w 5)]+[C 5 .(\dot{Y}-l 5 . \dot{\theta}-\dot{Y} w 5)] \\
& +[K 6 .(Y-l 6 . \theta-Y w 6)]+[C 6 .(\dot{Y}-l 6 . \dot{\theta}-\dot{Y} w 6)] \\
& +[K 7 .(Y-l 7 . \theta-Y w 7)]+[C 7 .(\dot{Y}-l 7 . \dot{\theta}-\dot{Y} w 7)] \\
& +[K p .(Y-Y p-X p . \theta)]+[C p .(\dot{Y}-\dot{Y} p-X p . \dot{\theta})]=0 \tag{6}
\end{align*}
$$

Chassis (Pitch)
$I y . \ddot{\theta}+[l 1 . K 1 .(Y+l 1 . \theta-Y w 1)]+[l 1 . C 1 .(\dot{Y}+l 1 . \dot{\theta}-\dot{Y} w 1)]$
$+[l 2 . K 2 .(Y+l 2 . \theta-Y w 2)]+[l 2 . C 2 .(\dot{Y}+l 2 . \dot{\theta}-\dot{Y} w 2)]$
$+[l 3 . K 3 .(Y+l 3 . \theta-Y w 3)]+[l 3 . C 3 .(\dot{Y}+l 3 . \dot{\theta}-\dot{Y} w 3)]$
$+[l 4 . K 4 .(Y+l 4 . \theta-Y w 4)]+[l 4 . C 4 .(\dot{Y}+l 4 . \dot{\theta}-\dot{Y} w 4)]$
$+[l 5 . K 5 .(Y-l 5 . \theta-Y w 5)]+[l 5 . C 5 .(\dot{Y}-l 5 . \dot{\theta}-\dot{Y} w 5)]$
$+[l 6 \quad . K 6 .(Y-l 6 . \theta-Y w 6)]+[l 6 . C 6 .(\dot{Y}-l 6 . \dot{\theta}-\dot{Y} w 6)]$
$+[l 7 . K 7 .(Y-l 7 . \theta-Y w 7)]+[l 7 . C 7 .(\dot{Y}-l 7 . \dot{\theta}-\dot{Y} w 7)]$
$-[K p . X p .(Y-Y p-X p . \theta)]-[C p . X p .(\dot{Y}-\dot{Y} p-X p . \dot{\theta})]=0$
Vertical displacement of wheels:
Wheel 1

$$
\begin{align*}
& (m w 1 . \ddot{Y} w 1)+[K w 1 .(Y w 1-a 1)] \\
& +[C w 1 \cdot(\dot{Y} w 1-\dot{a} 1)]+[K 1 \cdot(Y w 1-Y 1)]  \tag{8}\\
& +[C 1 \cdot(\dot{Y} w 1-\dot{Y} 1)]=K w 1 \cdot A \cdot \sin \omega t+C w 1 \cdot A \cdot \omega \cdot \cos \omega t
\end{align*}
$$

Wheel 2

$$
\begin{align*}
& (m w 2 . \ddot{Y} w 2)+[K w 2 \cdot(Y w 2-a 2)] \\
& +[C w 2 \cdot(\dot{Y} w 2-\dot{a} 2)]+[K 2 \cdot(Y w 2-Y 2)] \\
& +[C 2 \cdot(\dot{Y} w 2-\dot{Y} 2)]=K w 2 \cdot A \cdot \sin (\omega(t-\alpha 1))  \tag{9}\\
& +C w 2 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 1))
\end{align*}
$$

Wheel 3

$$
\begin{align*}
& (m w 3 \cdot \ddot{Y} w 3)+[K w 3 \cdot(Y w 3-a 3)] \\
+ & {[C w 3 \cdot(\dot{Y} w 3-\dot{a} 3)]+[K 3 \cdot(Y w 3-Y 3)] } \\
+ & {[C 3 \cdot(\dot{Y} w 3-Y \text { Y} 3)]=K w 3 \cdot A \cdot \sin (\omega(t-\alpha 2)) }  \tag{10}\\
+ & C w 3 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 2))
\end{align*}
$$

Wheel 4

$$
\begin{align*}
& (m w 4 \cdot \ddot{Y} w 4)+[K w 4 \cdot(Y w 4-a 4)]+[C w 4 \cdot(\dot{Y} w 4-\dot{a} 4)] \\
& +[K 4 \cdot(Y w 4-Y 4)]+[C 4 \cdot(\dot{Y} w 4-\dot{Y} 4)]  \tag{11}\\
& =K w 4 \cdot A \cdot \sin (\omega(t-\alpha 3))+C w 4 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 3))
\end{align*}
$$

Wheel 5

$$
m w 5 \cdot \ddot{Y} w 5+[K w 5 \cdot(Y w 5-a 5)]+[C w 5 \cdot(\dot{Y} w 5-\dot{a} 5)]
$$

$$
\begin{equation*}
+[K 5 \cdot(Y w 5-Y 5)]+[C 5 \cdot(\dot{Y} w 5-\dot{Y} 5)] \tag{12}
\end{equation*}
$$

$$
=K w 5 \cdot A \cdot \sin (\omega(t-\alpha 4))+C w 5 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 4))
$$

Wheel 6

$$
\begin{align*}
& (m w 6 \cdot \ddot{Y} w 6)+[K w 6 \cdot(Y w 6-a 6)]+[C w 6 \cdot(\dot{Y} w 6-\dot{a} 6)] \\
& +[K 6 \cdot(Y w 6-Y 6)]+[C 6 \cdot(\dot{Y} w 6-\dot{Y} 6)]  \tag{13}\\
& =K w 6 \cdot A \cdot \sin (\omega(t-\alpha 5))+C w 6 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 5))
\end{align*}
$$

Wheel 7

$$
\begin{align*}
& (m w 7 \cdot \ddot{Y} w 7)+[K w 7 \cdot(Y w 7-a 7)]+[C w 7 \cdot(\dot{Y} w 7-\dot{a} 7)] \\
& +[K 7 \cdot(Y w 7-Y 7)]+[C 7 \cdot(\dot{Y} w 7-\dot{Y} 7)]  \tag{14}\\
& =K w 7 \cdot A \cdot \sin (\omega(t-\alpha 6))+C w 7 \cdot A \cdot \omega \cdot \cos (\omega(t-\alpha 6))
\end{align*}
$$

The set of Auxiliary equations for chassis \& wheels (used while deriving their bounce and pitch equations as shown above), for small values of angle $\theta$ are:
$Y 1=Y+l 1 \cdot \sin \theta=Y+l 1 . \theta$ and $\dot{Y} 1=\dot{Y}+(l 1 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 1 \cdot \dot{\theta}$ $Y 2=Y+l 2 . \sin \theta=Y+l 2 . \theta$ and $\dot{Y} 2=\dot{Y}+(l 2 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 2 . \dot{\theta}$ $Y 3=Y+l 3 \cdot \sin \theta=Y+l 3 . \theta$ and $\dot{Y} 3=\dot{Y}+(l 3 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 3 \cdot \dot{\theta}$ $Y 4=Y+l 4 \cdot \sin \theta=Y+l 4 . \theta$ and $\dot{Y} 4=\dot{Y}+(l 4 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 4 . \dot{\theta}$ $Y 5=Y-l 5 \cdot \sin \theta=Y-l 5 . \theta$ and $\dot{Y} 5=\dot{Y}+(15 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+15 \cdot \dot{\theta}$ $Y 6=Y-l 6 . \sin \theta=Y-l 6 . \theta$ and $\dot{Y} 6=\dot{Y}+(l 6 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 6 \cdot \dot{\theta}$ $Y 7=Y-l 7 \cdot \sin \theta=Y-l 7 . \theta$ and $\quad \dot{Y} 7=\dot{Y}+(l 7 \cdot \cos \theta) \cdot \dot{\theta}=\dot{Y}+l 7 . \dot{\theta}$

Following state variables have been used:

$$
\begin{aligned}
& Y h=X 1 ; \dot{Y} h=X 2 ; Y u t=X 3 ; \dot{Y} u t=X 4 ; \\
& Y l t=X 5 ; \dot{Y} l t=X 6 ; Y t h=X 7 ; \dot{Y} t h=X 8 \\
& Y p=X 9 ; \dot{Y} p=X 10 ; Y=X 11 ; \dot{Y}=X 12 ; \\
& \dot{\theta}=X 13 ; \dot{\theta}=X 14 ; Y w 1=X 15 ; \dot{Y} w 1=X 16 ; \\
& Y w 2=X 17 ; \dot{Y} w 2=X 18 ; Y w 3=X 19 \\
& \dot{Y} w 3=X 20 ; Y w 4=X 21 ; \dot{Y} w 4=X 22 ; Y w 5=X 23 ; \\
& \dot{Y} w 5=X 24 ; Y w 6=X 25 ; \dot{Y} w 6=X 26 ; \\
& Y w 7=X 27 ; \dot{Y} w 7=X 28
\end{aligned}
$$

Substituting the above variables in Eqns (1) to (14) and writing the equations in state space representation form,

$$
\begin{equation*}
\dot{X}=A \cdot X+B \cdot Q \tag{16}
\end{equation*}
$$

where $A$ is System Matrix such that

$$
\begin{aligned}
A= & {[A 1 A 2 A 3 A 4 A 5 A 6 A 7 A 8 A 9 A 10 A 11} \\
& A 12 A 13 A 14 A 15 A 16 A 17 A 18 A 19 A 20 \\
& A 21 A 22 A 23 A 24 A 25 A 26 A 27 A 28]^{T}
\end{aligned}
$$

$X$ is State Matrix such that

$$
\begin{aligned}
X= & {[X 1 X 2 X 3 X 4 X 5 X 6 \times 7 X 8 X 9 X 10 X 11} \\
& X 12 X 13 X 14 X 15 X 16 X 17 X 18 X 19 X 20 \\
& X 21 X 22 X 23 \times 24 X 25 X 26 X 27 X 28]^{T}
\end{aligned}
$$

$B$ is Input Matrix such that,
$B=[B 1 B 2 B 3 B 4 B 5 B 6 B 7]$ wherein
$B 1=[000000000000000(\mathrm{Kwl} / \mathrm{mwl}) 000000000000]^{T}$
$B 2=[00000000000000000(K w 2 / m w 2) 0000000000]^{T}$
$B 3=[0000000000000000000(K w 3 / m w 3) 00000000]^{T}$
$B 4=[000000000000000000000(\mathrm{Kw} 4 / \mathrm{mw} 4) 000000]^{T}$
$B 5=[00000000000000000000000(K w 5 / m w 5) 0000]^{T}$
$B 6=[0000000000000000000000000(\mathrm{Kw6} / \mathrm{mw} 6) 00]^{T}$
$B 7=[000000000000000000000000000(\mathrm{Kw7} / \mathrm{mw} 7)]^{T}$
$Q$ is Input (Scalar) such that $Q=[a 1 a 2 a 3 a 4 a 5 a 6 a 7]^{T}$ where $a 1, . . a 7$ refer to the road input vertical displacement of wheels 1-7 (m), which are defined such that:

$$
\begin{aligned}
& a 1=A^{\prime} \cdot \sin (\omega t) ; a 2=A^{\prime} \cdot \sin (\omega(t-\alpha 1)) ; \\
& a 3=A^{\prime} \cdot \sin (\omega(t-\alpha 2)) ; a 4=A^{\prime} \cdot \sin (\omega(t-\alpha 3)) ; \\
& a 5=A^{\prime} \cdot \sin (\omega(t-\alpha 4)) ; a 6=A^{\prime} \cdot \sin (\omega(t-\alpha 5)) ; \\
& a 7=A^{\prime} \cdot \sin (\omega(t-\alpha 6)) ;
\end{aligned}
$$

Now for the sinusoidal road profile considered by Shirahatt ${ }^{11}$, et al., amplitude and pitch values of the sinusoidal proving ground used for case study in the current work are
implemented as amplitude $\left(A^{\prime}\right)=0.15 \mathrm{~m}$ and pitch $(\lambda)=4 \mathrm{~m}$ as illustrated below. A vehicle velocity of $\mathrm{V}=11.11 \mathrm{~m} / \mathrm{s}$. is also derived corresponding to a vehicle speed of $40 \mathrm{~km} / \mathrm{hr}$. Here, $\omega$ refers to the circular frequency of the displacement applied (rad/s) and ' $\alpha 1, \ldots . \alpha 7$ ' refer to the time lag between the concerned road wheel and first road wheel, which also is equal to the phase angle between the input displacements applied to them. Taking the typical case of movement of adjacent road wheels 1 and 2, it may be observed that they follow the same trajectory with a time delay of which may be defined, by referring to Fig. 1 as:

$$
\alpha 1=\frac{(l 1-l 2)}{V}=\frac{(2.5-1.7)}{11.11}=0.072 s
$$

Same way, the time lag between other road wheels and first road wheel may be arrived at.

Forcing circular frequency $\omega$ of the displacement applied may be used, for the above vehicle velocity as:

$$
\omega=2 . \pi \cdot f=2 . \pi \cdot\left(\frac{V}{\lambda}\right)=2 . \pi \cdot(11.11 / 4)=17.45 \mathrm{rad} / \mathrm{s}
$$

Now, for the sinusoidal road profile conditions, for the present case, as function of time and displacements of the first set of adjacent wheels 1 and 2 as function of time will become ${ }^{11}$ :

$$
\begin{aligned}
& a 1(t)=\left\{\begin{array}{c}
\frac{A^{\prime}}{2}\left(1-\cos (\omega t), \text { if } 0 \leq t \leq \frac{2 \lambda}{V}\right. \text { and } \\
0, \text { otherwise }
\end{array}\right. \\
& a 2(t)=\left\{\begin{array}{c}
\frac{A^{\prime}}{2}\left(1-\cos \left(\omega(t-\alpha 1), \text { if } 0 \leq t \leq \frac{2 \lambda}{V}\right. \text { and }\right. \\
0, \text { otherwise }
\end{array}\right.
\end{aligned}
$$

Same is applicable for the other sets of adjacent road wheels also.

Now from the governing Eqns (1) to (15), the system matrices may be defined as:

$$
\begin{aligned}
A 1= & {[0100000000000000000000000000] } \\
A 2= & {[(-K h / m h)(-C h / m h)(\text { Kh/mh })(\text { Ch } / m h)} \\
& 000000000000000000000000] \\
A 3= & {[0001000000000000000000000000] } \\
A 4= & {[(- \text { Kh / mut })(\text { Ch } / m u t)((- \text { Kh }- \text { Kut }- \text { Ktth } / m u t))} \\
& ((-C h-C u t-\text { Ctth }) / m u t)(\text { Kut } / m u t)(C u t / m u t) \\
& (\text { Ktth } / m u t)(\text { Ctth } / m u t) 00000000000000000000] \\
A 5= & {[0000010000000000000000000000] } \\
A 6= & {[00(\text { Kut /mlt })(\text { Cut } / m l t)((- \text { Kut }- \text { Klt }) / m l t)} \\
& ((-C u t-C l t) / m l t)(\text { Klt } / m l t)
\end{aligned}
$$

(Clt / mlt $) 00000000000000000000$ ]
$A 7=[0000000100000000000000000000]$
$A 8=[00($ Ktth $/ \mathrm{mth})($ Ctth $/ \mathrm{mth})(\mathrm{Klt} / \mathrm{mth})$
$($ Clt / mth $)((-K t h-K t t h-K l t) / m t h)$ ( $(-$ Cth - Ctth $-C l t) / m t h)($ Kth $/ m t h)$ (Cth/mth)000000000000000000]
$A 9=[0000000001000000000000000000]$

$$
A 10=[000000(\text { Kth } / m p)(\text { Cth } / m p)
$$

$$
((-K p-K t h / m p))((-C p-C t h) / m p)(K p / m p)
$$

$$
(C p / m p)(K p . x p / m p)
$$

$$
(C p . x p / m p) 00000000000000]
$$

$A 11=\left[\begin{array}{lllllllllllllllllllll}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$

$$
\begin{aligned}
& A 12=\left[\begin{array}{llllll}
0 & 0 & 0 & 0 & 0 & 0
\end{array} 00(K p / M)(C p / M)\right. \\
& ((-K 1-K 2-K 3-K 4-K 5-K 6-K 7-K p) / M) \\
& +((-C 1-C 2-C 3-C 4-C 5-C 6-C 7-C p) / M) \\
& +\left(-\binom{K 1 * l l+K 2 * l 2+K 3 * l 3+K 4 * l 4-}{K 5 * l 5-K 6 * l 6-K 7 * l 7-K p * X p} / M\right) \\
& +\left(-\binom{C 1 * l 1+C 2 * l 2+C 3 * l 3+C 4 * l 4-}{C 5 * l 5-C 6 * l 6-C 7 * l 7-C p * X p} / M\right) \\
& +(K 1 / M)(C 1 / M)(K 2 / M)(C 2 / M)(K 3 / M) \\
& (C 3 / M)(K 4 / M)(C 4 / M)(K 5 / M) \\
& +(C 5 / M)(K 6 / M)(C 6 / M)(K 7 / M)(C 7 / M)]
\end{aligned}
$$

$A 13=\left[\begin{array}{llllllllllllllllllllllll}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right)$

$$
\begin{aligned}
& A 14=[00000000(-K p * X p / I y)(-C p * X p / I y) \\
& +\left(\left(-\binom{K 1 * l 1+K 2 * l 2+K 3 * l 3+K 4 * l 4-}{K 5 * l 5-K 6 * l 6-K 7 * l 7-K p * X p}\right) / I y\right) \\
& +\left(\left(-\binom{C 1 * l 1+C 2 * l 2+C 3 * l 3+C 4 * l 4-}{C 5 * l 5-C 6 * l 6-C 7 * l 7-C p * X p}\right) / I y\right) \\
& +(-(K 1 * l 1 * l 1+K 2 * l 2 * l 2+K 3 * l 3 * l 3+ \\
& K 4 * l 4 * l 4+K 5 * l 5 * l 5+K 6 * l 6 * l 6 \\
& +K 7 * l 7 * l 7+K p * X p * X p) / I y) \\
& (-(C 1 * l 1 * l 1+C 2 * l 2 * l 2+C 3 * l 3 * l 3+C 4 * l 4 * l 4+ \\
& C 5 * l 5 * l 5+C 6 * l 6 * l 6+C 7 * l 7 * l 7+C p * X p * X p / I y)) \\
& (K 1 * l 1 / I y)(C 1 * l l / I y)(K 2 * l 2 / I y)(C 2 * l 2 / I y) \\
& (K 3 * l 3 / I y)(C 3 * l 3 / I y)(K 4 * l 4 / I y) \\
& (C 4 * l 4 / I y)(-(K 5 * l 5 / I y))(-C 5 * l 5 / I y) \\
& (-K 6 * l 6 / I y)(-C 6 * l 6 / I y) \\
& (-K 7 * l 7 / I y)(-C 7 * l 7 / I y)]
\end{aligned}
$$

$A 15=[0000000000000001000000000000]$

```
\(A 16=[0000000000(K 1 / m w 1)(C 1 / m w 1)\)
\((K 1 . l 1 / m w 1)(C 1 . l 1 / m w 1)((-K 1-K w 1) / m w 1)\)
((-C1-Cw1)/mw1)000000000000 ]
\(A 17=[0000000000000000010000000000]\)
\(A 18=[0000000000(\mathrm{~K} 2 / \mathrm{mw} 2)\)
        \((C 2 / m w 2)(K 2 . l 2 / m w 2)\)
        (C2.l2/mw2)00((-K2-Kw2)/mw2)
        ( \((-C 2-C w 2) / m w 2) 0000000000]\)
\(A 19=[0000000000000000000100000000]\)
\(A 20=[0000000000(K 3 / m w 3)(C 3 / m w 3)\)
        (K3.l3/mw3)(C3.l3/mw3)0000
        \(((-K 3-K w 3) / m w 3)\)
        ((-C3-Cw3)/mw3) 00000000]
\(A 21=[0000000000000000000001000000]\)
\(A 22=[0000000000(K 4 / m w 4)(C 4 / m w 4)\)
        (K4.14/mw4) (C4.14/mw4)
        \(000000((-K 4-K w 4) / m w 4)\)
        ((-C4-Cw4)/mw4)000000]
\(A 23=[0000000000000000000000010000]\)
\(A 24=[0000000000(K 5 / m w 5)(C 5 / m w 5)\)
    (-K5.15/mw5)(-C5.15/mw5)
    \(00000000((-K 5-K w 5) / m w 5)\)
    ((-C5-Cw5)/mw5) 0000]
\(A 25=[0000000000000000000000000100]\)
\(A 26=[0000000000(K 6 / m w 6)(C 6 / m w 6)\)
    (-K6.l6/mw6) (-C6.l6/mw6)
    \(0000000000((-K 6-K w 6) / m w 6)\)
    ((-C6-Cw6)/mw6) 00]
\(A 27=[0000000000000000000000000001]\)
\(A 28=[0000000000(K 7 / m w 7)(C 7 / m w 7)\)
    \((-K 7.17 / m w 7)(-C 7.17 / m w 7)\)
    \(000000000000((-K 7-K w 7) / m w 7)\)
    ((-C7-Cw7)/mw7)]
```


### 2.3 Numerical Simulation and Validation

The natural frequencies and the frequency domain vibration responses of various lumped masses of the vehicleoccupant composite model are obtained using transfer functions obtained from the state space variables formed, utilising the governing equations of motion (1) - (15), by subjecting the
model to idealised road inputs simulating the ground reaction forces using the approach suggested by Michael ${ }^{15}$. Matlab 2014 a software package has been used in these simulations. Sample vibration response graphs are given in results section. Natural frequency of major mass segment viz. the chassis thus obtained is compared with the value established by an earlier research work, as discussed in the results section, to validate the model.

## 3. OPTIMAL AFV SEAT SUSPENSION DESIGN USING INVARIANT POINTS THEORY

### 3.1 Numerical Simulations

Now the study is focused on crew seat location and the optimised values of seat suspension parameters are obtained by visualising a 2 dof occupant-seat suspension model of Fig. 2, using the two methods of Invariant points theory proposed by Liu ${ }^{7}$, et al.


Figure 2. Occupant-seat suspension model.
Method 1 of invariant points theory involves the use of direct mathematical formula. Method 2 involves the formulation of objective function, selection of lower and upper bounds of parameter values based on earlier studies and the application of genetic algorithm (G A) toolbox of optimisation App of Matlab 2014a software. G.A, which is stochastic technique search method, not only increases the probability of finding the global optimum solution but also avoids convergence to a local minimum which is a drawback of gradient based methods ${ }^{6}$. G.As are global search methods and are based on the Darwins principle of natural selection and genetic modification ${ }^{16}$. They operate with a population of possible solutions (individuals) of the optimisation problem. Also they work using three operators viz. selection, cross over and mutation ${ }^{17}$. Here, G.A is applied to search for the optimal parameters of vehicle seat suspension in order to minimise bodily vibration response of crew to achieve his best comfort during vehicle operation.

Global Optimisation Toolbox of Matlab 2014a provides functions that search for global solutions to problems which contain multiple maxima or minima. The toolbox includes solvers like genetic algorithm, pattern search, etc., The genetic algorithm is a method for solving both constrained and unconstrained optimisation problems. As mentioned above, it is based on natural selection, the process that drives biological
evolution. The algorithm repeatedly modifies a population of individual solutions. It selects individuals at random from the current population, at each step, to be parents and uses them to produce the children for the next generation. The population 'evolves' toward an optimal solution over successive generations. As mentioned earlier, it uses three main types of rules viz. selection, cross over and mutation at each step to create the next generation from the current population ${ }^{18}$.

Optimised parameter values of seat suspension for ride comfort are chosen from the results obtained at the end of optimisation process using G.A solver. To illustrate the improvement obtained in ride comfort with the optimised parameter values, acceleration response of body for the current and optimised parameter values are obtained using the above said occupant-seat suspension model. Then the results are compared.

### 3.2 Validation of Formulated 2 dof Occupant-seat Suspension Model through Case Study

Acquiring time domain data using data acquisition set up has been quite familiar since long. Use of MEMS sensor as part of data acquisition set up has been reported by Patel ${ }^{19}$, et al.. Though modern sensors like MEMS are widely functional now-a-days, use of sensors like piezo electric accelerometers are still popular due to their ruggedness as required in the current case study. Hence by using the said piezo electric accelerometer, case study has been carried out, by acquiring data on seat level vibration of the vehicle, as part of present work, to validate the formulated 2 dof Occupant-seat suspension model of Fig. 2, before using the same for carrying out the Invariant Points Theory based optimisation study.

For this, first, the acceleration responses are obtained for the model of Fig. 2 through simulation study, for the idealised sinusoidal road input of amplitude $\left(A^{\prime}\right)=0.15 \mathrm{~m}$ and pitch $(\lambda)=4 \mathrm{~m}$ for vehicle speed of $40 \mathrm{~km} / \mathrm{hr}$. Matlab code written for the purpose is used for this. Then the case study is carried out by running the AFV in the Sinusoidal test track, with the accelerometer duly mounted beneath the Crew Seat of AFV, as shown in Fig. 3. Said accelerometer is stud mounted to the installation kit welded in the required location of the vehicle. Layout of data acquisition set up is given in Fig. 4. Running of test vehicle on test track is as shown in Fig. 5.


Figure 3. Mounting of accelerometer.


Figure 4. Layout of data acquisition setup.


Figure 5. Running of test vehicle in test track.
Sensor, pre-amplifier, recorder, storage medium/ display are the components common to the data acquisition setup. Sensor's function is to detect the mechanical movement and convert it into a specific, usable electric signal. Accelerometer is the sensor used here. The required filtering, gain and cable drive capability are provided by the pre-amplifier. Spectrum analyser is the display system used in lab level analysis. In the current study, vibration data are stored in ASCII codes in time domain on to a laptop connected to the data acquisition system, by fixing a sampling rate of 100 kHz and sampling period of 100 ms . The recorded data are further processed \& analysed in the lab using Signal Processing techniques to obtain the seat level acceleration. Power for the instruments during data acquisition is provided by an inverter which is supplied with 24 V DC from battery placed on rear of the tank.

From the vibration data acquired, acceleration response at seat level is obtained, by using the Matlab code developed for this purpose. Final seat level vibration values are obtained by passing the output through a low pass filter with upper cut-off frequency as 100 Hz , based on region of interest for our study, aimed at human comfort analysis. Acceleration levels thus obtained are compared with those obtained through simulation study, to validate the optimisation model formulated, as mentioned in the results section.

### 3.3 Optimisation using Invariant Points Theory based Objective Function

For implementing the optimisation methodology based on Invariant points theory, as proposed by Liu ${ }^{7}$, et al., for the Occupant-seat suspension model of Fig. 2, the parameters are defined such that the terms $m_{s}, m_{t}, K_{s}, C_{s}$ and $K_{t}$ represent seated driver (sprung) mass, driver seat (un sprung) mass, stiffness and damping coefficients of human body segment viz. 'thigh with pelvis' and stiffness of seat suspension respectively. Also the terms $X_{s}, X_{t}$ and $X_{r}$ refer to the state variables which
stand for sprung and un-sprung mass displacements and road input respectively. For the ride comfort of the crew of AFV., desired objective is proposed as the minimisation of objective function formed by the combination of these three state variables ${ }^{7}$, et al..

### 3.3.1 Method 1

Under method 1 of the above approach, expressions for optimal suspension parameters, as arrived at by the above researchers ${ }^{7}$ are:

$$
\begin{align*}
& K s=\left(K_{t} \cdot m_{s} \cdot m_{t}\right) /\left(m_{s}+m_{t}\right)^{2} \text { and } \\
& C s=\sqrt{K_{t} \cdot m_{s}^{3} \cdot m_{t}\left(m_{s}+2 m_{t}\right) /\left(m_{s} \cdot m_{t}\right)^{4}} \tag{17}
\end{align*}
$$

With reference to Fig. 2, for a 2 dof occupant-seat suspension model, a group of nominal seat suspension parameters obtained from earlier studies are considered such that: $m_{s}=61 \mathrm{~kg}, m_{t}=10 \mathrm{~kg}, C_{s}=2475 \mathrm{Ns} / \mathrm{m}, K_{s}=49340 \mathrm{~N} / \mathrm{m}$ and $K_{t}=90000 \mathrm{~N} / \mathrm{m}$.

### 3.3.2 Method 2

Under the method 2 of Invariant points theory proposed by the authors, the objective function is selected as:

$$
\begin{equation*}
F=b_{1} Y_{1}^{2}+b_{2} Y_{2}^{2}+b_{3} Y_{3}^{2}+b_{4} Y_{4}^{2} \tag{18}
\end{equation*}
$$

where in

$$
\begin{align*}
Y_{1}= & C_{s}^{2} K_{t}+K_{s}\left(-K_{t} \cdot m_{s}+K_{s} \cdot m_{t}\right)  \tag{19}\\
Y_{2}= & C_{s}^{2} K_{t}\left(m_{s}+2 m_{t}\right)+2 K_{s} m_{t}\left(-K_{t} m_{s}+K_{s} \cdot m_{t}\right)  \tag{20}\\
Y_{3}= & -2 K_{s}^{2}\left(m_{s}+m_{t}\right)\left[-K_{t} m_{s} m_{t}+K_{s}\left(\left(m_{s}+m_{t}\right)^{2}\right)\right]  \tag{21}\\
& +C_{s}^{2} K_{t}\left[-2 K_{s}\left(m_{s}+m_{t}\right)^{2}+K_{t} m_{s}\left(m_{s}+m_{t}\right)\right] \\
Y 4= & -K_{t} m_{s} m_{t}+K_{s}\left(m_{s}+m_{t}\right)^{2} \tag{22}
\end{align*}
$$

where as $b 1, b 2, b 3$ and $b 4$ are positive weighing coefficients for the local maximum values, selecting the value of $b_{i}=1(i=1,2,3,4)$, as suggested by the authors. Now the optimal values are obtained under method 2 using optimal toolbox as suggested by the said researchers ${ }^{7}$, by solving the equations with initial values and value ranges of $K_{s}$ and $C_{s}$, as described below. G.A is the optimisation algorithm used for this purpose.

### 3.3.2.1 Optimisation based on G.A using Method 2

To obtain the optimal values using G.A following the method 2 illustrated above, first a function file has been created in Matlab 2014a to define the objective function of Eqns. (18) to (22) of method 2. Then the lower and upper bounds of suspension parameters corresponding to the same nominal values of method 1 mentioned earlier, are fixed as:

$$
\begin{align*}
& 60 \leq m_{s} \leq 62 ; 8 \leq m_{t} \leq 12 ; 1980 \leq C_{s} \leq 2970 \\
& 39472 \leq K_{s} \leq 59208 \& 72000 \leq K_{t} \leq 108000 \tag{23}
\end{align*}
$$

Now setting G.A parameters like population size of 50 , number of variables as 5 , number of generations as 5000 , fitness scaling function as 'rank', selection function as 'uniform', cross over technique as 'heuristic', probability of cross over as
0.8 and objective function accuracy of $1 \mathrm{e}^{-15}$, retaining default values wherever necessary, optimal values are obtained at $185^{\text {th }}$ iteration.

## 4. RESULTS AND DISCUSSION

### 4.1 Validation of Vehicle-occupant Composite Model using Established Results

Table 4 shows the values of natural frequencies $\left(f_{n}\right)$ of various lumped masses of the vehicle-occupant composite model for the idealised road inputs, simulating the ground reaction forces, obtained through the current mathematical modelling and simulation study. From the fact that the value of natural frequency of the major mass segment i.e Chassis mass M thus obtained ( 1.2357 Hz ), reasonably agrees with the value obtained in an earlier analysis carried out by Balamurugan ${ }^{20}$ $(1.2449 \mathrm{~Hz})$, validity of vehicle-occupant composite model formulated as part of present work, is established.

Table 4. Natural frequencies ( fn ) of various masses obtained through modelling and simulation

| Mass | $\mathbf{f n}(\mathbf{H z})$ | Mass | $\mathbf{f n}(\mathbf{H z})$ | Mass | $\mathbf{f n}(\mathbf{H z})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $m h$ | 3.9955 | $M$ | 1.2357 | $m w 5$ | 0.6227 |
| $m u t$ | 2.8060 | $m w 1$ | 0.7318 | $m w 6$ | 3.6079 |
| $m l t$ | 2.1361 | $m w 2$ | 0.0391 | $m w 7$ | 0.6227 |
| $m t h$ | 2.2460 | $m w 3$ | 0.6227 |  |  |
| $m p$ | 1.4021 | $m w 4$ | 3.6079 |  |  |

### 4.2 Frequency Domain Responses

Frequency domain vibration responses of various masses of the vehicle-occupant composite model formulated, are obtained using Matlab 2014a software, by subjecting the model to idealised road inputs at seven wheel stations, thus simulating the ground reaction forces. Figs. 6 and 7 show such graphs for selected masses viz. head and chassis (bump), respectively, with the magnitude in db . The notation followed for output graphs is such that z13 refers to vibration response of state 1 due to road input at wheel station 3, and so on. As there are 7 road input forces and 14 displacement output state variables considered in the current study, a total of 98 transfer functions are to be investigated. However due to the similarity in the input forces $1,3,5, \& 7$ and $2,4, \& 6$ of the system considered, it is observed that the transfer functions and thus the resulting response output graphs are found to be similar in certain cases, as in Figs. 6 and 7, for instance, $Z 11=Z 13=Z 15=Z 17$ and $Z 111=Z 113=Z 115=Z 117$, following the above said notation. This is also supported by the results obtained for the similar case of 3 mass system proposed by Hatch ${ }^{15}$.

### 4.3 Validation of occupant-seat Suspension Model through Case Study

Seat level acceleration levels are obtained through simulation study and case study as mentioned earlier. The results are illustrated graphically in Figs. 8(a) and 8(b), respectively. The reasonable agreement between the two results demonstrates the validity of the occupant-seat suspension model formulated in the present work.


Figure 6. Response of head to road inputs at wheel stations 1, 3,5 , and 7 .


Figure 7. Response of chassis (bump) to road.

### 4.4 Optimisation of AFV Seat Suspension Parameters through Invariant Points Theory Approach

### 4.4.1 Method 1 of Invariant Points Theory

Based on this method 1 of Invariant Points Theory, one could obtain the two crucial optimal parameter values corresponding to the nominal values as: $K s^{\prime}=10891 \mathrm{~N} / \mathrm{m}$ and $C s^{\prime}=806.94 \mathrm{Ns} / \mathrm{m}$. However, as these values differ much from the nominal values, they are not chosen as optimal values. It is decided that the applicability of method 1, i.e. application of direct mathematical formula, to the current problem needs further assessment.

### 4.4.2 Method 2 of Invariant Points Theory

Results of the optimisation study carried out using the G.A toolbox of optimisation App of Matlab 2014a are:

$$
\begin{aligned}
& m_{s}=61 \mathrm{~kg}, m_{t}=8 \mathrm{~kg}, C_{s}=1980 \mathrm{Ns} / \mathrm{m} \\
& K_{s}=39473 \mathrm{~N} / \mathrm{m} \& K_{t}=72000 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$



Figure 8. Comparison of Seat level acceleration response values obtained through modelling and simulation study with the case study results (a) Acceleration ( $\mathrm{m} / \mathbf{s}^{2}$ ) at seat level (obtained through modeling and simulation) for old parameter values (b) Acceleration ( $\mathrm{m} / \mathrm{s}^{2}$ ) at seat level (obtained through case study) for old parameter values.

To illustrate the improvement obtained in ride comfort with the optimised parameter values, acceleration responses of seat obtained for the current (Fig. 8 (a)) and optimised (Fig. 9) parameter values of the above said occupant-seat suspension model are compared. From the two acceleration values, it is obvious that with the optimised seat suspension parameter values, ride comfort of crew is improved through minimisation in the acceleration response of seat of occupant considerably.

## 5. CONCLUSIONS AND RECOMMENDATION

A mathematical model of the AFV-occupant composite system is developed, analysed by simulation study and validated. The said model is quite suitable for obtaining the vibration responses of various lumped masses of AFV as well as the occupant, which will aid in designing heavy vehicle suspension parameters based on vehicle - occupant composite model study rather than that of the simple vehicle model study,


Figure 9. Acceleration ( $\mathrm{m} / \mathrm{s}^{2}$ ) at seat level (obtained through modelling and simulation) for optimised seat suspension parameter values.
whose necessity is supported by the reports of National Institute of Agricultural Engineering, U.K ${ }^{8}$.

Then the optimised seat suspension parameter values which minimise the vibration responses of seat of crew are obtained through modern optimisation technique viz. G.A.

Thus a cost-effective solution for improving the ergonomic level of AFV crew, with simple modifications in his seat suspension, implementing the optimised seat suspension parameters, has been arrived at.

Concept behind the current optimisation work restricted to vehicle seat suspension may be extended to the whole AFV suspension system and the benefit achieved with respect to the crew comfort may be assessed, as future work.

## REFERENCES

1. Dhir, A. \& Sankar, S. Ride dynamics of high - speed tracked vehicles: Simulation with field validation. Vehicle Sys. Dynamics, 1994, 23, 379-409. doi: 10.1080/00423119408969067
2. Wael Abbas, Ashraf Emam, Saeed Badran, Mohamed shebl \& Ossama Abouelatta. Optimal seat and suspension design for a half-car with driver model using genetic algorithm. Intelligent Control and Automation, 2013, 4, 199-205. doi: 10.4236/ica.2013.42024.
3. InternationalOrganizationforStandardization.Mechanical vibration and shock evaluation of human exposure to whole- body vibration-Part1: General Requirements. Geneva, Switzerland. ISO, 2631-1.
4. Ishbir Singh; Nigam, S.P \& V.H Saranc, V. H. Modal analysis of human body vibration model for Indian subjects under sitting posture. Ergonomics, October 2014. doi: 10.1080/00140139.2014.961567.
5. Harris Cyril, M; and Allan G. Piersol. 2002. In Harris' shock and vibration handbook. 5th ed. New York: McGraw-Hill.
6. Wael, Abbas; Ossama, B. Abouelatta; Magdy, El-Azab; Mamdouh, El-Saidy \& Adel, A. Megahed. Optimal seat suspension design using genetic algorithms. J. Mech.

Eng. Automation, 2011, 1, 44-52.
7. Jin, Liu; Yongjun, Shen \& Shaopu, Yang. Parameters optimization of passive vehicle suspension based on Invariant points theory. Int. J. Smart Sensing Intell. Sys., 2013, 6(5), 2182-2199.
8. Patil, M.K.; \& Palanichamy, M.S. A mathematical model of tractor-occupant system with a new seat suspension for minimization of vibration response. Appl. Math. Modelling, 1988, 12, 63-71.
doi: 10.1016/0307-904X(88)90024-8
9. Ramamurthy, N.V. \& Patil, Mothiram K. Human body vibration response minimization by relaxation seat suspension system for an armoured fighting vehicle. In Proceedings of the International Conference on Biomedical Engineering (ICBME), IISc., Bangalore, India, Dec 2124, (2001), 97-100.
10. Rakheja, S; Afework, Y \& Sankar, S. An analytical and experimental investigation of the driver-seat-suspension system. Vehicle Sys. Dynamics, 1994, 23, 501-524.
doi: 10.1080/00423119408969072
11. Shirahatt, A.; Prasad, P.S.S.; Panzade, P. \& Kulkarni, M.M. Optimal design of passenger car suspension for ride and road holding. J. Braz. Soc. Mech. Sci. Eng., 2008, 30(1), 66-76.
doi: 10.1590/S1678-58782008000100010
12. Engineering Design Hand Book, Automotive series, Automotive suspensions, HQ, US Army Material Command, AMC Pamphlet, AMCP 706-356, April 1967, 2574, 8-23-8-26.
13. Banerjee, Saayan \& Balamurugan, V. Nonlinear ride dynamics mathematical model of tracked vehicle. In Proc. of International Conference on Multi Body Dynamics, Vijayawada, India, 2011, 385-398.
14. Chandramouli, P. Feasibility study report on Semi-active suspension system for AFV. Report no. DRDO-CVRD-CAR-002-2005 (2005).
15. Hatch, Michael R. In Vibration simulation using Matlab and Ansys, Chapman \& Hall/CRC press LLC, Florida, U.S, 2001.
16. Likaj, R.; Shala, A.; Bruqi, M. \& Bajrami, X. Optimal design and analysis of vehicle suspension system. In DAAAM International Scientific Book, edited by B. Katalinic. DAAAM International, Vienna, Austria. 2014, Ch. 07. pp. 087-108.
doi: 10.2507/daaam.scibook.2014.07.
17. Zhongzhe, Chi; Yuping, He \& Greg, F. Naterer. Design optimization of vehicle suspensions with a quarter-vehicle model. Trans. CSME, 2008, 32(2).
18. Global optimization toolbox user's guide http://www. mathworks.com, (Accessed on 30 September 2017).
19. Patel, Viral K. \& Patel, Maitri N. Development of smart sensing unit for vibration measurement by embedding accelerometer with the Arduino Microcontroller, Instrumentation Science, 2017, 6(1), 1-7. doi: 10.5923/j.instrument.20170601.01.
20. Balamurugan, V. Dynamic analysis of a military tracked vehicle. Def. Sci. J., 2000, 50(2), 155-165.
doi: $10.14429 / \mathrm{dsj} .50 .3410$

## CONTRIBUTORS

Mr N.V. Ramamurthy received his BE (Mech. Engg.) from College of Engg., Guindy, Chennai, India, his MTech (Applied Mechanics) from IITM., Chennai, India and pursuing his PhD in Mech. Engg. at SRM University, Kattankulathur. Currently he is working as Tech. Officer 'C' at CVRDE, DRDO, Min. of Defence, Chennai, India. His current research interests include Vehicular vibration and Ergonomics.
In the current work, he made exhaustive literature survey, carried out mathematical modelling, simulation, model validation using earlier research findings, organising case study activity for model validation and manuscript preparation.

Dr B.K. Vinayagam, received his MSc, (Engineering) in Machine tool design \& manufacturing and his PhD in Flexible Manufacturing System. Currently he is working as Professor in the Department of Mechatronics Engg. at SRM Univ., Kattankulathur, India. His current research interests include Robotics \& Vehicular vibrations.

In the present work, he provided valuable inputs to the author in the current topic of research and guided him in the modeling, simulation \& analysis tasks to obtain the final outputs \& organising the research findings.

Dr J. Roopchand, received his BE (Mechanical Engineering) from College of Engineering, Guindy, Chennai, India, his MTech (Machine Design) from IITM., Chennai, India and his Ph.D in Mechanical Engg. From Anna Univ., Chennai, India. Currently he is working as Addl. Director at CVRDE, DRDO, Min. of Defence, Chennai, India. His current research interests include Vehicular vibration and Terra mechanics.
In the present study, he finalised the methodology for case study including trial run of test vehicle in proving ground and provided complete guidance to the author in the data acquisition process during the case study.

