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Image Inpainting and Enhancement using Fractional Order Variational Model

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ABSTRACT

The intention of image inpainting is to complete or fill the corrupted or missing zones of an image by considering the knowledge from the source region. A novel fractional order variational image inpainting model in reference to Caputo definition is introduced in this article. First, the fractional differential, and its numerical methods are represented according to Caputo definition. Then, a fractional differential mask is represented in 8-directions. The complex diffusivity function is also defined to preserve the edges. Finally, the missing regions are filled by using variational model with fractional differentials of 8-directions. The simulation results and analysis display that the new model not only inpaints the missing regions, but also heightens the contrast of the image. The inpainted images have better visual quality than other fractional differential filters.

Keywords: Fractional calculus; Image inpainting; PDE models; Variational model

1. INTRODUCTION

Digital image completion, or inpainting, is used to complete or replace the corrupted or missing zones of an image by using the knowledge from the known regions, such that a neutral observer would not notice any changes. There are diverse important applications of the digital image inpainting techniques, such as: damaged painting reconstruction, photo restoration, superimposed text removal, object removal, image compression and coding.

The image inpainting approaches are branched into the three groups: exemplar-based inpainting[1,2], diffusion-based inpainting[3-13], and hybrid inpainting[14]. Exemplar-based inpainting technique repeatedly synthesises the unknown area by a most identical patch in the known area. An influential exemplar-based inpainting approach was developed by Antonio[1], *et al.*. Many other innovations improving the speed and efficacy of the Antonio's proposal have been amplified[2].

Diffusion-based image inpainting refers to the technique of completing, which employs the information around the damaged region to estimate isophotes, and propagates information from outside region to inside region by propagation. It utilises the partial differential equation (PDE) based and variational based restoration methods. The PDE techniques follow isophote directions in the image to perform the restoration process. The first PDE-based image completion method was introduced by Bertalmio[6], *et al.*. The first variational method to the image completion was introduced by Nitzberg and Mumford[3], and the second variational model to image completion was proposed by Masnou and Morel[5], based on interpreting the level lines with minimal curvature. A famous variational model for image inpainting was introduced by Chan and Shen[7]. Their variational framework completes the damaged areas by minimising the total variation (TV), while retaining approximately the ground truth image in the source regions. This method adopts an Euler–Lagrange (E-L) equation and anisotropic (non-linear) diffusion which depends on the isophotes strength. It fails to connect broken edges. The same authors extended the TV model in curvature driven diffusion (CDD) model. It is based on the geometric information of the isophotes. It modifies the coefficient of conductivity to be stronger when the isophotes have large curvature. Quick curvature driven diffusion is proposed by Xu[10], *et al.* to reduce the computational complexity of the CDD model. Biradar and Kohir[11] applied a simple method based on a nonlinear median filter to diffuse median value from exterior to interior regions. Barbu[13] proposed a fast converging second order nonlinear diffusion to image inpainting.

Recently, fractional order PDEs have been studied in computer vision. The fractional derivative[15,16] finds a major role in digital image processing[16-23]. It is the generalised form of integer order derivative. Fractional derivative is defined by many mathematicians like Riemann-Liouville, Grunwald-Letnikov, and Caputo. It exhibits the non-local property, as the fractional derivative at a pixel depends on the whole image and not just the neighbourhood pixel values. It is very useful for edge preservation and enhancement of the image. Zhang[17,18], *et al.* proposed p-Laplace fractional order variational image inpainting based on Grunwald-Letnikov and Riemann-Liouville definitions. The inpainting process of these models is based on the fractional differential filter in four directions and the diffusion process is controlled by the p-Laplacian fractional

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order gradient. The total of the coefficients in the fractional differential filter mask derived from these two definitions are not equal to zero, whereas for the Caputo definition, it is close to zero, which is very much required for the contrast enhancement. The following changes are proposed in this article. The diffusion process is controlled by the complex diffusion coefficient[24,25], which is more generalised and efficient to preserve the edges and the Caputo fractional differential filter is considered in 8-directions because it possesses anti-rotational capability[22].

2. PRELIMINARIES

Given an image, $u_0 \in L_2(\Omega)$ with $\Omega \subset R^2$ an inpainting or missing domain having boundary $\delta\Omega$, and E as surrounding domain nearby $\delta\Omega$. The problem is to recover the ground truth image u from the degraded image u_0. The diffusion models are propagating local information with smoothness constraints from exterior to the interior of the missing regions.

2.1. PDE based Inpainting Models

The simplest PDE based inpainting formulation concerns isotropic diffusion which arises from heat flow equation.

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u \\ u_{(t=0)} = u_0 \end{cases} \tag{1}$$

where Δu denotes image Laplacian. The PDE progression is parameterised with a time variable t, which describes the continuous evolution of the function u. It minimises these changes in all inclinations (directions) and performs as a low pass filter. Thus, this model gets blurred in the neighbourhood of contours and edges. Non-linear (anisotropic) diffusion models are useful to retain prominent edges and sharpness. The heat flow Eqn. (1) can be rewritten as

$$\frac{\partial u}{\partial t} = div(\nabla u) = \nabla.(\nabla u) \tag{2}$$

where $div(.)$ represents divergence operator. This led to Perona and Malik[4] to suggest a non-linear expansion of the heat flow equation by introducing a diffusion coefficient.

$$\frac{\partial u}{\partial t} = div\left(c\left(|\nabla u|\right)\nabla u\right) \tag{3}$$

The propagation (diffusion) mechanism is diluted close to the edges and is forceful in homogeneous regions.

2.2 Variational Image Inpainting Models

Variational formulation is another image regularisation technique, where the image is identified as a function of bounded variation (BV). Variational models aim at minimising energy functional. In total variation (TV) model[7], the energy function comes from the TV - norm and it minimizes TV energy of the image in the missing region under some constraints of fidelity to image observations.

$$\min_{u \in BV(\Omega)} \left\{ J(u) = \int_{E\cup\Omega} |\nabla u|\, dxdy + \frac{\lambda_\Omega}{2} \int_\Omega (u-u_0)^2\, dxdy \right\} \tag{4}$$

First part of the equation denotes the regularisation, the second part of the equation denotes the fidelity, and λ_Ω denotes the regularisation parameter defined as

$$\lambda_\Omega = \begin{cases} \lambda, (x,y) \in E \bigcup \Omega \\ 0, (x,y) \notin E \bigcup \Omega \end{cases}$$

The E-L energy minimisation equation with Neumann boundary condition associated to Eqn. (4) is

$$\frac{\partial u}{\partial t} = -\nabla.\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda_\Omega (u - u_0) \tag{5}$$

Even though TV model is stable and provide exclusive solution, the textures and prominent information are rigorously smoothed out. The performance of the inpainting can be improved by combining fractional calculus to the integer order TV model. The fractional order variational image inpainting model is proposed by Zhang[17,18], *et al.*. The energy minimisation equation of that model is

$$\frac{\partial u}{\partial t} = \overline{(-1)^\alpha} div^\alpha \left(\frac{\nabla^\alpha u}{\left(|\nabla^\alpha u|\right)^{2-p}} \right) + \lambda_\Omega (u-u_0), p \in [1,2] \tag{6}$$

where ∇^α is a fractional order derivative. Zhang[17,18], *et al.* considered Grunwald-Letnikov, Riemann-Liouville fractional derivative definitions in their work. The total of filter coefficients based on Grunwald-Letnikov and Riemann-Liouville definitions are not equal to zero. Thereby, the contrast of the image is reduced.

3. PROPOSED MODEL

The performance of inpainting process and contrast enhancement can be upgraded by introducing the following modifications in the fractional order variational model given in the Eqn. (6).

- The fractional derivative according to Caputo definition.
- The 8-directional fractional differential filter.
- The complex edge stopping or diffusivity function.

The major advantage of Caputo fractional differential filter over other fractional differential filters is that the total of the filter coefficients is close to zero and the fractional differential masks have anti-rotation capability[22]. Since there are only eight neighbouring points for each pixel, the fractional mask in the eight directions has well anti-rotative and it can enhance the texture well and inpaint the curvy missing regions of the image. Eight fractional differential masks which are respectively on the directions of positive x-direction $(x+)$, negative x-direction $(x-)$, positive y-direction $(y+)$, negative y-direction $(y-)$, right upward diagonal (rud), left down diagonal (lud), right down diagonal (rdd), and left upward diagonal (lud).

The diffusion process is controlled by complex diffusion coefficient[24,25], which is more generalised and efficient to preserve the edges in the process of restoration. Hence, the diffusion strength, $\left(|\nabla u|\right)^{p-2}$ in Eqn. (6) is replaced by the influence of complex diffusion coefficient in the proposed model. The E-L energy minimisation equation of the proposed model is represented as

$$\frac{\partial u}{\partial t} = \overline{(-1)^\alpha} div^\alpha \left(c\left(\operatorname{Im} ag(u)\right)\right)\nabla^\alpha u + \lambda_\Omega (u-u_0) \tag{7}$$

where $c(.)$ is complex edge stopping function, defined as

$$c(s)=\frac{e^{j\theta}}{1+\left(\frac{s}{k_2}\right)^2} \tag{8}$$

where k_2 is edge stopping specification and the value ranges from 1 to 1.5. A subjective property of edge information is described by the small value of θ. For big values of θ the imaginary part feeds back to the real part creating ringing effect, which is unacceptable effect. Here for testing the inpainting performance the value of θ is chosen as $\pi/30$. The discrete representation of the first term of Eqn. (7) in 8-directions is

$$\begin{aligned} div^{\alpha}\left(c\left(\operatorname{Im}ag(u)\right)\nabla^{\alpha}u\right) &= \nabla^{\alpha}_{x-}\left(c\left(\operatorname{Im}ag\left(\nabla^{\alpha}_{x+}u\right)\right)\nabla^{\alpha}_{x+}u\right)\\ &+\nabla^{\alpha}_{y-}\left(c\left(\operatorname{Im}ag\left(\nabla^{\alpha}_{y+}u\right)\right)\nabla^{\alpha}_{y+}u\right)\\ &+\nabla^{\alpha}_{ldd}\left(c\left(\operatorname{Im}ag\left(\nabla^{\alpha}_{rud}u\right)\right)\nabla^{\alpha}_{rud}u\right)\\ &+\nabla^{\alpha}_{rdd}\left(c\left(\operatorname{Im}ag\left(\nabla^{\alpha}_{lud}u\right)\right)\nabla^{\alpha}_{lud}u\right) \end{aligned} \tag{9}$$

The computation of this model is based on the gradient descent technique.

3.1 Construction of Caputo Fractional Differential Filter

The definition of fractional derivative[15] in the Caputo sense of a 1D signal $u(x)$ existing in the duration $[0,x]$ and for any real number α is

$$\nabla^{\alpha}u(x)=\frac{1}{\Gamma(m-\alpha)}\int_0^x\frac{u^{(m)}(\xi)}{(x-\xi)^{\alpha-m+1}}d\xi,(m-1)\le\alpha\le m \tag{10}$$

where Γ is a Gamma function. For processing, one may transform the continuous sum (integral) to the discrete sum of products. Split the integral period[22] $[0,x]$ into N equal segments. The $N+1$ causal points are

$$\begin{cases} u_N \equiv u(0)\\ u_{N-1}\equiv u\left(x/N\right)\\ \vdots\\ u_k \equiv u\left(x-kx/N\right)\\ \vdots\\ u_0\equiv u(x)\end{cases} \tag{11}$$

The Eqn. (10) is approximated for $m=2$ as

$$\nabla^{\alpha}u(x)\cong\frac{1}{\Gamma(2-\alpha)}\sum_{k=0}^{N-1}\int_{kx/N}^{(kx+x)/N}\frac{u^{(2)}(\xi)}{(x-\xi)^{\alpha-1}}d\xi,1\le\alpha<2 \tag{12}$$

By referring to the difference expression of second order differentiation[22], one has

$$\int_{kx/N}^{(kx+x)/N}\frac{u^{(2)}(x-\xi)}{\xi^{\alpha-1}}d\xi=\frac{u\left(x+\frac{x}{N}-\frac{kx}{N}\right)-2u\left(x-\frac{kx}{N}\right)+u\left(x-\frac{x}{N}-\frac{kx}{N}\right)}{\left(x/N\right)^2}$$

$$\int_{kx/N}^{(kx+x)/N}\frac{d\xi}{\xi^{\alpha-1}}=\frac{\left(N/x\right)^{\alpha}}{2-\alpha}(u_{k-1}-2u_k+u_{k+1})$$
$$((k+1)^{2-\alpha}-k^{2-\alpha}) \tag{13}$$

Substituting Eqn. (13) in Eqn. (12), one get

$$\nabla^{\alpha}u(x)\cong\frac{x^{-\alpha}N^{\alpha}}{\Gamma(3-\alpha)}\sum_{k=0}^{N-1}(u_{k-1}-2u_k+u_{k+1})((k+1)^{2-\alpha}-k^{2-\alpha}),1\le\alpha<2 \tag{14}$$

By substituting the values of k and $k=n\le N-1$ (n is odd number) in the Eqn. (14), the filter coefficients, $C^{\alpha}_{u_k}$ can be constructed as given below

$$\begin{cases} C^{\alpha}_{u_{-1}}=\dfrac{1}{\Gamma(3-\alpha)}\\ C^{\alpha}_{u_0}=\dfrac{2^{2-\alpha}-3}{\Gamma(3-\alpha)}\\ C^{\alpha}_{u_1}=\dfrac{3-3(2)^{2-\alpha}-3^{2-\alpha}}{\Gamma(3-\alpha)}\\ \vdots\\ C^{\alpha}_{u_k}=\dfrac{-(k-1)^{2-\alpha}-3k^{2-\alpha}-3(k+1)^{2-\alpha}-(k+2)^{2-\alpha}}{\Gamma(3-\alpha)}\\ \vdots\\ C^{\alpha}_{u_{n-2}}=\dfrac{-(n-3)^{2-\alpha}-3(n-2)^{2-\alpha}-3(n-1)^{2-\alpha}-n^{2-\alpha}}{\Gamma(3-\alpha)}\\ C^{\alpha}_{u_{n-1}}=\dfrac{-2n^{2-\alpha}+3(n-1)^{2-\alpha}-(n-2)^{2-\alpha}}{\Gamma(3-\alpha)}\\ C^{\alpha}_{u_n}=\dfrac{n^{2-\alpha}-(n-1)^{2-\alpha}}{\Gamma(3-\alpha)}\end{cases} \tag{15}$$

Therefore, the anterior $n+2$ approximate backward difference of fractional partial differential can be represented in x-direction and y-direction respectively as

$$\begin{aligned}\frac{\partial^{\alpha}u(x,y)}{\partial x^{\alpha}}&=C^{\alpha}_{u_{-1}}u(x+1,y)+C^{\alpha}_{u_0}u(x,y)+C^{\alpha}_{u_1}u(x-1,y)+\cdots\\ &+C^{\alpha}_{u_k}u(x-k,y)+\cdots+C^{\alpha}_{u_{n-2}}u(x-n+2,y)\\ &+C^{\alpha}_{u_{n-1}}u(x-n+1,y)+C^{\alpha}_{u_n}u(x-n,y)\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial^{\alpha}u(x,y)}{\partial y^{\alpha}}&=C^{\alpha}_{u_{-1}}u(x,y+1)+C^{\alpha}_{u_0}u(x,y)+C^{\alpha}_{u_1}u(x,y-1)+\cdots\\ &+C^{\alpha}_{u_k}u(x,y-k)+\cdots+C^{\alpha}_{u_{n-2}}u(x,y-n+2)\\ &+C^{\alpha}_{u_{n-1}}u(x,y-n+1)+C^{\alpha}_{u_n}u(x,y-n)\end{aligned} \tag{17}$$

Similarly, the fractional differential for the given fractional order subsequently on the symmetric eight directions can be executed and represented in Fig. 1. These are positive x-direction, negative x-direction, positive y-direction, negative y-direction, right upward diagonal, left down diagonal, right down diagonal and left upward diagonal correspondingly denoted by $F^{\alpha}_{x+},F^{\alpha}_{x-},F^{\alpha}_{y+},F^{\alpha}_{y-},F^{\alpha}_{rud},F^{\alpha}_{ldd},F^{\alpha}_{rdd},F^{\alpha}_{lud}$. One may recognise that, $C^{\alpha}_{u_{-1}}$ is the weight of the fractional differential filter for the given α on non-causal pixel u_{-1} and $C^{\alpha}_{u_0}$ is the weight of the fractional differential filter of interest pixel. In general, $C^{\alpha}_{u_k}$ is the weight of the fractional differential filter for given value of k. If the image is to be processed with a nonlinear filter, the values of its pixels are calculated with a $n\times n$ size mask using

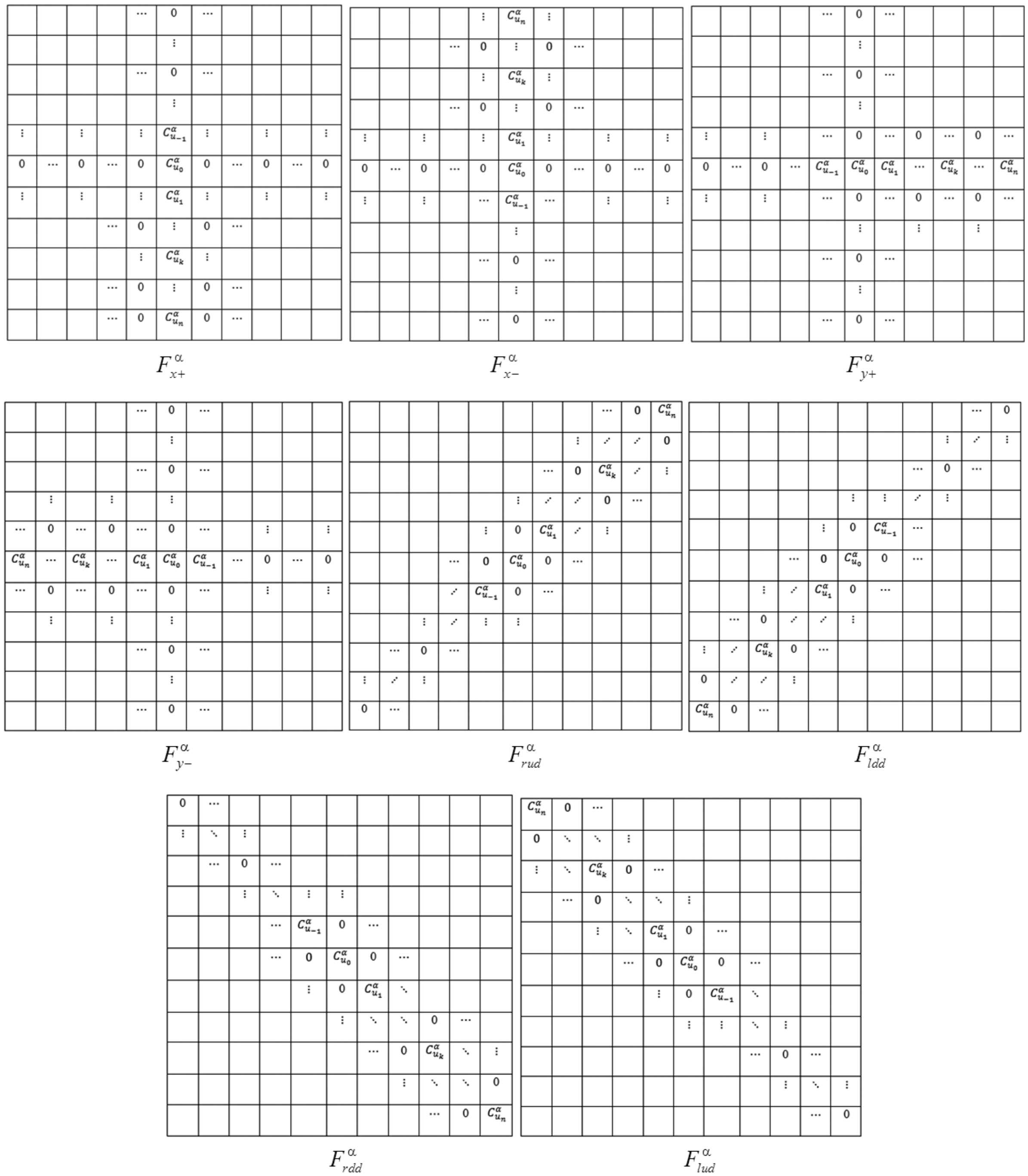

Figure 1. Fractional differential filters on eight directions.

$$\nabla_{dir}^{\alpha} u(x,y) = \sum_{i=-\frac{(n-1)}{2}}^{\frac{(n-1)}{2}} \sum_{j=-\frac{(n-1)}{2}}^{\frac{(n-1)}{2}} F_{dir}^{\alpha}(i,j)\, u(x+i, y+j) \qquad (18)$$

where $dir = x+, x-, y+, y-, ldd, udd, lud, rud$.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The proposed technique has been exercised on large collections of images affected by missing regions. The USC-SIPI[26] image database is used in our experiments. The proposed technique provides an effective restoration of the degraded image, completing successfully the missing zones. It also preserves the image details, and reduces the undesirable effects, such as image blurring, stair-casing and speckle effects. The optimal image reconstruction results are achieved by the proper selection of fractional order. This value is detected by trial and error, through empirical observation. In this work, $\alpha = 1.6$ is considered for optimal reconstruction result. The performance of this model has been quantified by using the well-known metrics, such as peak signal to noise ratio[27] (PSNR), mean structural similarity[28] (MSSIM), mutual information[29] (MI), and normalised cross-correlation[27] (NCC). All models are implemented under 64-bit Windows 8 and MATLAB R2013b running on a laptop with Intel Core i3, 1.40 GHz and 4 GB of memory.

A text removing example, using the proposed technique is described in Fig. 2, where some method comparison results are displayed. The images of that figure depict the inpainting results achieved by various inpainting techniques on the cameraman image. The text is superimposed on the image and the different inpainting techniques are applied.

To understand the inpainting capacity and loss in contrast, the residual images $u_0 - u + 256$ are considered. One can observe that, the inpainting is not done properly in the heterogeneous regions by the state of the art models, such as Chan & Shen[7] model, and two Zhang[17,18], *et al.* models. These models do not preserve edges effectively and produce loss in contrast. The inpainting regions after applying the proposed model are filled effectively than the other models. The performance metrics of these models are registered in Table 1. As one could observe in that table, the performance measures of proposed inpainting technique achieve the highest values. One more observation is that, the proposed model works well, even if the image has partially textured regions, but the other fractional order variational models are not. The logic is that, Zhang[17,18], *et al.* models ($\alpha = 1.8$) are close to fourth order PDE, whereas the proposed model ($\alpha = 1.6$) is close to third order PDE. The performance of the proposed model based on the fractional differential filter in 4 directions which is used by Zhang[18], *et al.* is also applied. The diffusion process of the pixel intensity in the missing regions depends upon only 4-neighbourhood pixels. Hence, the propagation of the pixels from the known pixels into the missing regions in the diagonal direction is less.

Table 1. Comparison of various variational inpainting models for superimposed text removal (Cameraman Image with text, PSNR = 19.41 dB)

Model	PSNR	MSSIM	MI	NCC
Chan and Shen[7]	26.48	0.7039	2.6809	0.9816
Zhang[17], *et al.*	31.53	0.9453	3.8743	0.9941
Zhang[18], *et al.*	32.15	0.9620	4.2673	0.9949
Proposed model	33.03	0.9736	5.5312	0.9958

The proposed technique is also applied to remove the handwritten text from the image. It is compared with isotropic diffusion, Bertalmio, PM diffusion, You and Kaveh, Zhang models with respect to residual images as shown in Fig. 3. It is found that, the proposed reconstruction technique produces the result without any loss of contrast and blur. This technique is also applied for the removal of text and scratches in the presence of Gaussian noise, $\sigma = 10\%$ and $\sigma = 15\%$ on Peppers and House images. The proposed model inpaints well, even the known regions are affected by the noise and it can also denoise the non-inpainting regions without introducing any unintended effects i.e., edge blur, staircasing effect, and loss in contrast.

The simulation results are listed in Tables 1-4. It is observed that the proposed model is associated with higher PSNR, MI, MSSIM, and NCC values in comparison to other techniques for all four sample images in consideration. The values of MSSIM and NCC associated with the proposed model, which are very close to one, indicate that the proposed scheme is well capable of structures and high MI value indicates that the edge preservation of testing images. One could examine that,

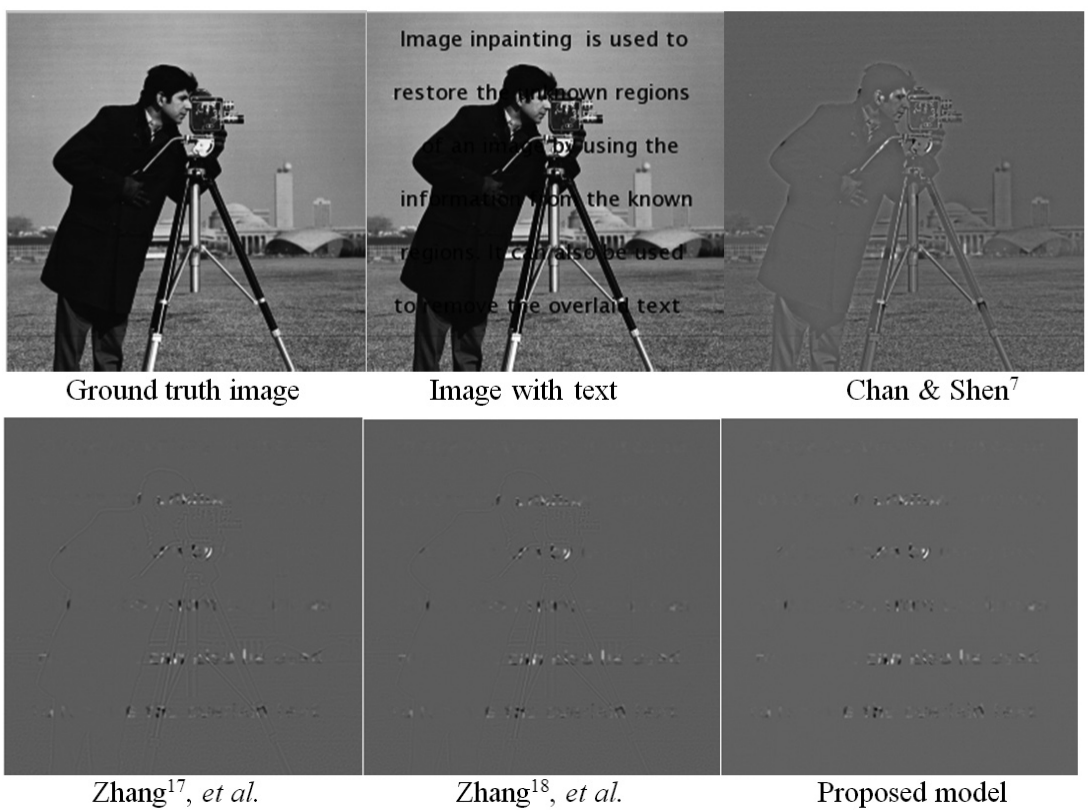

Figure 2. Comparison of various variational inpainting models for superimposed text removal (Residual of inpainted images with ground truth image).

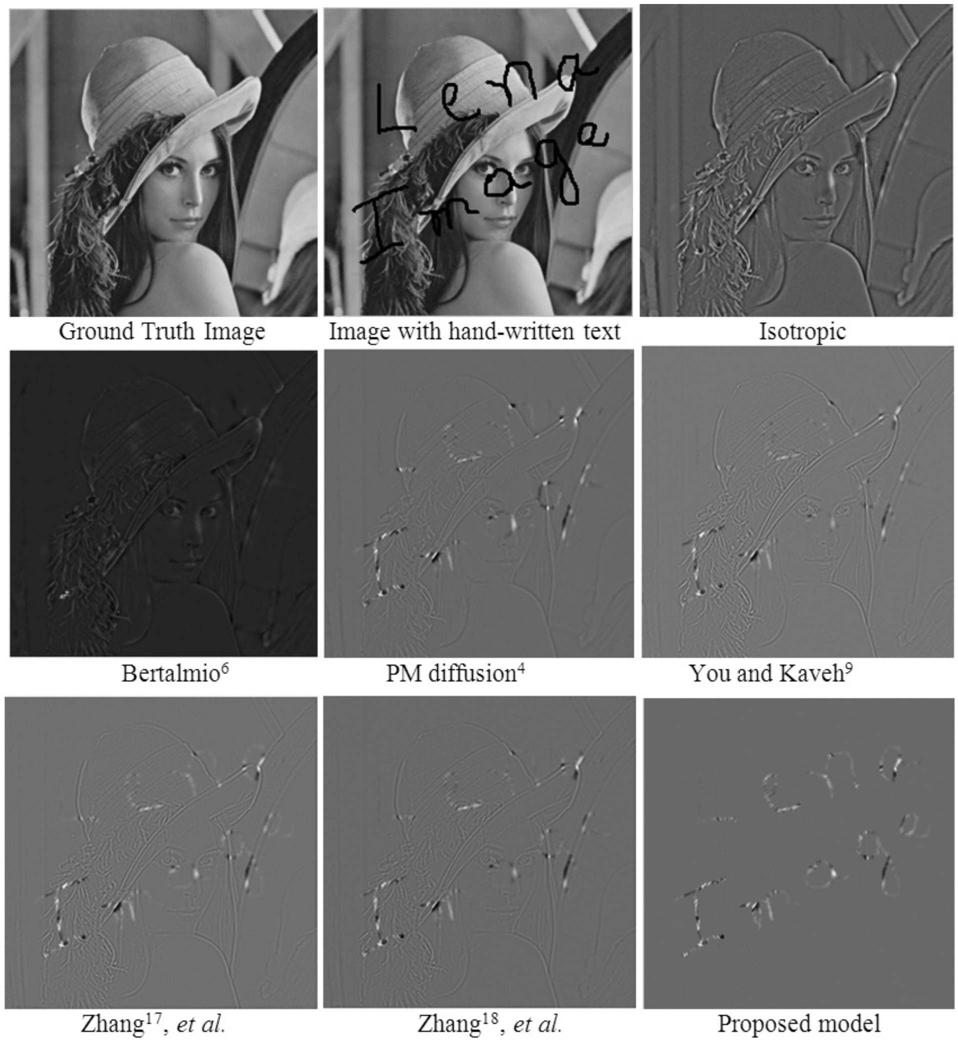

Figure 3. Comparison of various inpainting models for hand-written text removal (Residual of inpainted images with ground truth image).

Table 2. Comparison of various image inpainting models for hand-written text removal (Damaged Lena Image, PSNR = 19.38 dB)

Model	PSNR	MSSIM	MI	NCC
Isotropic diffusion	29.23	0.7805	2.8031	0.9838
Bertalmio[6]	26.14	0.6088	2.4475	0.9644
PM diffusion[4]	37.01	0.9602	4.3553	0.9971
You and Kaveh[9]	34.74	0.9257	3.6613	0.9951
Zhang[17], *et al.*	34.80	0.9352	3.6702	0.9952
Zhang[18], *et al.*	35.86	0.9481	3.9062	0.9962
Proposed model	38.56	0.9793	5.2906	0.9980

the proposed inpainting technique obtains the better objective quality and visual quality.

5. CONCLUSIONS

In this paper, Caputo fractional differential filter is proposed and it is applied to inpaint the missing regions. The edge-stopping function is also applied to preserve the edges of the image, which depends on the fractional order gradient. The

Table 3. Comparison of various image inpainting models for scratch removal in the presence of Gaussian noise ($\sigma = 15\%$)(damaged house image, PSNR = 16.11 dB)

Model	PSNR	MSSIM	MI	NCC
Chan and Shen[7]	27.01	0.6173	2.0553	0.9771
You and Kaveh[9]	28.77	0.6318	2.1281	0.9823
Zhang[17], *et al.*	29.45	0.6377	2.0738	0.9848
Zhang[18], *et al.*	30.34	0.6390	2.1250	0.9875
Proposed model	30.99	0.6494	2.2456	0.9878

Table 4. Comparison of various image inpainting models for text removal in the presence of Gaussian noise ($\sigma = 10\%$) (damaged peppers image, PSNR = 15.89 dB)

Model	PSNR	MSSIM	MI	NCC
Chan and Shen[7]	25.20	0.6022	2.2091	0.9523
You and Kaveh[9]	25.32	0.6122	2.4134	0.9768
Zhang[17], *et al.*	27.65	0.6297	2.4793	0.9805
Zhang[18], *et al.*	27.84	0.6311	2.4927	0.9814
Proposed model	29.84	0.7180	2.8535	0.9826

sum of the Caputo fractional differential filter coefficients is zero. Thereby, the loss of contrast of the image is minimised, while effectively inpainting the missing regions. The idea of inpainting the missing regions using the proposed model performs the better visual quality and objective quality.

The proposed fractional order variational method may be applied for denoising problems of the astronomical and microscopic images. The proposed framework may be implemented in future work to the problem of completing the damaged or missing wavelet coefficients in view of lossy image transmission.

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