Defence Science Journal, Vol. 67, No. 3, May 2017, pp. 332-338, DOI : 10.14429/dsj.67.10657

Predictive Missile Guidance with Online Trajectory Learning

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ABSTRACT

This study presents a predictive guidance scheme for tactical missiles. The modern day targets, with improved manoeuverability, have revealed insufficient performance of the conventional guidance laws. The underlying cause of this poor performance is the reactive nature of the conventional guidance laws such as proportional navigation (PN) and pure pursuit (PP). Predictive guidance offers an alternative approach to the classical methods by taking proactive actions by estimating target's future trajectory. However, most of the existing predictive guidance approaches assume that the interceptor have a model of the target dynamics. A guidance strategy is developed in this study, that can learn the target dynamics iteratively and adapt the interceptor actions accordingly. A recursive least squares (RLS) estimation algorithm is employed for learning and estimating the possible future target positions, and a fixed horizon nonlinear program is employed for selecting the optimal interception action. Monte-Carlo simulations show that the guidance algorithm introduced in this work demonstrates a significantly improved performance compared to the alternatives in terms of interception time and miss distance.

Keywords: Predictive guidance; Trajectory learning; Tactical missile; Recursive least squares; Parameter estimation; Optimal interception

NOMENCLATURE

V_c	Closing velocity
N	Effective navigation ratio
$\dot{\lambda}$	Line-of-sight (LOS) rate
t_k	The kth time instant
t_f	The terminal time instant
h	The length of each time interval
r_{TM}	Interceptor-target relative range
ϕ	LOS angle
x_M	X-coordinate of the interceptor
y_M	Y-coordinate of the interceptor
x_T	X-coordinate of the target
y_T	Y-coordinate of the target
υ_M	Speed of the interceptor
υ_T	Speed of the target
θ_M	Heading direction of the interceptor
θ_T	Heading direction of the target
$\Delta_M(k)$	$\theta_M(t_k)-\theta_M(t_{k-1})$
$\Delta_T(k)$	$\theta_T(t_k)-\theta_T(t_{k-1})$
C	Target motion parameter
L	Target position history buffer length
M	Estimation buffer length
$\hat{x}_M$	Estimated x-coordinate of the interceptor
$\hat{y}_M$	Estimated y-coordinate of the interceptor
$\hat{x}_T$	Estimated x-coordinate of the target
$\hat{y}_T$	Estimated y-coordinate of the target

Received : 16 September 2016, Revised : 16 November 2016
Accepted : 06 December 2016, Online published : 24 April 2017

1. INTRODUCTION

Various missile guidance laws have been developed and studied since the World War II. The proportional navigation[1] (PN) is quite celebrated among these laws, due to its simplicity and efficiency. First studies on modern guidance laws based on modern control and estimation theory were conducted during 1960's. However at that time, implementation of these methods was not feasible because of the computational restrictions. Yet, with the proceeding technology and increasing capability of the targets, the aforementioned attitude towards modern laws began to change during late 1970's[2-8].

The problem of coping with modern day targets (e.g., intercepting a hostile aircraft) with advanced manoeuverability skills, calls for utilisation of estimation techniques within guidance algorithms. By making use of these estimation methods, it is possible to predict the future trajectory of the target. The basic idea behind predictive guidance is enabling the interceptor to take advantage of the estimated future trajectory of the target and modify the guidance law according to these estimations. Talole & Banavar[9] have shown that the PN law modified with predictive control is superior to the PN law itself in terms of control effort. Prabhakar[10], *et al.* demonstrated that predictive guidance is capable of exhibiting a significantly improved performance. However, most of the existing predictive guidance laws assume full or partial knowledge of the target's dynamics. In this paper, estimations are based on a learning process, which utilises the noisy measurement of the target positions by passing them through a recursive least squares (RLS) estimation algorithm. Eventually, with this guidance strategy, the interceptor can intercept the target by demonstrating a better engagement performance than it can achieve with the alternative approaches.

1.1 Previous Work on Predictive Guidance

There has been a significant amount of previous work on predictive guidance for missiles. Abzug[11] shows that a control system can be designed by just taking the miss distance at the final time (t_f) into account. However this study does not yield a remarkable interception engagement performance for a manoeuvering target. Garber[12] solves the interception problem as an optimum control problem for a linear in homogeneous system with a quadratic performance index. Garber presents a closed form optimal guidance law, which includes a predicted miss distance term. However, Garber assumes full knowledge of target dynamics and do not provide any suggestions for estimation of the predicted term stated above.

Salmon[13] introduces 'multipoint guidance' as a precomputed form of predictive guidance for intercepting a ballistic target. Salmon points that every time a guidance command is calculated, the required numerical prediction of the target and interceptor trajectories is costly in terms of real time data processing. Moreover Salmon's efforts do not provide any solution for intercepting a manoeuvering target. Hecht & Troesch[14] develop interceptor control laws, which are linear functions of predicted terminal error terms. These predictive guidance laws are derived via the linearisation of the nonlinear equations of motion. Due to linearisation, this approach do not work efficiently for agile modern day targets. Furthermore, Hecht & Troesch conduct their simulation studies while assuming a stationary target.

Best & Norton[15] establishes their guidance framework with connections to model predictive control. Gaussian-sum approximation to the probability density function of the target was exploited in order to cope with the target position uncertainty. However this guidance framework comes with a significant computational load. Kim[16] presented receding horizon guidance laws (RHG) that works under inaccurate time-to-go information and the current target position. Kim proved that RHG is able to intercept the target within the given vicinity of the target at an appropriately selected instant after the detonation time. However, the dependency of this guidance algorithm to the current target position information constraints itself to a reactive nature.

Talole[17], *et al.* suggested a time delay control based approach for estimating the target acceleration. In this way, the target acceleration requirement of the guidance law was met. They showed that the continuous time nonlinear predictive control approach is effective for expressing the formulation of an optimal homing guidance law for tactical missiles. The guidance law they formulated was shown to exhibit better performance than PN via simulations. Yet, this guidance law requires the measurement of the relative range rate. In practice this means that higher cost sensing systems should be employed. Moreover their guidance law entails selection of filter gains and controller parameters by trial and error procedure by observing the simulated responses. These gains and parameters may have to be adjusted for different kinds of engagement scenarios.

Shaviv & Oshman[18] pointed out the affiliation of the existing missile guidance law designs methods to the separation theorem. They provided a composition of multiple model estimation and guidance in the frame work of general separation theorem. Particle filtering and a geometry based methodology are employed in the study. Authors gave both the discrete and continuous time formulations. However the guidance law they offer has a remarkable computation load as a prominent drawback.

Ma[19], *et al.* introduced learn and predict (LP) as a new guidance law which employs least squares (LS) algorithm for target position prediction. During the estimation and prediction phase of the LP, predetermined fixed size of matrices (i.e. buffers) are employed. LP is tested via non-manoeuvering and manoeuvering (by randomly changing flight path angle) target cases. The performance of LP is compared with some typical existing guidance laws like pursuit guidance and beamer rider guidance law. Comparisons validated the superiority of LP for the randomly manoeuvering target case. Yet, because of the fixed size buffers the algorithm partially constraints itself to a reactive nature. Furthermore, authors do not give any insight on how to select buffer size (L), which is critical to the success rate of the algorithm.

In this paper, authors introduced a new algorithm called online predictive guidance (OPG), which aims to address the shortcomings of the approaches discussed in the previous subsection. The main idea behind OPG is combining an RLS estimator with an optimal guidance law to estimate target's future positions from noisy measurements and alter the course of the intercepting missile to increase the hit accuracy. The contributions of this work can be summarised as follows:

- The developed RLS estimator learns the target dynamics incrementally. The guidance law does not require storage of a fixed size buffer for storing previous estimations, hence there is no need to tune a buffer size beforehand.
- OPG is computationally efficient and appropriate for real time implementation, due to low computational cost of RLS and the associated interceptor action optimisation algorithm.
- OPG does not require any sort of target model simplification. The algorithm works regardless of whether the target is non-manoeuvering or manoeuvering.
- Simulations studies confirm the remarkably improved engagement performance for the OPG, compared to alternative approaches.

2. PROBLEM FORMULATION AND INTERCEPTOR-TARGET MODELS

Let $\mathbf{X}(t_k) = [x_M(t_k), y_M(t_k), \dot{x}_M(t_k), \dot{y}_M(t_k)]$, $\mathbf{Y}(t_k) = [x_T(t_k), y_T(t_k), \dot{x}_T(t_k), \dot{y}_T(t_k)] \in \mathbb{R}^4$ denote the state vectors of the interceptor and the target at the time instant t_k, where dotted terms represent the time derivatives. $x_M(t_k), y_M(t_k), \dot{x}_M(t_k), \dot{y}_M(t_k)$ are the interceptor Cartesian coordinates and their rates, while $x_T(t_k), y_T(t_k), \dot{x}_T(t_k), \dot{y}_T(t_k)$ are the target Cartesian coordinates and their rates.

Here the problem is to design a guidance law to compute the heading direction $\theta_M(t_k)$ of the interceptor at the kth time instant, such that eventually at some time instant t_f, the following inequalities will be satisfied:

$$\Delta_r(k) < 0, \quad (1)$$

$$r_{TM}(k) < R_c, \quad (2)$$

where $\Delta_r(k)$ is the difference between sequential interceptor-target distances $\Delta_r(k) = r_{TM}(t_k) - r_{TM}(t_{k+1})$ and R_c is the critical interceptor-target distance chosen by the designer. By employing inequality 2, interceptor-target distance reduction is guaranteed while satisfying a certain miss distance limit. Otherwise it is occasionally possible for the simulations to terminate when $r_{TM}(k) >> R_c$, even when inequality 1 is satisfied. The following assumptions have been made for the development of the guidance law:

- Only planar (two dimensional) engagement scenarios are considered.
- The missile and the target speeds are assumed to be constant.
- The missile can achieve the commanded reference flight path angle immediately in the next time step.
- Interceptor measures both its own position and the target's position with additive uniform noise.

The first three assumptions are quite common in the missile guidance literature. We consider uniform distribution noise in order to imitate noisy data provided by real sensors as much as possible while considering simplicity of implementation. More advanced sensor noise models can be taken into account, however this is beyond the scope of this paper. Under the assumptions stated above and considering the engagement geometry given in the Fig. 1, the kinematic model is given in Eqn. (3)

$$\begin{aligned} x_T(k+1) &= x_T(k) + (t_{k+1} - t_k)\upsilon_T \cos\theta_T(k) \\ y_T(k+1) &= y_T(k) + (t_{k+1} - t_k)\upsilon_T \sin\theta_T(k) \\ x_M(k+1) &= x_M(k) + (t_{k+1} - t_k)\upsilon_M \cos\theta_M(k) \\ y_M(k+1) &= y_M(k) + (t_{k+1} - t_k)\upsilon_M \sin\theta_M(k) \end{aligned} \quad (3)$$

where $\theta_M(k), \theta_T(k), \upsilon_M, \upsilon_T$ are the interceptor's and target's flight path angles at the time instant t_k, and the speeds of the missile and the target, respectively.

The following discrete pure pursuit (PP) law, Eqn. (4), and proportional navigation (PN) law, Eqn. (5), are implemented for the comparisons in Section 4.

$$\theta_M(k) = \phi(k) \quad (4)$$

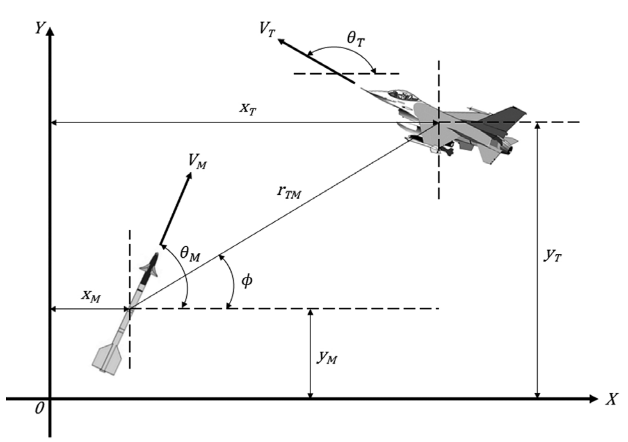


Figure 1. The missile-target engagement geometry.

$$\theta_M(k) = \theta_M(k-1) + \Delta_M(k) \quad (5)$$

where

$$\Delta_M(k) = N\dot{\lambda}h \quad (6)$$

In Eqn. (6) $N, \dot{\lambda}, h$ are the effective navigation ratio, line-of-sight (LOS) rate, and the length of each time interval.

3. GUIDANCE ALGORITHM

Online predictive guidance is formed by three main constituents: Learning, prediction, and guidance. (a) Learning phase of the guidance, the missile is gathering the position information of the target by processing noisy position measurements. Herewith, by utilising a recursive least squares (RLS) estimator, the position observations enables estimation of the kinematic parameters of the target, which in turn enables predicting the future positions of the target at the (b) Prediction phase. Finally, a low dimensional nonlinear program is solved at the (c) Guidance phase, for selecting the optimal interceptor flight path angle. To terminate the algorithm, at the end of each time step inequalities (1) and (2) are checked whether they are satisfied or not.

3.1 Learning Phase

We first write the future states of the target as a function of the current states:

$$\begin{aligned} x_T(k+1) &= x_T(k) + h\upsilon_T \cos\theta_T(k) \\ y_M(k+1) &= y_M(k) + (t_{k+1} - t_k)\upsilon_M \sin\theta_M(k) \\ \theta_T(k) &= \theta_T(k-1) + \Delta_T(k) \end{aligned} \quad (7)$$

where

$$\Delta_T(k) = Ch.$$

Here C is a parameter that describes the evolution of the target's states. Note that perfect knowledge of C enables prediction of the target's future states. See the work conducted by Ma[19], *et al.* for the original description of this representation of target dynamics. The following information of the target's position is being recorded at each time step:

$$\{(x_T(k-j), y_T(k-j)); j = 1, 2, \cdots\}$$

$$\begin{aligned} \Delta_x(k,j) &\triangleq x_T(k-j) - x_T(k-j-1) \\ &= V_T \cos\theta_T(k-j-1) \\ \Delta_y(k,j) &\triangleq y_T(k-j) - y_T(k-j-1) \\ &= V_T \sin\theta_T(k-j-1) \end{aligned}$$

where $V_T = \upsilon_T h.$ Plugging in Eqn. (7) to the above:

$$\begin{aligned} \Delta_x(k,j) &= V_T \cos(\theta_T(k-j-2) + Ch) \\ &= V_T \cos\theta_T(k-j-2)\cos(Ch) \\ &\quad - V_T \sin\theta_T(k-j-2)\sin(Ch) \\ &= \Delta_x(k,j-1)\cos(Ch) \\ &\quad - \Delta_y(k,j-1)\sin(Ch) \end{aligned} \quad (8)$$

similarly,

$$\begin{aligned} \Delta_x(k,j) &= \Delta_y(k,j-1)\cos(Ch) \\ &\quad + \Delta_x(k,j-1)\sin(Ch) \end{aligned} \quad (9)$$

Writing down Eqns. (8) and (9) in matrix form:

$$\begin{bmatrix} \Delta_x(k,0) & -\Delta_y(k,0) \\ \Delta_y(k,0) & \Delta_x(k,0) \\ \Delta_x(k,1) & -\Delta_y(k,1) \\ \Delta_y(k,1) & \Delta_x(k,1) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \cos(Ch) \\ \sin(Ch) \end{bmatrix} = \begin{bmatrix} \Delta_x(k,1) \\ \Delta_y(k,1) \\ \Delta_x(k,2) \\ \Delta_y(k,2) \\ \vdots \end{bmatrix} \quad (10)$$

Equation (10) presents the least squares problem for the estimation of the parameter C.

Consider $\xi_1 = \cos(Ch), \xi_2 = \sin(Ch)$.

Hence

$$\mathbf{A}_0 = \begin{bmatrix} \Delta_x(k,0) & -\Delta_y(k,0) \\ \Delta_y(k,0) & \Delta_x(k,0) \end{bmatrix}, \mathbf{b}_0 = \begin{bmatrix} \Delta_x(k,1) \\ \Delta_y(k,1) \end{bmatrix},$$

$$\mathbf{A}_1 = \begin{bmatrix} \Delta_x(k,1) & -\Delta_y(k,1) \\ \Delta_y(k,1) & \Delta_x(k,1) \end{bmatrix}, \mathbf{b}_1 = \begin{bmatrix} \Delta_x(k,2) \\ \Delta_y(k,2) \end{bmatrix},$$

$$\vdots$$

where the least squares (LS) solution for the first data set given as,

$$\hat{\xi}_0 = (\mathbf{A}_0^T \mathbf{A}_0)^{-1} \mathbf{A}_0^T \mathbf{b}_0$$

From the LS solution, it is straightforward to derive the RLS solution for each time step,

$$\hat{\xi}(k+1) = \hat{\xi}(k) + (A(k+1)^T A(k+1))^{-1} A(k+1)^T \\ (b(k+1) - A(k+1)^T \hat{\xi}(k)) \quad (11)$$

whereas

$$\left\| b(k+1) - A(k+1)^T \hat{\xi}(k) \right\| \quad (12)$$

is the norm of the innovation[20]. This term plays an important role in the interpretation of the results; innovation per time step is a measure of the quality of the knowledge the interceptor possesses regarding the target's dynamics.

3.2 Prediction Phase

In this phase, the target's future positions within M (i.e. prediction horizon) steps is estimated by the latest $\hat{\xi}(k)$ obtained from Eqn. (11), as follows:

$$\hat{x}_T(k \mid k) = x_T(k)$$

$$\hat{y}_T(k \mid k) = y_T(k)$$

For any $j = 1, 2, \cdots, M$,

$$\Delta_x(k, j) = V_T \cos(\theta_T(k + j - 2) + Ch)$$

$$\Delta_y(k, j) = V_T \sin(\theta_T(k + j - 2) + Ch)$$

The predicted target coordinates, within M steps further, can be estimated by the following expressions:

$$\hat{x}_T(k + j \mid k) = \hat{x}_T(k) + \hat{\Delta}_x(k, j)$$

$$\hat{y}_T(k + j \mid k) = \hat{y}_T(k) + \hat{\Delta}_y(k, j)$$

3.3 Guidance Phase

So far, the learning phase is completed by obtaining the parameter C and then related position estimations are computed in the prediction phase. Next step is to solve the optimisation problem in order to determine the interceptor's flight path angle for guidance. Predicted interceptor positions within next M time steps are defined as follows:

$$\hat{x}_M(k + j \mid k) = \hat{x}_M(k) + V_M \cos(\theta_M(k + j \mid k)) j$$

$$\hat{y}_M(k + j \mid k) = \hat{y}_M(k) + V_M \sin(\theta_M(k + j \mid k)) j$$

The following optimisation problem is considered for computing the interceptor path angle, which minimises the relative distance between the target and the interceptor:

$$\underset{\theta_M}{\text{minimize}} \qquad \hat{r}(k + j \mid k) = \sqrt{\chi}$$

where

$$\chi = (\hat{x}_T(k + j \mid k) - \hat{x}_M(k + j \mid k))^2 \\ + (\hat{y}_T(k + j \mid k) - \hat{y}_M(k + j \mid k))^2.$$

4. SIMULATION RESULTS

To assess the performance of OPG, Monte-Carlo simulations are conducted to mitigate for the uncertainty in the measurement model. Different target manoeuvering cases are considered while assuming receding and approaching target scenarios. Each simulation case has been repeated for 100 times. The resulting miss distances and interception times have been averaged over these 100 simulations for the comparison of the PP, PN, LP, and the OPG. The simulation times are capped at 550 time steps.

Initial conditions for the simulations and the related simulation parameters for receding and approaching target scenarios are given in Table 1. Note that the states of the target and interception models have dimensionless units.

Table 1. Simulation initialisation parameters and their values for receding and approaching target scenarios

Simulation parameter	Receding target scenario	Approaching target scenario
$(x_T(0), y_T(0))$	(1000,1000)	(2000,2000)
$(x_M(0), y_M(0))$	(0,0)	(0,0)
υ_T	100	-100
υ_M	150	150
$\theta_T(0)$	0	0
$\theta_M(0)$	$\phi(0)$	$\phi(0)$
N	3	3
L	10	10
M	5	5
R_C	20	20
h	0.05	0.05

Three different manoeuvering target types, which are illustrated in Figs. 2-4, where the missile employs OPG for the receding target case, are considered in simulations:

- Non-manoeuvering target: The target maintains its initial flight path angle throughout the simulation.
- Coordinated turn: The target follows a circular trajectory in the plane with a fixed flight path angle rate.
- Switching coordinated turn: The target follows circular trajectories and switches the direction of the turn every 100 simulation steps.

Engagement performance results of OPG, LP, PN, and PP are presented in the tables below in terms of interception time and

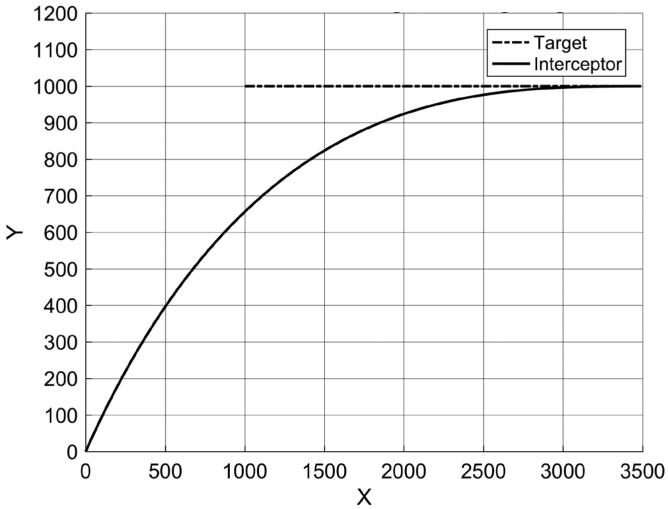


Figure 2. Intercept trajectory where the missile employs OPG for the non-manoeuvering target case in the receding target scenario.

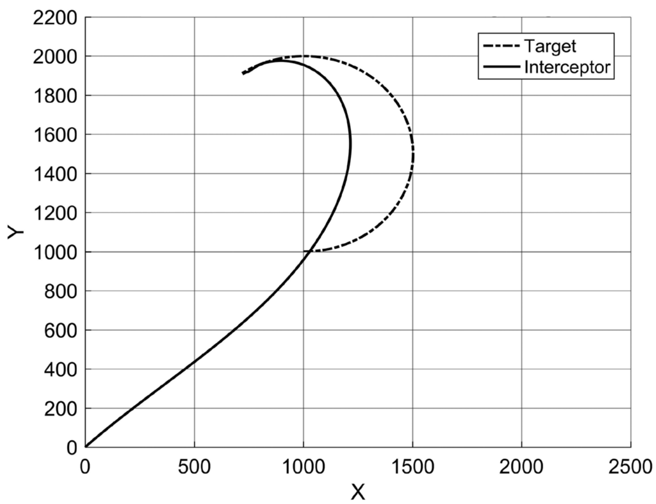


Figure 3. Intercept trajectory where the missile employs OPG for the coordinated turn case in the receding target scenario.

miss distance. Table 2 presents all three manoeuver cases in the receding target scenario. Here it is observed that OPG, LP, and PN have similar interception times for the non-manoeuvering target case; however PP demonstrates a poor engagement performance in terms of both interception time and miss distance compared to OPG, LP, and PN. Moreover, the miss distances of OPG and PN are close to each other, with OPG achieving slightly better performance. In addition, it should be also noted that the performance of OPG is superior to its predictive counterpart LP. When the case where the target is executing a coordinated turn is considered, the superiority of OPG is observed again; OPG maintains remarkably better performance compared to PN and PP, both in terms of miss distance and interception time. Moreover OPG

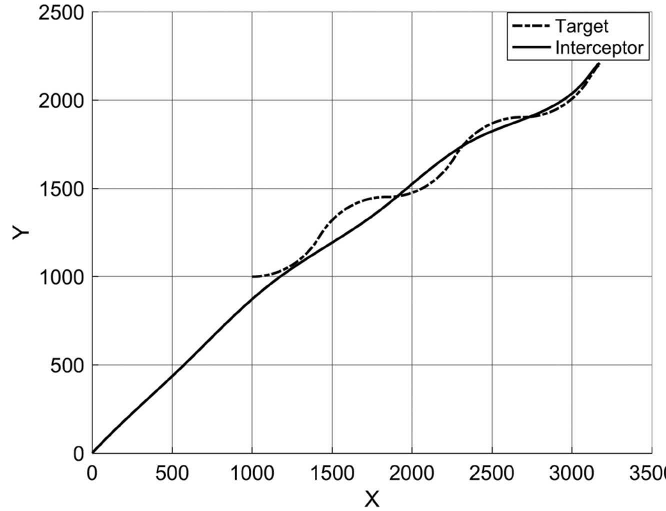


Figure 4. Intercept trajectory where the missile employs OPG for the switching coordinated turn case in the receding target scenario.

performs obviously better than LP in terms of miss distance while achieving similar interception time. For the switching coordinated turn case, all four approaches have similar interception times; however OPG achieves significantly less miss distance compared to LP, PN, and PP. Table 3 presents all three manoeuver cases in the approaching target scenario. More or less similar assessments with the receding target scenario can be made for the approaching target scenario when the results presented by Table 3 are considered. Although the miss distance results slightly increased, OPG still demonstrates the best performance by far in terms of miss distance. Here one also observes that interception times are almost the same except the interception time of PN for the non-manoeuvering case. Fig. 5 depicts the interceptor lateral acceleration time histories of the considered guidance laws for the coordinated turn case in the receding target scenario. According to Fig. 5, OPG and LP exhibit similar lateral acceleration behaviour while PN and PP remarkably differ from these two approaches. PP's chattering acceleration time history reveals the reason of the poor performance. Relative smoothness of the OPG acceleration time history also should be noted. This is because of the RLS estimator employed by OPG. Although OPG generates greater magnitude acceleration signals compared to PN, OPG achieves a better engagement performance eventually. Similar situation is observed for the rest of the cases and scenarios. Overall, the simulation results show that OPG is indeed the superior guidance algorithm compared to both classical approaches

Table 2. Receding target scenario simulation results

Guidance law	Non-manoeuvering target		Coordinated turn		Switching coordinated turn	
	Interception time (s)	Missed distance	Interception time (s)	Missed distance	Interception time (s)	Missed distance
OPG	23.95	0.29	18.82	4.87	26.05	3.25
LP	24.85	6.11	18.77	10.31	25.95	10.19
PN	23.60	0.60	22.30	15.33	26.34	12.86
PP	27.50	240.90	27.50	81.62	27.50	230.90

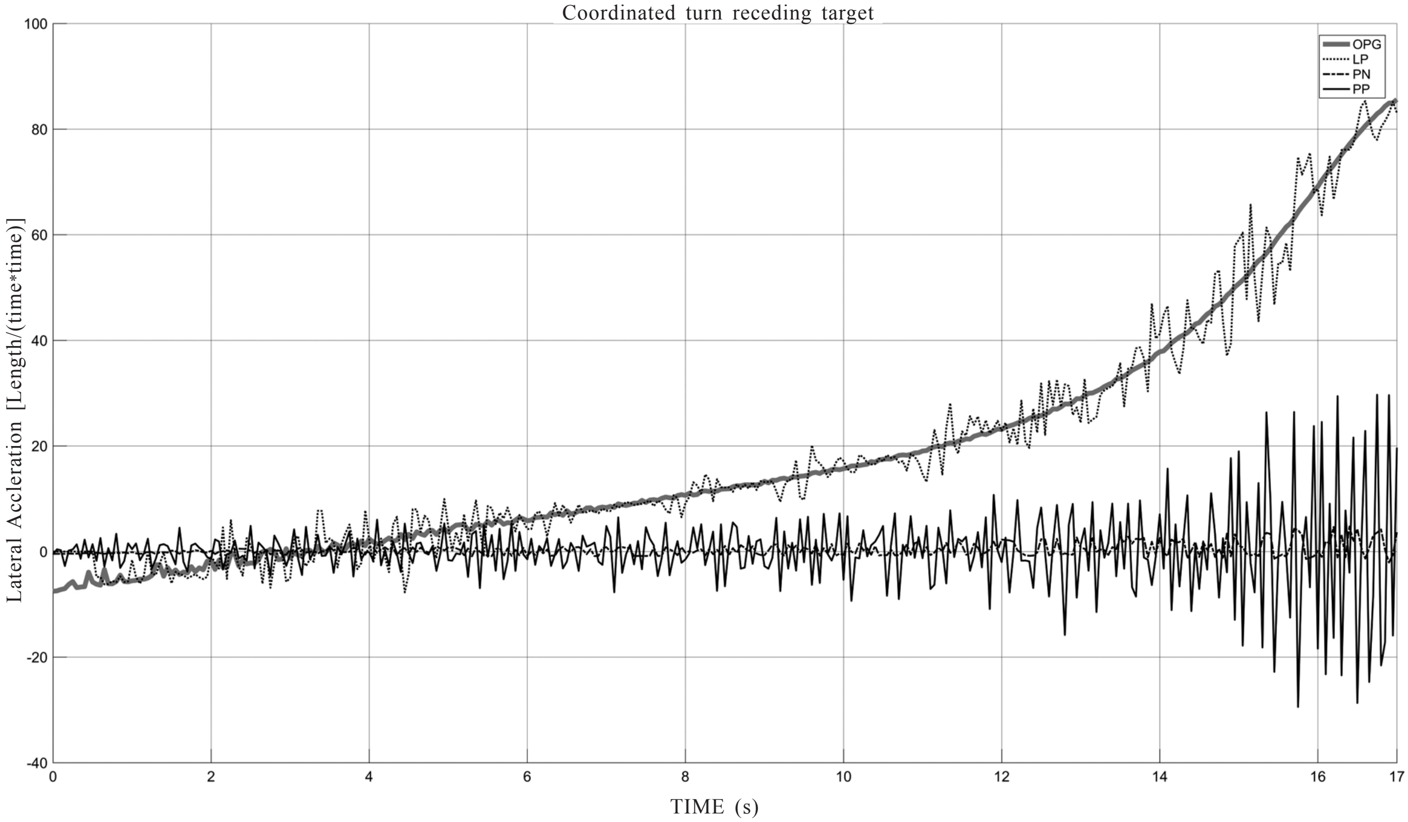


Figure 5. Interceptor lateral acceleration time histories of the considered guidance laws for the coordinated turn case in the receding target scenario.

Table 3. Approaching target scenario simulation results

Guidance law	Non-manoeuvering target		Coordinated turn		Switching coordinated turn	
	Interception time (s)	Missed distance	Interception time (s)	Missed distance	Interception time (s)	Missed distance
OPG	17.55	0.91	17.77	4.92	12.74	3.35
LP	17.50	7.10	17.67	10.46	12.64	11.26
PN	14.45	3.78	19.97	14.55	12.92	3.44
PP	27.50	93.30	27.50	151.67	27.50	41.49

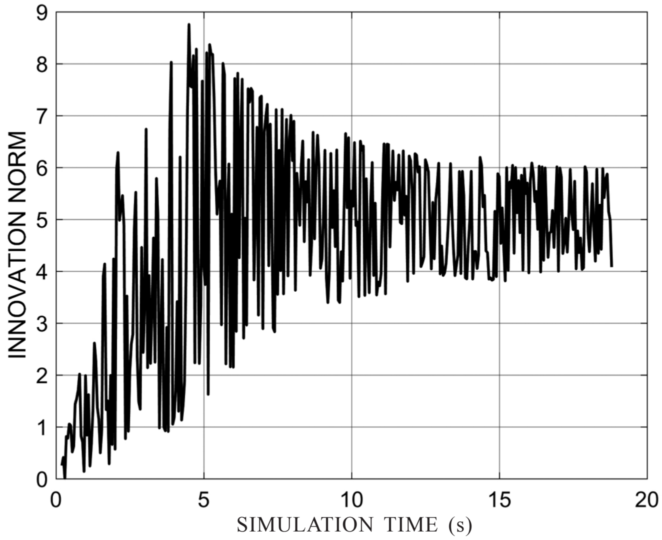
and modern approaches such as LP, especially for the cases where targets are manoeuvering. In addition, simulation results show that the manoeuver type of the target has a remarkable influence on the interceptor-target engagement performance. In particular, the manoeuver type directly affects how the learning process evolves. This fact can be observed by monitoring the innovation norm (Eqn. (12)) versus time. Fig. 6(a) displays the innovation norm versus time for the case where the target executes a coordinated turn. Figure reveals that innovation norm increases steadily in the beginning, which is due to interceptor gaining new information on the targets trajectory at each time step. After this initial phase, the innovation norm settles down to variations in a constant bound, since the learning process gets saturated after a while. Note that innovation norm never goes to zero because of the measurement noise. On the other hand, inspecting the same plot for the switching coordinated turn case in Fig. 6(b), we see that as the target changes its turning direction, the learning process starts from the scratch as it can be seen from the cyclic behaviour of the innovation norm. These results show that OPG can adapt to changes in the manoeuvering pattern of the target, which makes it especially useful against agile targets that employ advanced evasion tactics.

5. CONCLUSION AND FUTURE WORK

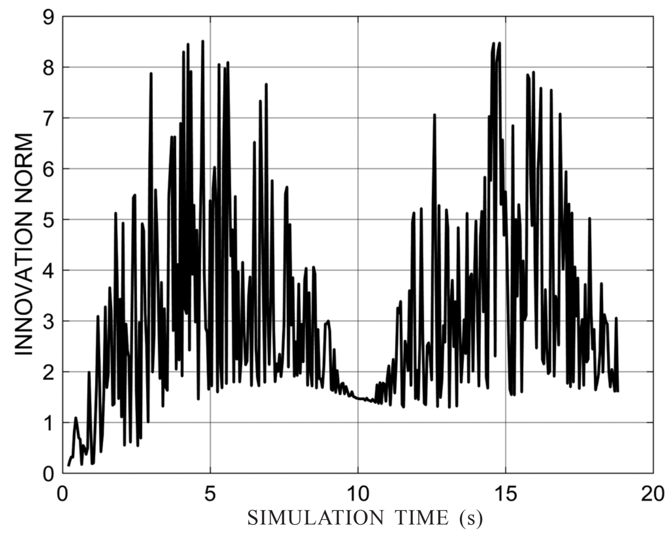
In this work the development of a computationally efficient predictive guidance scheme, for tactical missiles, has been presented. The developed algorithm presents a fusion of recursive least squares learning and optimal guidance. Through the simulation studies, it has been put forth how the predictive guidance strategy contributes to the missile's objectives in terms of interception time and miss distance. Moreover, we have verified through the simulations that the guidance scheme introduced demonstrates a significantly better performance than the alternatives. Future works involve further studies on learning algorithms, probabilistic methods and considering more intricate target manoeuvers.

REFERENCES

1. Nesline, F. W. & Zarchan , P. A new look at classical vs modern homing missile guidance. *J. Guidance, Control, Dynamics,* 1981, **4**(1), 78–85. doi:10.2514/3.56054
2. Shneydor, Neryahu A. Missile guidance and pursuit: kinematics, dynamics and control. Horwood Publishing, England, 1998.



(a)



(b)

Figure 6. Innovation norm time histories for coordinated turn (a) and switching coordinated turn (b) cases in the receding target scenarios.

3. Bryson, Arthur E. & Ho, Yu-Chi. Applied optimal control: optimisation, estimation and control. Taylor & Francis, U.S.A., 1975.
4. Blakelock, John H. Automatic control of aircraft and missiles. John Wiley & Sons, U.S.A., 1991.
5. Zarchan, Paul. Tactical and strategic missile guidance. AIAA, U.S.A., 2002.
6. Siouris, George M. Missile guidance and control systems. Springer, U.S.A., 2004.
7. Yanushevsky, Rafael. Modern missile guidance. CRC Press, U.S.A., 2007.
8. Balakrishnan, S.; Tsourdos, A. & White B. Advances in missile guidance, control, and estimation. CRC Press, U.S.A., 2012.
9. Talole, S. & Banavar, R. N. Proportional navigation through predictive control. *J. Guidance, Control, Dynamics*, 1998, **21**(6), 1004–1006. doi: 10.2514/2.4339
10. Prabhakar, N.; Kumar, I.D.; Tata, S.K. & Vaithiyanathan, V. A predictive explicit guidance scheme for ballistic missiles. *Def. Sci. J.*, 2013, **63**(5), 456–461. doi: 10.14429/dsj.63.2575
11. Abzug, M. Final-value homing missile guidance. *J. Spacecraft Rockets*, 1967, **4**(2), 279–280. doi: 10.2514/3.28850
12. Garber, V. Optimum intercept laws for accelerating targets. *AIAA Journal*, 1968, **6**(11), 2196–2198. doi: 10.2514/3.4962
13. Salmon, D.M. Multipoint guidance-an efficient implementation of predictive guidance. *AIAA Journal*, 1973, **11**(12), 1749–1755. doi: 10.2514/3.50681
14. Hecht, C. & Troesch, A. Predictive guidance for interceptors with time lag in acceleration. *IEEE Trans. Automatic Control*, 1980. **25**(2), 270–274. doi: 10.1109/TAC.1980.1102289
15. Best, R. & Norton, J. Predictive missile guidance. *J. Guidance, Control, Dynamics*, 2000, **23**(3), 539–546. doi: 10.2514/2.4562
16. Kim, K.B. Receding horizon guidance laws with the current target information. *Decision and Control, 2001. In* Proceedings of the 40th IEEE Conference, 2001, **2**, 1523–1528. doi: 10.1109/CDC.2001.981111
17. Talole, S.; Phadke, S. & Singh, R. Predictive homing guidance using time delay control. *In* Proceedings of the AIAA guidance, navigation, and control conference and exhibit, 2005, 3122–3130. doi: 10.2514/6.2005-6159
18. Shaviv, I. G. & Oshman, Y. Estimation-guided guidance. *In* Proceedings of the AIAA Guidance, Navigation and Control Conference, 2006. doi: 10.2514/6.2006-6217.
19. Ma, H.; Wang, M.; Fu, M. & Yang, C. A new discrete-time guidance law base on trajectory learning and prediction. *AIAA Guidance, Navigation, Control Conference*, 2012, pp. 4471. doi: 10.2514/6.2012-4471
20. Chong, E.K. & Zak, S.H. An introduction to optimisation. John Wiley & Sons, U.S.A., 2013.

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