

## Modified Alternative-signal Technique for Sequential Optimisation for PAPR Reduction in OFDM-OQAM Systems

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### ABSTRACT

A modified alternative signal technique for reducing peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing systems employing offset quadrature amplitude modulation (OFDM-OQAM) is proposed. Lower PAPR reduces the complexity of digital to analog converters and results in increasing the efficiency of power amplifiers. The main objective of the algorithm is to decrease PAPR with low complexity. The alternative signal method involves the individual alternative signal (AS-I) and combined alternative signal (AS-C) algorithms. Both the algorithms decrease the peak to average power ratio of OFDM-OQAM signals and AS-C algorithm performs better in decreasing PAPR. However the complexity of AS-C algorithm is very high compared to that of AS-I. To achieve reduction in PAPR with low complexity, modified alternative signal technique with sequential optimisation (MAS-S) is proposed. The quantitative PAPR analysis and complexity analysis of the proposed algorithm are carried out. It is demonstrated that MAS-S algorithm simultaneously achieves PAPR reduction and low complexity.

**Keywords:** Alternative signal; Orthogonal frequency division multiplexing; Peak to average power ratio; Complementary cumulative distribution function

### 1. INTRODUCTION

Orthogonal frequency division multiplexing which employs offset quadrature amplitude modulation (OFDM-OQAM) is an efficient scheme suitable for long-term evolution (LTE) and other wireless systems due to its high spectral efficiency. Orthogonal frequency division multiplexing is a scheme best suited to mitigate frequency selective fading as suggested by Jiang<sup>1</sup>. Reduction of peak-to-average power ratio (PAPR) for OFDM-OQAM modulation is proposed by Skrzypczak<sup>2</sup>. Here Offset Selected mapping (OSLM) is introduced which does not yield good PAPR reduction performance. Li and Wang<sup>3</sup> proposed novel selected mapping schemes with reduced complexity which results in substantial reduction of computational complexity. But here PAPR reduction is inferior to that of conventional SLM scheme.

A parallel artificial bee colony algorithm is cited by Taspinar and Yildirim<sup>4</sup> which results in good PAPR reduction and low complexity but requires more number of IFFT operations. Yang and Hu<sup>5</sup> have developed swapped SLM scheme for PAPR reduction in OFDM. Sohn<sup>6</sup> proposed a new genetic algorithm using SLM approach which results in desirable PAPR reduction with low complexity. PAPR reduction technique using Modified widely linear selected mapping scheme is presented by Yang and Siu<sup>7</sup> which requires less IFFT operations thereby reducing the complexity. However the PAPR reduction performance is inferior to that of conventional SLM scheme.

PAPR reduction using clipping and filtering was proposed by Islam<sup>8</sup> while PAPR reduction scheme for OFDM using peak windowing and clipping is suggested by Singh and Fidele<sup>9</sup>. Clipping based iterative PAPR reduction techniques has been cited by Kollar and Varga<sup>10</sup>. All these schemes do not consider the computational complexity reduction. Even though clipping is easier to implement, it may result in in-band and out-of-band interferences while eliminating the orthogonality among the subcarriers. PAPR reduction by partial transmit sequence (PTS) Technique is suggested by various authors<sup>11-14</sup>. PTS makes use of iterative routine to find the phase factors which are optimum. A novel segmental PTS scheme is proposed by Ye and Li<sup>11</sup> which divides the overlapped OFDM-OQAM signals into number of segments, then some disjoint sub-blocks are divided and multiplied with different phase factors in each segment. But this method does not achieve significant reduction in complexity. Hou and Ge<sup>12</sup> have presented a new PTS algorithm with lower computational complexity where complexity is reduced considerably. However the PAPR reduction performance is same as that of conventional PTS scheme. PTS sub-blocking technique using only partial IFFT's for complexity reduction is cited by Ghassemi<sup>14</sup> which achieves complexity reduction. But the PAPR performance is same as that of original PTS scheme which results in increase in the complexity of power amplifiers thereby reducing the efficiency.

Siohan and Siclet<sup>15</sup> have performed the analysis and design of OFDM-OQAM systems based on filter bank theory. The importance of filter bank multicarrier (FBMC) which

is an effective solution for multiuser multicarrier systems is suggested by Farhang<sup>16</sup>. PAPR reduction using Zadoff-chu matrix transform (ZCMF) pre-coding based OFDM system is presented by Baig<sup>17</sup> to allow the Radio frequency amplifier to operate near its saturation level. Hasan<sup>18</sup> proposed a technique for PAPR reduction using Linear predictive coding (LPC) which uses signal whitening property of LPC as a pre-processing step in OFDM systems. However both of the coding schemes proposed by Baig<sup>17</sup> and Hasan<sup>18</sup> do not consider the computational complexity. For obtaining the better codes and preparing huge look up tables for encoding and decoding, complexity becomes more. PAPR reduction scheme in generalised multicarrier signals using modified active constellation extension (ACE) is suggested by Kliks and Bogucka<sup>19</sup>. PAPR reduction of space frequency coded OFDM systems using the active constellation extension are presented by Naeiny and Marvasti<sup>20</sup>. But the complexity of these suggested methods increases due to repeated search. A tone reservation algorithm based on cross-entropy method for decreasing PAPR is proposed by Chen and Chiu<sup>21</sup> where the cross-entropy method is introduced to determine the suboptimum values of the peak reduction carriers. But this scheme does not reduce PAPR and computational complexity at the same time. Lu and Qu<sup>22</sup> have cited PAPR reduction of FBMC-OQAM signals using sliding window Tone Reservation where the computational complexity factor is not considered. Optimised Iterative clipping and filtering for PAPR reduction of OFDM signals is considered by Wang and Luo<sup>24</sup> while simplified approach to optimised Iterative clipping and filtering is cited by Hong and Tang<sup>25</sup>. Though it reduces out-of-band radiation and reduces PAPR, the complexity of the methods is not at all considered. Xin and Wan<sup>26</sup> have developed efficient tone reservation PAPR system with optimal clipping for OFDM systems.

Sliding window tone reservation technique for PAPR reduction of OFDM-OQAM signals is proposed by Shixian and Qu<sup>27</sup> where as a novel mutiblock tone reservation scheme for PAPR reduction in OQAM-OFDM systems has been proposed by Jiang and Luo<sup>28</sup>. Overlapped segmental active constellation extension for PAPR reduction of OFDM/OQAM system is proposed by Sandeep<sup>29</sup>. Cyclic spectral analysis of OFDM/OQAM signals has been performed by Desimir and

Selena<sup>30</sup>. But either PAPR performance analysis or complexity analysis have not been performed in these schemes. Ge and Li<sup>31</sup> have suggested PAPR reduction using Non-linear companding transforms and Reduction in PAPR by applying both PTS and SLM algorithms has been suggested by Chen<sup>32</sup>. Qu and Jiang<sup>33</sup> have proposed multi block optimisation for PAPR reduction of FBMC-OQAM signals. However the computational complexity of this scheme has not been discussed.

In this study, modified alternative signal technique with sequential optimisation (MAS-S) for reducing PAPR in OFDM-OQAM systems is proposed. We developed first individual AS (AS-I) and combined AS (AS-C) algorithms. AS-I decreases PAPR of every OQAM-OFDM symbol individually and achieves less complexity. But AS-I algorithm has poor PAPR performance. So we employ AS-C algorithm which reduces PAPR by jointly applying to all OFDM-OQAM symbols. However the complexity of AS-C algorithm increases exponentially with the number of OFDM-OQAM symbols. To balance PAPR reduction performance and complexity, we develop Modified Alternative signal technique with sequential optimisation (MAS-S). We performed the quantitative PAPR analysis of the proposed algorithm to show that it achieves good PAPR reduction. We also perform the complexity analysis of MAS-S algorithm to prove its low complexity.

The results and outcomes of our developed technique have widespread applications in wireless OFDM-OQAM systems which have the inherent advantage of high spectrum efficiency. Reduction in PAPR increases the efficiency of power amplifiers and enhances the performance of the OFDM-OQAM systems. This in turn results in providing reliable wireless transmission and hence the findings of our proposed technique find application in defence equipment especially in designing wireless standards such as LTE.

## 2. CHARACTERISATION OF OFDM-OQAM SYSTEM

The transmitter section of OFDM-OQAM is shown in Fig. 1. After modulation using QAM, the serially inputted QAM data symbols are converted to parallel form by serial in parallel out block to a matrix  $\hat{S}$  expressed as

$$\hat{S} = [S^0 \quad S^1 \quad \dots \quad S^{M-1}] \quad (1)$$

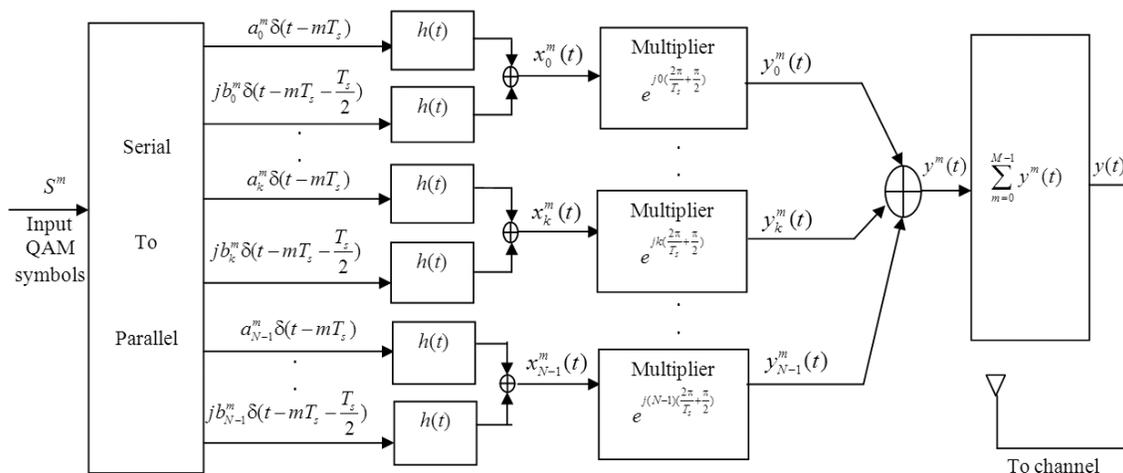


Figure 1. Block diagram of OFDM-OQAM transmitter.

Here  $M$  represents the number of blocks of data and  $S^m$  is the  $m^{\text{th}}$  block given by

$$S^m = [S_0^m \ S_1^m \ \dots \ S_{N-1}^m]^T \quad (2)$$

where  $T$  specifies the transpose,  $N$  represents the number of subcarriers. Now  $S_k^m$  signifies the QAM data symbol for  $m^{\text{th}}$  block and  $k^{\text{th}}$  subcarrier. It is expressed as

$$S_k^m = a_k^m + jb_k^m \quad (3)$$

With  $a_k^m$  and  $b_k^m$  denoting the real and imaginary parts of  $S_k^m$ .

The real and imaginary parts of  $S_k^m$  are offset by  $T_s/2$  where  $T_s$  signifies the symbol period and passed through a prototype filter having the impulse response. The output of the filter is given by

$$x_k^m(t) = a_k^m \delta(t - mT_s) * h(t) + jb_k^m \delta(t - mT_s - \frac{T_s}{2}) * h(t) \quad (4)$$

In Eqn. (4), ‘\*’ denotes convolution. The real part  $a_k^m \delta(t - mT_s)$  and the offset imaginary part  $b_k^m \delta(t - mT_s - \frac{T_s}{2})$  are convolved with the impulse response of the prototype filter and then summed up to obtain the outputs  $x_0^m(t), x_k^m(t), \dots, x_{N-1}^m(t)$  as shown in Fig. 1.

The prototype filter that we incorporate in the transmitter section of OFDM/OQAM system is a low pass filter suitable for generating the required output. The impulse response of this filter is specified in detail in Section 2.1.

$x_k^m(t)$  is then modulated with  $N$  subcarriers which are orthogonal to obtain  $y_k^m(t)$  defined as

$$y_k^m(t) = x_k^m(t) e^{jk(\frac{2\pi t}{T_s} + \frac{\pi}{2})}, k = 0, 1, \dots, N-1 \quad (5)$$

The OFDM-OQAM symbol of  $m^{\text{th}}$  block is generated by adding  $y_k^m(t)$  over  $N$  subcarriers as

$$y^m(t) = \sum_{k=0}^{N-1} y_k^m(t) \quad (6)$$

The required signal  $y(t)$  is obtained by adding all the  $M$  symbols as

$$y(t) = \sum_{m=0}^{M-1} y^m(t) \quad (7)$$

The block diagram of OFDM-OQAM demodulator is as shown in Fig. 2. After passing through the channel, the transmitted signal  $y(t)$  corrupted with additive white Gaussian noise (AWGN) and Rayleigh fading is given as input to the demodulator. So, the input to the demodulator becomes  $y(t) + n(t)$ . This signal is passed through a set of correlation demodulators which is the combination of three stages namely the multiplier, integrator and sampler.

The multiplier performs the multiplication of the incoming signal with the conjugate factors while the integrator is used to integrate and obtain the required signal in one symbol period. The sampler is used to sample and convert the continuous time to discrete time signals. After the parallel to serial conversion, the signal is passed through the base band demodulator which uses a threshold for obtaining the output QAM symbols. If the input to the baseband demodulator is greater than threshold, the output is symbol ‘1’ whereas if the input less than the defined threshold, it is symbol ‘0’ and the final output is  $\hat{S}^m$ .

## 2.1 Impulse Response of Prototype Filter

The prototype filter as considered by Siohan<sup>15</sup> is a low-pass filter suitable for OFDM/OQAM system for generating the required OFDM/OQAM symbols. It takes into account both the real and offset imaginary parts as inputs. It is given by

$$h(t) = d(0) + 2 \sum_{i=1}^{p-1} (-1)^i d(i) \cos(\frac{2\pi i t}{pN}) \quad (8)$$

where  $p = 4$  and  $d(i)$  denote the normalised constants. The values of  $d(i)$  for  $i = 0, 1, 2, 3$  are given by

$$\begin{aligned} d(0) &= 1, d(1) = 0.972 \\ d(2) &= 0.707, d(3) = 0.235 \end{aligned} \quad (9)$$

The length of impulse response is  $pT_s$  and  $N$  represents the number of subcarriers.

## 3. CONCEPT OF PAPR AND CCDF

### 3.1 Peak to Average power ratio

The peak to average power ratio (PAPR) of the OFDM signal is defined as the ratio of maximum instantaneous power to the average power. It is formulated mathematically as

$$PAPR[y(t)] = \frac{\max[|y(t)|^2]}{P_{avg}}, 0 \leq t \leq NT \quad (10)$$

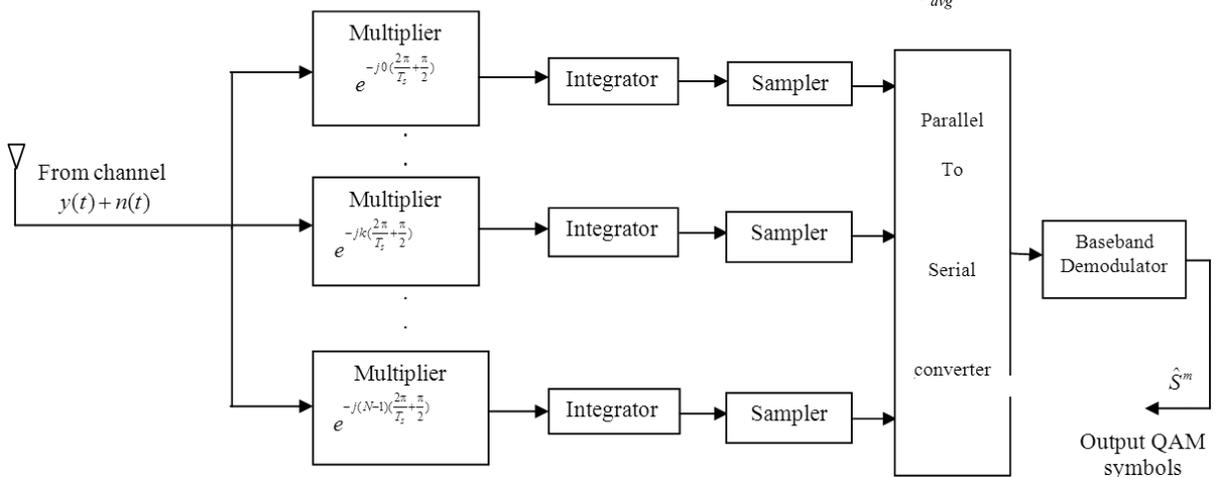


Figure 2. Block diagram of OFDM-OQAM receiver or demodulator.

where  $P_{avg} = E[|y(t)|^2]$  represents the average power. The maximum power is evaluated in the interval  $[0, NT]$ . PAPR is generally expressed in dB.

### 3.2 Complementary Cumulative Distribution

#### Function

If the distribution of power of output OFDM signals is found, the probability that the instantaneous power exceeds the predefined threshold value can be obtained. It is done by finding the CCDF for various values of PAPR.

The CCDF is formulated and expressed as

$$CCDF = \Pr(PAPR > PAPR_0) \quad (11)$$

where  $PAPR_0$  represents the threshold value of PAPR.

## 4. ALTERNATIVE-SIGNAL TECHNIQUE

Three algorithms corresponding to Alternative signal method were developed. First individual AS (AS-I), then combined AS (AS-C) and finally the desired modified AS with sequential optimisation (MAS-S).

### 4.1 AS-I Algorithm

AS-I algorithm decreases the Peak to Average power ratio by selecting the phase factor which is optimum and then it is applied to the individual  $M$  number of symbols. Let the phase factors be represented as

$$\tilde{C} = \{c^0 \quad c^1 \dots \quad c^{U-1}\} \quad (12)$$

Here  $U$  denotes the size of phase factor matrix  $\tilde{C}$  defined in Eqn. (12).

The phase factor for  $u^{th}$  block where  $0 \leq u \leq U-1$  is represented as

$$c^u = [c_0^u \quad c_1^u \dots \quad c_{N-1}^u]^T \quad (13)$$

Here  $c^u = e^{j(\frac{2\pi i}{w})}$ ,  $i = 0, 1, \dots, w-1$  and  $w$  is an integer.

By applying the AS-I algorithm, the phase factors are multiplied with the symbols  $y_k^m(t)$  generated in Eqn. (5) to obtain required output. If we denote the product as  $\tilde{y}_k^m(t)$  it is expressed as

$$\tilde{y}_k^m(t) = y_k^m(t)c_k^u \quad (14)$$

The generated new OFDM-OQAM symbol is given by

$$\tilde{y}^m(t) = \sum_{k=0}^{N-1} \tilde{y}_k^m(t)c_k^u \quad (15)$$

Hence the PAPR reduction problem for AS-I algorithm can be expressed as

$$PAPR_{AS-I} = \frac{\max[\sum_{k=0}^{N-1} |y_k^m(t)c_k^u|^2]}{E[\sum_{k=0}^{N-1} |y_k^m(t)c_k^u|^2]}, 0 \leq t \leq NT \quad (16)$$

### 4.2 AS-C Algorithm

To enhance the PAPR performance, we develop combined AS (AS-C) algorithm which reduces the PAPR by jointly applying to all 'M' symbols.

The new symbol  $\tilde{y}^m(t)$  is generated as specified in Eqn. (15). Then AS-C algorithm is applied jointly to all  $M$  symbols to decrease PAPR. The PAPR reduction problem for AS-C algorithm is given by

$$PAPR_{AS-C} = \frac{\max[\sum_{m=0}^{M-1} \sum_{k=0}^{N-1} |y_k^m(t)c_k^u|^2]}{E[\sum_{m=0}^{M-1} \sum_{k=0}^{N-1} |y_k^m(t)c_k^u|^2]} \quad (17)$$

### 4.3 MAS-S Algorithm

To simultaneously reduce PAPR and complexity, we propose Modified AS algorithm with sequential optimisation (MAS-S). Figure 3 shows the block diagram of MAS-S algorithm. In the  $m^{th}$  block, we decrease the PAPR of  $y^m(t)$  by taking into account the previous OFDM-OQAM symbols given by  $y^0(t), y^1(t), \dots, y^{M-1}(t)$ .

For the  $0^{th}$  block, the OFDM-OQAM output symbol generated  $y^0(t)$  is given as input. It is multiplied with the phase factor which is optimum and obtain the new OFDM-OQAM symbol having minimum PAPR which is denoted as  $\hat{y}^0(t)$ . Now the output of the  $0^{th}$  block is given as input to the  $1^{st}$  block

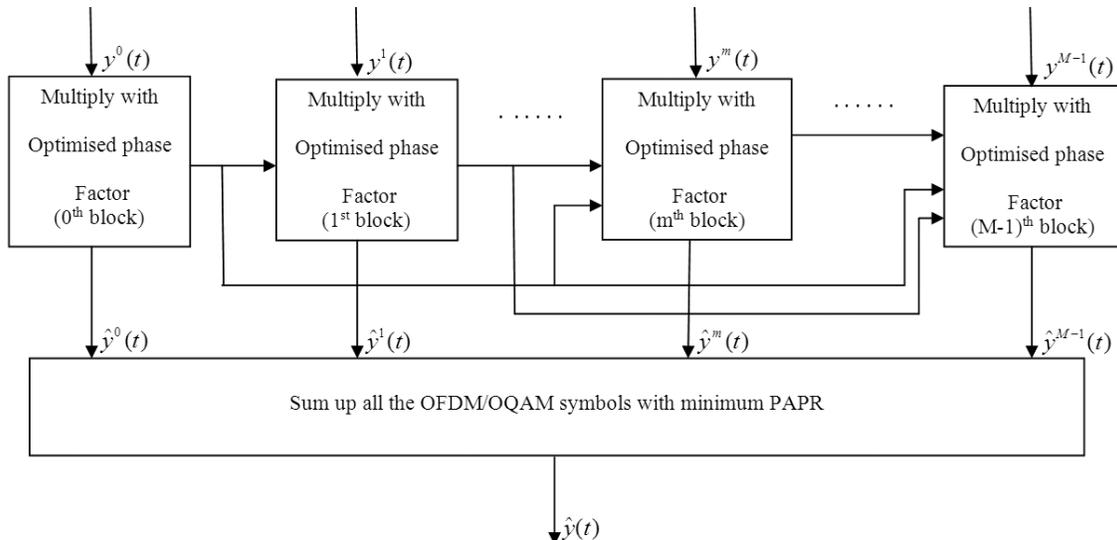


Figure 3. Block diagram of MAS-S technique.

where the PAPR reduction problem is formulated as

$$PAPR_1 = \frac{\max [ |\hat{y}^0(t) + \sum_{k=0}^{N-1} y_k^1(t) c_k^u|^2 ]}{E[ |\hat{y}^0(t) + \sum_{k=0}^{N-1} y_k^1(t) c_k^u|^2 ]} \quad (18)$$

The new OFDM/OQAM symbol having minimum PAPR at the output of the 1<sup>st</sup> block is  $\hat{y}^1(t)$ . Next  $\hat{y}^0(t)$  and  $\hat{y}^1(t)$  are given as inputs to the 2<sup>nd</sup> block to obtain the new symbol with minimum PAPR denoted as  $\hat{y}^2(t)$ . Now this procedure is repeated until the (M-1)<sup>th</sup> block. The final PAPR optimisation problem in the m<sup>th</sup> block can be formulated as

$$PAPR_{MAS-S} = \frac{\max [ |\sum_{p=0}^{m-1} \hat{y}^p(t) + \sum_{k=0}^{N-1} y_k^m(t) c_k^u|^2 ]}{E[ |\sum_{p=0}^{m-1} \hat{y}^p(t) + \sum_{k=0}^{N-1} y_k^m(t) c_k^u|^2 ]} \quad (19)$$

Our objective is to minimise the  $PAPR_{MAS-S}$  as specified in Eqn. (19) using sequential optimisation. The MAS-S algorithm can be described as follows

Step 1: Obtain all the OFDM-OQAM symbols as given by  $y^0(t), y^1(t), \dots, y^m(t), \dots, y^{M-1}(t)$ . and apply them as inputs.

Step 2: Multiply  $y^0(t)$  with the optimised phase factor and the symbol with minimum PAPR is denoted as  $\hat{y}^0(t)$ . It is given as input to the 1<sup>st</sup> block to obtain  $\hat{y}^1(t)$ .

Step 3: In the m<sup>th</sup> block, solve the PAPR optimisation problem as specified in Eqn. (19) and denote the new symbol with the minimum PAPR as  $\hat{y}^m(t)$ .

Step 4: Increment  $m$  by one. If  $m \leq M-1$  goto Step 3. Otherwise, determine  $\hat{y}(t) = \sum_{m=0}^{M-1} \hat{y}^m(t)$  and display the final output.

Next we show the MAS-S algorithm steps described above in a flowchart representation. Figure 4 illustrates the flowchart of MAS-S algorithm.

First the OFDM/OQAM symbols are given as inputs to the blocks as shown in Fig. 4. After multiplying the symbols with optimum phase factors, the PAPR optimisation problem as specified in Eqn. (19) is performed for every block. The condition is tested by incrementing  $m$  by one. If  $m > M-1$ , then the desired output is displayed by summing up all the OFDM/OQAM symbols with minimum PAPR. Otherwise the procedure from multiplication with optimised phase factors and PAPR optimisation is repeated.

#### 4.4 Quantitative Analysis of PAPR

##### 4.4.1 Analytical Expression for CCDF of AS-I, AS-C and MAS-S Algorithms

Analytical expression for CCDF in terms of PAPR for the AS-I, AS-C and MAS-S algorithms is derived here. The amplitude of an OFDM-OQAM signal follows Rayleigh distribution while the power has chi-square distribution with zero mean. Let  $N$  be the number of subcarriers for the OFDM-OQAM system. The cumulative distribution function (CDF) of the system is expressed as

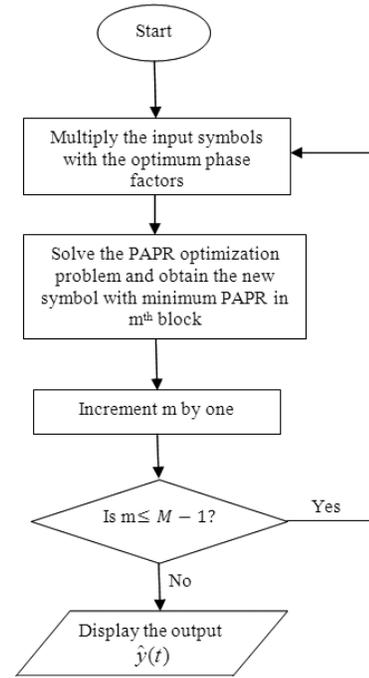


Figure 4. Flowchart of MAS-S algorithm.

$$F(z) = \int_0^z f_s(s) ds \quad (20)$$

where  $f_s(s)$  is the Rayleigh distributed probability density function. So, Eqn. (20) can be represented as

$$F(z) = \int_0^z \frac{s}{\sigma^2} \exp\left(-\frac{s^2}{2\sigma^2}\right) ds \quad (21)$$

where  $\sigma^2$  is the variance and  $\sigma$  is the standard deviation of the Random variable 'S'.

The integral in Eqn. (21) can be evaluated by making the appropriate substitution as

$$\frac{s^2}{2\sigma^2} = k \quad (22)$$

The integral in Eqn. (22) becomes

$$F(z) = \int_{k=0}^{\frac{z^2}{2\sigma^2}} \exp(-k) dk \quad (23)$$

Evaluating the integral in Eqn. (23) yields the result as

$$F(z) = 1 - \exp\left(-\frac{z^2}{2\sigma^2}\right) \quad (24)$$

The above equation can be expressed as

$$F(z) = 1 - \exp(-p_0) \quad (25)$$

where  $p_0 = \frac{z^2}{2\sigma^2}$  represents the PAPR threshold. Assuming that the signal samples are mutually independent, the CDF of an OFDM data block with  $N$  subcarriers is given by

$$F(z) = [1 - \exp(-p_0)]^N \quad (26)$$

From Eqn. (26), the CDF can be expressed as

$$P(PAPR \leq p_0) = [1 - \exp(-p_0)]^N \quad (27)$$

The complementary CDF (CCDF) is given by

$$P(PAPR > p_0) = \{1 - [1 - \exp(-p_0)]\}^N \quad (28)$$

The CCDF of PAPR for oversampled signal as suggested in<sup>23</sup> can be expressed as

$$P(PAPR > p_0) = \{1 - [1 - \exp(-p_0)]\}^{N\alpha} \quad (29)$$

In Eqn. (29)  $\alpha$  is a parameter related to oversampled signal. It is also specified in Ref.<sup>26</sup> that  $\alpha=2.8$  is a good approximation for the oversampling factor of 4.

Equation (29) can be represented as

$$CCDF = \{1 - [1 - \exp(-p_0)]\}^{N\alpha} \quad (30)$$

This expression derived in Eqn. (30) serves as the expression for CCDF in terms of PAPR for AS-I, AS-C and MAS-S algorithms.

#### 4.4.2 PAPR Analysis of AS-I, AS-C and MAS-S Algorithms

Expression for PAPR for MAS-S algorithm is derived by considering the optimised phase factors given by  $c_u^k = [1 \ -1 \ j \ -j]$  and compare it with the case for AS-I algorithm where the phase factors are  $c_u^k = [1 \ -1]$ .

##### Approach I

Here the phase factors are  $c_u^k = [1 \ -1 \ j \ -j]$  which specifies MAS-S method. When the phase factors are multiplied with the OFDM-OQAM symbols having constant amplitude of  $A$ , the resultant symbols are given by

$$\tilde{y}_k(t) = [A \ -A \ jA \ -jA] \quad (31)$$

To determine the expression for PAPR, we determine the peak power and average power separately.

The peak power of  $\tilde{y}_k(t)$  is given by

$$P_{peak} = \sum_{k=0}^3 |\tilde{y}_k(t)|^2 = 4A^2 \quad (32)$$

Let  $P_1, P_2, P_3, P_4$  be the individual powers of the respective OFDM/OQAM symbols in Eqn. (31). Then the average power is given by

$$P_{avg} = E[|\tilde{y}_k(t)|^2] = A^2(P_1 + P_2 + P_3 + P_4) \quad (33)$$

The expression for PAPR is given by

$$PAPR = \frac{P_{peak}}{P_{avg}} = \frac{4A^2}{A^2(P_1 + P_2 + P_3 + P_4)} \quad (34)$$

##### Approach II

In this approach, the phase factors are  $c_u^k = [1 \ -1]$  which represents the phase factors for AS-I algorithm. Here the OFDM-OQAM symbols are multiplied with the phase factors to obtain the new symbols as

$$\tilde{y}_k'(t) = [A \ -A] \quad (35)$$

The peak power of  $\tilde{y}_k'(t)$  is given by

$$P'_{peak} = \sum_{k=0}^1 |\tilde{y}_k'(t)|^2 = 2A^2 \quad (36)$$

If  $P_1, P_2$  are the individual powers of the respective OFDM/OQAM symbols in Eqn. (35), then the average power is given by

$$P'_{avg} = E[|\tilde{y}_k'(t)|^2] = A^2(P_1 + P_2) \quad (37)$$

The expression for PAPR is given by

$$PAPR' = \frac{P'_{peak}}{P'_{avg}} = \frac{2A^2}{A^2(P_1 + P_2)} \quad (38)$$

From Approach I, we observe that the PAPR expression attains lower value compared to the PAPR obtained in Approach II. So we infer that MAS-S algorithm which makes use of the optimised phase sequence gives lower PAPR compared to that of AS-I algorithm.

#### 4.5 Total Complexity of the Proposed MAS-S, AS-C and AS-I Algorithms

In this section, we perform the complexity analysis of the proposed MAS-S, AS-C and AS-I algorithms. We derive the expressions for total complexity of the three algorithms. The total complexity is the sum of the computational complexity and searching complexity. The computational complexity is equal to the number of complex additions and complex multiplications required. The searching complexity is equal to the total number of searches required for implementing the required algorithm.

##### 4.5.1 Total complexity of the Proposed MAS-S Algorithm

###### 4.5.1.1 Computational Complexity

Consider the proposed MAS-S algorithm that has 'N' subcarriers and 'V' sub-blocks. The total number of IFFT operations required for our proposed method is the sum of number of complex additions and complex multiplications. We know that for a standard IFFT flow graph, the number of complex additions required for an IFFT length of  $N$  is  $M \log_2 N$  and the number of complex multiplications is  $(1/2)M \log_2 N$ .

For  $V$  sub-blocks, the number of complex additions and multiplications required are  $V \log_2 N$  and  $(1/2)V \log_2 N$  respectively. So, the computational complexity of the proposed MAS-S algorithm is equal to the sum of number of complex additions and complex multiplications required which is given by  $T_1 = (3/2)V \log_2 N$ .

###### 4.5.1.2 Searching Complexity

Comprehensive search is performed for searching the optimum phase factors  $c^u$ . For every OFDM-OQAM symbol, the searches required are  $U$ . So, the complexity required for searching all  $M$  symbols for  $m = 0, 1, \dots, (M-1)$  is  $T_2 = UM$ .

Hence the total complexity of the proposed MAS-S algorithm is the sum of computational complexity and searching complexity. It is given by

$$T_{MAS-S} = T_1 + T_2 = \frac{3VN \log_2 N}{2} + UM \quad (39)$$

##### 4.5.2 Total complexity of the AS-C Algorithm

###### 4.5.2.1 Computational Complexity

The computational complexity is same as the proposed MAS-S algorithm which is equal to  $T_1' = (3/2)V \log_2 N$ .

#### 4.5.2.2 Searching Complexity

Here also exhaustive search is performed to search for the optimum value of phase factors. For each symbol,  $U$  searches are required. So, the complexity required for searching all the  $M$  symbols for  $m = 0, 1, \dots, (M-1)$  exponentially increases and is equal to  $T_2' = UM$ .

Total complexity of AS-C algorithm is equal to the sum of computational complexity and searching complexity which is given by

$$T_{AS-C} = T_1' + T_2' = \frac{3VN \log_2 N}{2} + UM \quad (40)$$

#### 4.5.3 Total complexity of the AS-I Algorithm

The total complexity of AS-I algorithm is same as the proposed MAS-S algorithm which is expressed as

$$T_{AS-I} = T_1 + T_2 = \frac{3VN \log_2 N}{2} + UM \quad (41)$$

From Eqns. (39) and (40), it is evident that though the computational complexity of MAS-S algorithm is same as that of AS-C algorithms, the searching complexity is very much less than that of AS-C algorithm which is exponentially increasing. Hence we can observe that the total complexity of MAS-S algorithm is very less than that of AS-C algorithm.

Though the total complexity of AS-I algorithm is same as that of MAS-S algorithm, AS-I algorithm has poor PAPR reduction performance than that of MAS-S algorithm. PAPR performances of these three algorithms were discussed in Section 5.

#### 4.5.4 Searching Complexity Calculations for Proposed MAS-S Algorithm and AS-C Algorithm

Here we show the searching complexity calculations for AS-C and MAS-S algorithms. Since the complexity required for AS-I and MAS-S algorithms is same, we consider the complexity for AS-C and MAS-S algorithms. We consider  $U = 2$  and the number of OFDM-OQAM symbols  $M = 5, 10, 15$  in the calculations. We use the expressions  $U^M$  and  $UM$  for complexity of AS-C and MAS-S algorithms as specified in Sections 4.5.2 and 4.5.1.

Table 1 shows the complexity calculations for AS-C and MAS-S algorithms for  $U = 2$  and  $M = 5, 10, 15$ . We observe that the complexity required increases exponentially for AS-C algorithm while it is very less for MAS-S algorithm. The complexity values are 10, 20, 30 for  $M = 5, 10, 15$  for MAS-S algorithm while for AS-C algorithm, the complexity values required are 32, 1,024, 32,768 for  $M = 5, 10, 15$ , respectively.

**Table 1. Complexity of AS-C and MAS-S algorithms with  $U=2$  and  $M=5, 10, 15$**

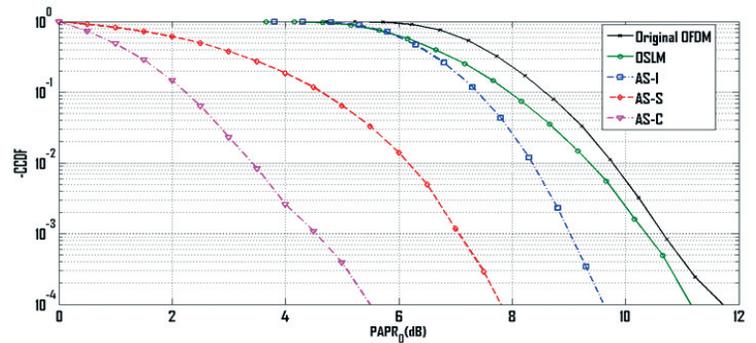
Number of Subcarriers (M)	Complexity required	
	AS-C	MAS-S
5	32	10
10	1,024	20
15	32,768	30

The complexity results justify the lower complexity of MAS-S algorithm as compared to that of AS-C algorithm.

## 5. RESULTS AND DISCUSSIONS

To observe the PAPR reduction performances of the three AS algorithms namely AS-I, AS-C and MAS-S, Monte Carlo simulations have been performed using MATLAB software. We consider 4-QAM, 8 QAM and QPSK modulation schemes with the number of subcarriers  $N = 64$ . The number of sub-blocks is set to  $V = 2, 4, 6, 8$ . The oversampling factor  $L = 4$  and the impulse response of the prototype filter is considered as defined in Eqn. (8). The normalised constants are taken from  $d(0)$  to  $d(3)$  as specified in Eqn. (9). The number of OFDM/OQAM symbols generated is 50,000. We employ 4-QAM modulation in Figs. 5-7.

Figure 5 shows the CCDF vs. PAPR plot for the phase rotation factors  $c_u^k = [1 \ -1]$ . Here  $N = 64$ ,  $V = 2$  and  $L = 4$ . The figure shows the PAPR performances of the three algorithms Individual, Combined and Sequential AS methods. The CCDF vs. PAPR threshold of conventional OFDM is plotted first. We then show the CCDF plots for conventional partial transmit technique (C-PTS) and compare with AS-I, AS-C and AS-S algorithms.



**Figure 5. CCDF vs. PAPR plots for C-PTS, AS-I, AS-C and AS-S algorithms.**

The PAPR values at CCDF of  $10^{-4}$  for AS-I, AS-S and AS-C are 9.7 dB, 7.8 dB and 5.8 dB, respectively whereas for C-PTS, the PAPR value is about 10.8 dB. We observe that the three AS algorithms decrease PAPR effectively compared to C-PTS technique. Among the three AS algorithms, though AS-C algorithm reduces PAPR, it requires very high complexity and hence obviously more number of searches. Hence it is impractical to use AS-C method for PAPR reduction in OFDM/OQAM systems. AS-I algorithm results in poorer PAPR performance and performance of AS-S algorithm lies in between that of AS-I and AS-C algorithms. AS-S algorithm shows improved PAPR performance compared to that of AS-I algorithm.

Figure 6 shows the CCDF vs PAPR plot when the phase rotation factors are  $c_u^k = [1 \ -1 \ j \ -j]$ . It shows the PAPR performances of AS-I, AS-C and MAS-S algorithms. The CCDF vs. PAPR of conventional OFDM is plotted first. The values of PAPR at CCDF of  $10^{-4}$  are 9.8 dB, 6.8 dB and 5.7 dB respectively. It is clear that MAS-S algorithm has better PAPR reduction performance compared to that of AS-I algorithm.

In Fig. 6, even if AS-C algorithm reduces PAPR

compared to MAS-S, because of higher complexity, we do not consider the performance of the AS-C algorithm. The PAPR performance of MAS-S algorithm shown in Fig. 6 depicts clearly the reduction in PAPR as compared to that AS-I algorithm. Here we make use of optimised phase sequence which gets multiplied with the sequentially generated OFDM/OQAM symbols.

PAPR reduction performance for AS-I and MAS-S algorithms with  $V = 4, 6, 8$  are as shown in Fig. 7. The values of PAPR at CCDF of  $10^{-3}$  for AS-I algorithm with  $V = 4, 6, 8$  are about 9 dB, 8.7 dB and 8.2 dB where as the PAPR values for MAS-S algorithm are 7.6 dB, 7.1 dB, and 6 dB. We see that as the number of sub-blocks ‘V’ increases, PAPR decreases for both methods.

MAS-S algorithm has good PAPR performance compared to that of AS-I algorithm. It is evident from Eqn. (34) of Approach I that the PAPR of MAS-S technique is lower compared to that of Approach II.

Figure 8 illustrates the PAPR performances of the AS-I and MAS-S algorithms for 8-QAM modulation with optimum phase sequence. The PAPR values at CCDF of  $10^{-4}$  are 9.7 dB, and 8.4 dB. Because of very high searching complexity, it is impractical and we do not consider the PAPR performance of AS-C algorithm. We also observe that AS-I and MAS-S algorithms achieve reduction in PAPR compared to conventional PTS method. MAS-S algorithm achieves about 1.3 dB reduction in PAPR compared to AS-I algorithm. Next we consider the PAPR performance of AS-I and MAS-S algorithms with QPSK modulation.

Figure 9 shows the PAPR performances of the AS-I and MAS-S algorithms for QPSK modulation with optimum phase sequence. The PAPR values at CCDF of  $10^{-4}$  are 9.6 dB, and 7.3 dB. We infer that AS-I and MAS-S algorithms achieve reduction in PAPR compared to conventional PTS method. MAS-S algorithm shows considerable degradation in PAPR compared to AS-I algorithm. The performance of AS-C algorithm is not considered because of its very high complexity.

Figure 10 shows the bit error rate (BER) performance of AS-I and MAS-S algorithms. The demodulator as shown in Fig. 2 is employed for evaluating the BER performance. Here we employ QPSK modulation at the transmitter section. Rayleigh fading channel is considered with IFFT length or number of subcarriers  $N = 64$ . The oversampling factor is set to  $L = 4$ . It is evident from Fig. 10 that MAS-S algorithm achieves lesser bit error rate than that of AS-I algorithm and hence it has a good BER performance.

### 6. CONCLUSIONS

A modified Alternative-signal technique for PAPR reduction in OFDM-OQAM system has been proposed in this paper. It was observed that AS-I algorithm has less complexity but has poor PAPR reduction performance. AS-C algorithm has good PAPR performance but has very high complexity. MAS-S algorithm which makes use of sequential optimisation procedure has better PAPR performance and at the same time has low complexity. PAPR performances of AS-I and MAS-S algorithms for

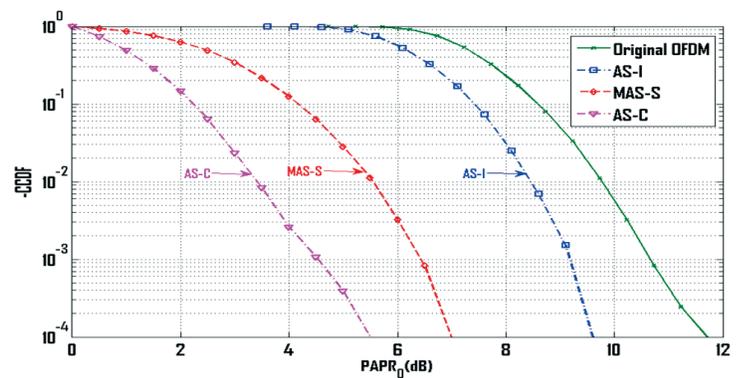


Figure 6. CCDF vs PAPR plots for AS-I, AS-C and MAS-S algorithms for 4-QAM modulation.

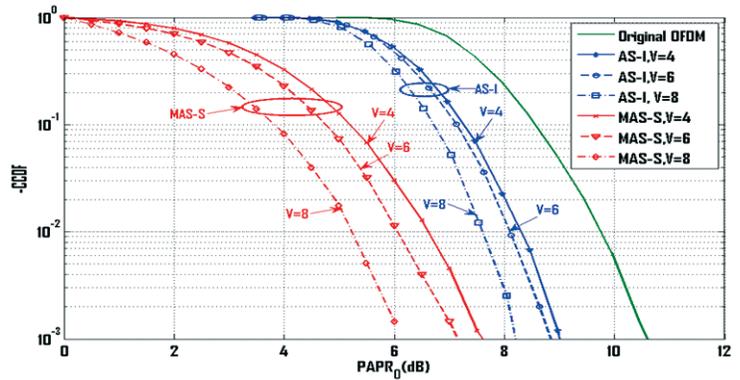


Figure 7. PAPR performance for AS-I and MAS-S algorithms with  $V = 4, 6, 8$  for 4-QAM modulation.

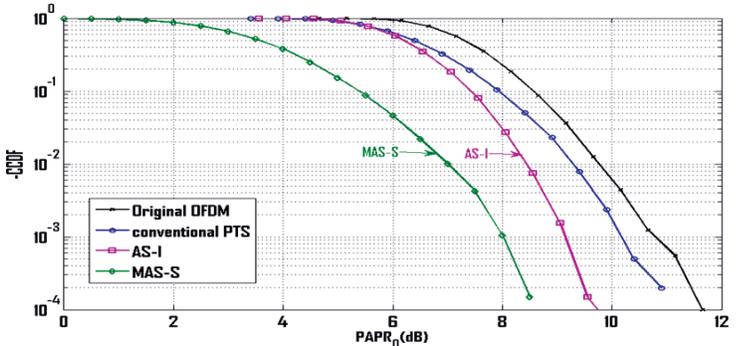


Figure 8. CCDF vs PAPR plots for AS-I and MAS-S algorithms for 8-QAM modulation.

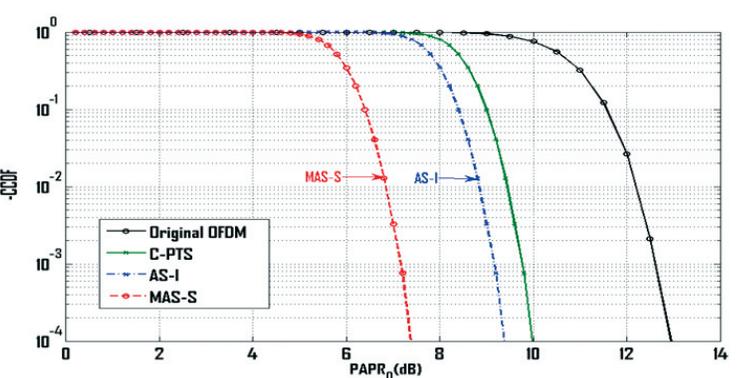


Figure 9. CCDF vs PAPR plots for AS-I and MAS-S algorithms for QPSK modulation.

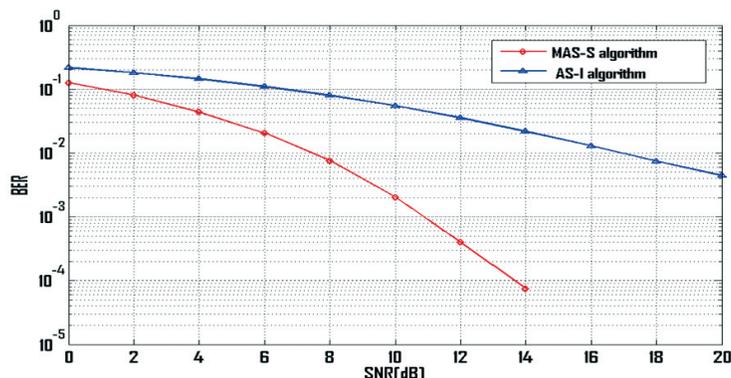


Figure 10. BER performance of AS-I and MAS-S algorithms.

QPSK, 4-QAM and 8-QAM modulation schemes have been evaluated. Simulation results convey that among the three algorithms, MAS-S algorithm is a good option for lowering PAPR in OFDM-OQAM systems since it provides better PAPR performance than that of AS-I algorithm and has very less complexity than that of AS-C algorithm. It is also seen that the proposed MAS-S algorithm has good BER performance compared to that of AS-I algorithm. MAS-S algorithm decreases the complexity of Digital to Analog converters and enhances the efficiency of power amplifier. The proposed technique is best suited to enhance the performance of wireless systems such as wireless interoperability for microwave access (WiMAX), LTE-Advanced and HIPERLAN.

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