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Effect of Yield Criterion on Stress Distribution and Maximum Safe Pressure for an Autofrettaged Gun Barrel

Amit Bhetiwal*, Sunil Kashikar, Haribhau Markale, and S.V. Gade

Gun Structures Design Group, Armament Research and Development Establishment, Pune - 411 021, India

E-mail: drdo.amitbhetiwal@gmail.com

ABSTRACT

World artillery in the present scenario is changing its role from defensive to aggressive nature and is attempting to achieve higher penetration into enemy targets. Even for an autofrettaged gun barrel, higher ranges requirement leads to higher barrel weight and its associated demerits. The design of gun barrel is based on the choice of yield criteria. Tresca yield criterion provides conservative design for a ductile barrel material. On the other hand, more accurate von Mises criterion presents complexity. The two criteria to evaluate various parameters required for design of an autofrettaged gun barrel are compared. The methodology for evaluation of maximum safe pressure, based on von Mises criterion, for an autofrettaged gun barrel is also included in the paper. Based on case study included in the article, for an autofrettaged gun barrel or pressure vessel with uniform thickness, a theoretical weight reduction of approximately 16 per cent is feasible with von Mises criterion as compared to Tresca criterion.

Keywords: Gun barrel; Autofrettage; Residual stress; Yield criteria; Maximum safe pressure

1. INTRODUCTION

Artillery gun barrels are intended to sustain high order of chamber pressure due to rapid burning and release of energy by gun propellant. The high pressure inside the gun barrel causes the projectile to move inside the bore and attain the desired launch velocity, known as muzzle velocity, to reach the desired range. The modern day artillery guns are no longer required only to perform exclusive defensive role, but also to penetrate deep in the enemy's territory and achieve deterrence. Thus, the longer range is, the higher muzzle velocity is required and consequently, higher the chamber pressure needs to be sustained by the gun barrel. Also, higher pressure levels causes decline in the fatigue life of the gun barrel. With the classical methods, this will lead to increase in the gun barrel weight and additional load to be borne by elevation drives. This cascading effect further leads to higher system weight and issues related to higher weight. In order to provide the ability to withstand high in-bore pressure and also to increase fatigue life of the gun barrel, autofrettage technique is utilised. In this technique, gun barrel is first forged and machined to slightly lesser internal diameter and higher outer diameter than desired. Then a high pressure is applied inside the gun barrel, either hydraulically or by ramming a swage through it, so that it makes the inner section (only upto certain radius) of the gun barrel to expand plastically. The elastic outer layers cause residual compressive stresses at the bore after removal of the pressure. The pressure which causes radial distribution of plastic and elastic layers is known as autofrettage pressure. During firing round the pressure in the bore has to overcome these compressive stresses before tensile stresses can be developed, thereby increasing safe working pressure and consequently, the fatigue lifetime[1].

The problem associated with barrel weight is greatly mitigated by the autofrettage process. However, the application of suitable strength theory may further be used for optimisation of barrel weight for a given safe limit. The safe limit of a gun barrel is defined in term of maximum safe pressure (MSP) that barrel can withstand without causing permanent deformation. Generally, barrel designer uses the third strength theory (Tresca criterion) as a basis for evaluation of MSP. However, barrel material being ductile, the fourth strength theory (von Mises criterion) should be more appropriate for evaluation of MSP[2]. The effect of yield criterion including the Bauschinger Effect has been discussed by Huang & Cui[3]. Clark[4], discussed two yield criteria namely Tresca criterion and von Mises criterion. It also states von Mises criterion as more accurate for prediction of bore yielding. However, a modified yield criterion has been described by Warren[5] and Hill[6] to avoid complexity of using von Mises criterion. Ab Ayob[7], *et al.* has included calculation of allowable internal pressure of autofrettage cylinder only by Tresca criterion. However, calculation of MSP based on von Mises criterion is not included in any of these studies.

Von Mises criterion, despite being more accurate and appropriate for ductile material, due to complexity of calculation has not been so popular among the pressure vessels and gun barrel designers. Instead, either Tresca criterion or modified yield criterion has been adopted. The calculation of MSP based on more accurate von Mises criterion for an autofrettaged gun barrel is also not available in the standards. Therefore, this

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paper aims at detailed discussion on evaluation of MSP based on both strength theories and their comparison with a linearly elastic perfectly plastic material model. The autofrettage process and methodology for calculation of overall stress in an autofrettaged barrel is also covered briefly in the paper.

2. AUTOFRETTAGE

For same material and thickness of gun barrel, the value of autofrettage pressure decides the depth of autofrettage i.e. radius of elastic-plastic junction or autofrettage radius. For the optimum autofrettage process, the autofrettage pressure should be selected so that the equivalent stress is minimum at the radius of elastic-plastic junction. This optimisation is discussed in detail and the methodology was applied to optimise the weight of the cylinder by Majzoobi and Ghomi[8]. ANSYS has been used by Abu[9], *et al.* to determine the residual stress for the optimum autofrettage radius. Various methods of predicting the residual stresses are also discussed by Gibson[10], *et al.*.

Zhu and Yang[1], in their study, have derived an analytical equation for optimum radius of elastic-plastic juncture for elastic perfectly plastic material. The optimum radius of elastic-plastic junction based on Tresca (IIIrd strength theory) and von Mises criterion (IVth strength theory) are given by Eqn. (1) and Eqn. (2)[1],

$$r_{jopt}^{III} = r_i * \exp\left(\frac{P}{\sigma_y}\right) \quad (1)$$

$$r_{jopt}^{IV} = r_i * \exp\left(\frac{\sqrt{3}P}{2\sigma_y}\right) \quad (2)$$

where r_i is inner radius of the gun barrel forging for autofrettage, r_o is outer radius of the gun barrel forging for autofrettage, P is internal working pressure in MPa, and σ_y is yield strength of the gun barrel material in MPa.

The optimum autofrettage pressures which corresponds to the above optimum radii of elastic-plastic junction for elastic perfectly plastic material are given by[1],

$$P_{aopt}^{III} = \frac{\sigma_y}{2}\left[1-\left(1-\frac{2P}{\sigma_y}\right)\exp\left(\frac{2P}{\sigma_y}\right)\right]+P \quad (3)$$

$$P_{aopt}^{IV} = \frac{\sigma_y}{\sqrt{3}}\left[1-\left(1-\frac{\sqrt{3}P}{\sigma_y}\right)\exp\left(\frac{\sqrt{3}P}{\sigma_y}\right)\right]+P \quad (4)$$

On apply the above optimum autofrettage pressure, following residual stresses will be generated on the gun barrel[11],

2.1 Residual Radial Stress

For plastic region, i.e. $r_i \le r \le r_{jopt}$

$$\sigma_{rr} = \sigma_y \left[\begin{array}{l} \dfrac{r_i^2}{r_o^2 - r_i^2}\left(1-\dfrac{r_o^2}{r^2}\right)\left\{\dfrac{r_{jopt}^2 - r_o^2}{2r_o^2} - \ln\left(\dfrac{r_{jopt}}{r_i}\right)\right\} \\ +\left\{\dfrac{r_{jopt}^2 - r_o^2}{2r_o^2} - \ln\dfrac{r_{jopt}}{r}\right\}\end{array}\right] \quad (5)$$

For elastic region, i.e. $r_{jopt} \le r \le r_o$

$$\sigma_{rr} = \sigma_y\left(1-\frac{r_o^2}{r^2}\right)\left[\left\{\frac{r_{jopt}^2}{2r_o^2} + \frac{r_i^2}{r_o^2 - r_i^2}\left(\frac{r_{jopt}^2 - r_o^2}{2r_o^2} - \ln\frac{r_{jopt}}{r}\right)\right\}\right] \quad (6)$$

2.2 Residual Hoop Stress

For plastic region, i.e. $r_i \le r \le r_{jopt}$

$$\sigma_{\theta r} = \sigma_y \left[\begin{array}{l} \dfrac{r_i^2}{r_o^2 - r_i^2}\left(1+\dfrac{r_o^2}{r^2}\right)\left\{\dfrac{r_{jopt}^2 - r_o^2}{2r_o^2} - \ln\left(\dfrac{r_{jopt}}{r_i}\right)\right\} \\ +\left\{\dfrac{r_{jopt}^2 + r_o^2}{2r_o^2} - \ln\dfrac{r_{jopt}}{r}\right\}\end{array}\right] \quad (7)$$

For elastic region, i.e. $r_{jopt} \le r \le r_o$,

$$\sigma_{\theta r} = \sigma_y\left(1+\frac{r_o^2}{r^2}\right)\left[\left\{\frac{r_{jopt}^2}{2r_o^2} + \frac{r_i^2}{r_o^2 - r_i^2}\left(\frac{r_{jopt}^2 - r_o^2}{2r_o^2} - \ln\frac{r_{jopt}}{r}\right)\right\}\right] \quad (8)$$

2.3 Working stress

Internal pressure build-up during the firing of the round causes the working stress, which can be evaluated by using Lame's equations[11] as follows:

For radial stress,

$$\sigma_r = \frac{P \times r_i^2}{r_o^2 - r_i^2}\left(1-\frac{r_o^2}{r^2}\right) \quad (9)$$

For tangential or hoop stress,

$$\sigma_\theta = \frac{P \times r_i^2}{r_o^2 - r_i^2}\left(1+\frac{r_o^2}{r^2}\right) \quad (10)$$

2.4 Overall Stress

During the firing of the rounds from an autofrettage gun barrel, the working stress gets superimposed on the existing residual stresses and stress distribution through the barrel thickness gets transformed. This overall stress distribution can be obtained by algebraic sum of stress due to internal working pressure and the residual stress caused by autofrettage.

Overall radial stress, $\sigma_{rT} = \sigma_r + \sigma_{rr}$ (11)

Overall hoop stress, $\sigma_{\theta T} = \sigma_\theta + \sigma_{\theta r}$ (12)

For a typical 155 mm artillery gun barrel, the stress distribution is as shown is Fig. 1.

Figure 1 clearly shows that the two strength theories yield different stress distributions. The effect on the optimum radius of elastic plastic junction and residual stress is significant. Whereas, it is observed that the difference is minimal in case of working stress. The working stress corresponds to the stress for a non-autofrettaged barrel. This signifies that the choice of failure theories i.e. either third (Tresca) or fourth (Mises) strength theory, is irrelevant in case of non-autofrettaged gun barrel.

Figure 2 shows distinct equivalent stresses based on two theories for an autofrettage barrel. Thus, it is clear that for an autofrettaged barrel, the choice of strength theory is significant.

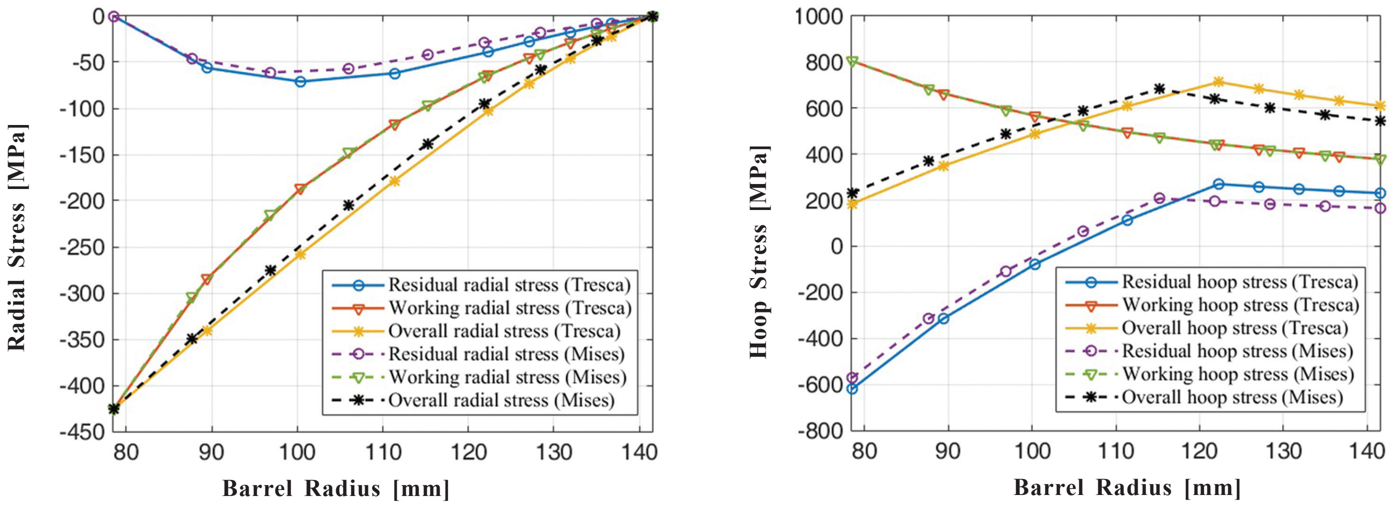

Figure 1. Typical stress distribution of an autofrettage gun barrel.

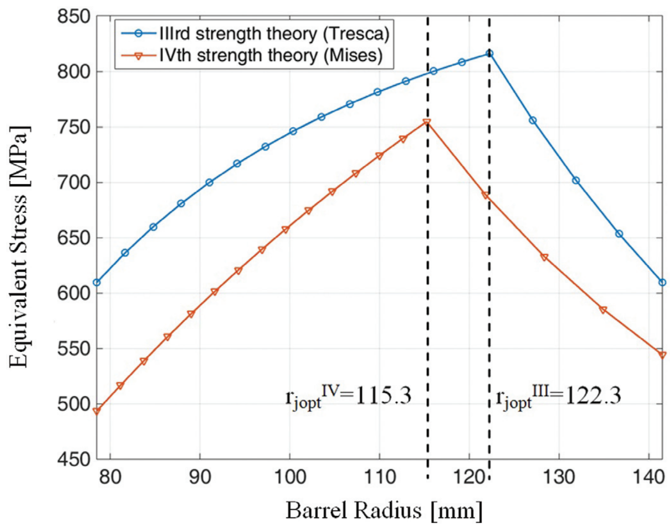

Figure 2. Comparison of equivalent stress based on IIIrd (Tresca) and IVth strength theories (Mises).

3. MAXIMUM SAFE PRESSURE

Various gun design pressure curves, along the length of the barrel, required for the design and evaluation of a gun barrel has been defined in STANAG 4110[12]. The important pressure curves are extreme service condition pressure (ESCP) and gun design pressure (GDP) curves. The process inside gun barrel during firing can be compared with the expansion process of IC engine, where the pressure is highest at the cylinder head and it reduces parabolically to the value of piston head cylinder. Similarly, in case of a gun barrel, the maximum gas pressure is at the breech end and it will reduce parabolically upto the maximum service pressure point. Thereafter, the pressure will further reduce adiabatically upto the pressure at muzzle end. This analogy is the basis for obtaining all gun design pressure curves. The ESCP is pressure curve with extreme ambient temperature, usually 60 °C. GDP curve is evaluated using ESCP curve by applying suitable safety criteria[12]. GDP curve is obtained merely by adding a suitable safety margin to each point on ESCP curve. The safety margin is defined in terms of the standard deviation in pressure. Thus, the safety margin is purely a statistical data and difficult to obtain with simulation. The standard deviation in pressure is defined[12] as 'an overall standard deviation for a specified system and represents the statistical distribution about the mean which is attributed to the summation of variances occurring between cannons, propellant lots, firing occasions and rounds'. GDP curve is further considered as base curve for the design of a gun barrel. Fig. 3 shows comparision of general variation of pressure for both ESCP and GDP curve.

The safe limit of a gun barrel is defined in term of maximum safe pressure (MSP). "The maximum safe pressure is the maximum pressure which the ordnance can withstand without causing permanent deformation sufficient to affect its operation or accuracy"[5]. Thus, subsequent to autofrettage, failure is considered to occur if again there is yielding of the inner surface beyond application of certain pressure. Therefore, Maximum Safe Pressure is the pressure beyond which yielding of an autofrettage barrel occurs at the inner surface. This implies that the equivalent stress at the inner radius is the basis of MSP. The factor of safety for a gun barrel is defined as the ratio of MSP to GDP.

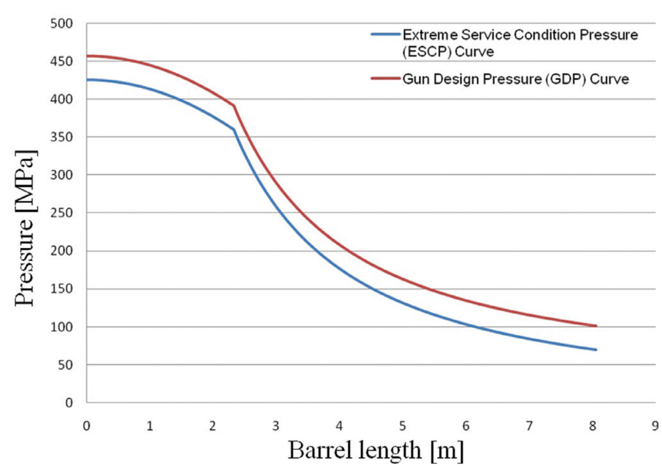

Figure 3. Comparison of ESCP and GDP curves for an autofrettage gun barrel.

In case of a thick walled tube, irrespective of end conditions, longitudinal stress (σ_z) is always the intermediate principal stress[2]. Thus, according to third strength theory (Tresca failure criterion) and fourth strength theory (von Mises failure criterion), the equivalent stresses at inner surface (i.e. $r=r_i$) are as follows[13]:

$$\sigma_{eq}^{III} = \sigma_{\theta T} - \sigma_{rT} \tag{13}$$

$$\sigma_{eq}^{IV} = \sqrt{\frac{1}{2}\left\{(\sigma_{\theta T} - \sigma_{rT})^2 + (\sigma_{rT} - \sigma_z)^2 + (\sigma_z - \sigma_{\theta T})^2\right\}} \tag{14}$$

where $\sigma_{\theta T}$ and σ_{rT} can be obtained by putting $r=r_i$ in Eqns. (5), (7), (9) to (12).

$$\sigma_{\theta T} = \left(\frac{k^2+1}{k^2-1}\right)P_{MSP} + \sigma_y\left[\left(\frac{k^2}{k^2-1}\right)\left\{\frac{m^2}{k^2} - 2\ln(m) - 1\right\} + 1\right] \tag{15}$$

$$\sigma_{rT} = -P_{MSP} \tag{16}$$

whereas σ_z at $r=r_i$ can be calculated as below[2]:

$$\sigma_z = \left(\frac{1-2\upsilon}{k^2-1}\right)P_{MSP} + 2\upsilon k\left\{\frac{m^2}{k^2} - 2\ln(m)\right\} \text{ .... Closed end} \tag{17}$$

$$= \left(\frac{-2\upsilon}{k^2-1}\right)P_{MSP} + 2\upsilon k\left\{\frac{m^2}{k^2} - 2\ln(m)\right\} \text{ ....Open end} \tag{18}$$

Presently, this study is focused on hydraulic autofrettaged process which is corresponding to closed end condition for a pressure vessel. Therefore, the expression for σ_z is chosen accordingly for further calculations.

Solving Eqn. (13) for P_{MSP} by putting $\sigma_{eq}^{III} = \sigma_y$ gives MSP based on third strength theory:

$$P_{MSP}^{III} = \frac{\sigma_y}{2}\left[2\ln(m) + \frac{k^2 - m^2}{k^2}\right] \tag{19}$$

For solving Eqn. (14) for MSP based on fourth strength theory, let's assume,

$$\sigma_z = A \times P_{MSP}^{IV} + B \tag{20}$$

$$\sigma_{\theta T} = C \times P_{MSP}^{IV} + \sigma_y D \tag{21}$$

where

$$A = \left(\frac{1-2\upsilon}{k^2-1}\right) \tag{22}$$

$$B = 2\upsilon k\left\{\frac{m^2}{k^2} - 2\ln(m)\right\} \tag{23}$$

$$C = \left(\frac{k^2+1}{k^2-1}\right) \tag{24}$$

$$D = \left[\left(\frac{k^2}{k^2-1}\right)\left\{\frac{m^2}{k^2} - 2\ln(m) - 1\right\} + 1\right] \tag{25}$$

Now solving Eqn. (14) for P by putting $\sigma_{eq}^{IV} = \sigma_y$ gives following quadratic equation in terms of P_{MSP}:

$$\begin{aligned} &2\left(A^2 + C^2 - AC + A + C + 1\right) \times {P_{MSP}^{IV}}^2 \\ &+2\left\{\sigma_y D(2C - A + 1) + B(2A - C + 1)\right\} \\ &\times P_{MSP}^{IV} + \left(2\sigma_2^2 D^2 - 2\sigma_2^2 + D^2 - 2\sigma_y BD\right) = 0 \end{aligned} \tag{26}$$

Further simplifying the Eqn. (26) gives,

$$a{P_{MSP}^{IV}}^2 + bP_{MSP}^{IV} + c = 0 \tag{27}$$

where

$$a = 2\left(A^2 + C^2 - AC + A + C + 1\right) \tag{28}$$

$$b = 2\left[\sigma_y D(2C - A + 1) + B(2A - C + 1)\right] \tag{29}$$

$$c = \left(2\sigma_2^2 D^2 - 2\sigma_2^2 + D^2 - 2\sigma_y BD\right) \tag{30}$$

Solving the quadratic Eqn. (27) gives P_{MSP}^{IV}. Only positive roots should be considered as the pressure in gun barrel acts internally.

4. DISCUSSION

For a case study of a 155 mm/52 Calibre autofrettaged gun barrel with following parameters, comparison of MSPs with the third and the fourth strength theories can be made.

$r_i = 78.5$ mm, $r_o = 141.5$ mm

$P = 426$ MPa

$\sigma_y = 960.7$ MPa

$P_{aopt}^{III} = 774.42$ MPa, $P_{aopt}^{IV} = 703.33$ MPa

$r_{jopt}^{III} = 122.31$ mm, $r_{jopt}^{III} = 115.25$ mm

$P_{MSP}^{III} = 547.48$ MPa, $P_{MSP}^{IV} = 578.42$ MPa

Based on the above results the optimum autofrettage pressure and optimum autofrettage radius is more in case of the third strength theory. It is also evident that the third strength theory is more conservative for design of an autofrettage gun barrel as compared to fourth strength theory. Whereas, the fourth strength theory is more realistic for a ductile material. Therefore, it becomes a choice for the designer to opt for conservative design and higher barrel weight or comparatively less conservative design and lesser barrel weight.

If the longitudinal stress is ignored, MSP based on the fourth strength theory is 581.89 MPa, which marginally greater i.e. 0.6 per cent increase in MSP. This is the reason why generally longitudinal stress is ignored for design of a pressure vessel including an autofrettaged gun barrel, irrespective of the end condition.

Other than working pressure and yield strength of material, the MSP is highly dependent on the barrel thickness which is usually represented by the ratio of outer radius to inner radius, denoted by k. The comparison of MSP based on the third and the fourth strength theories is depicted in the Fig. 4.

Figure 5 shows the percentage variation of MSP based on von Mises criterion with respect to Tresca criterion. It is evident from Fig. 4 that in the present case study Tresca criterion is conservative only upto certain value of k i.e. approximately 3.1 and beyond this value, von Mises becomes from conservative.

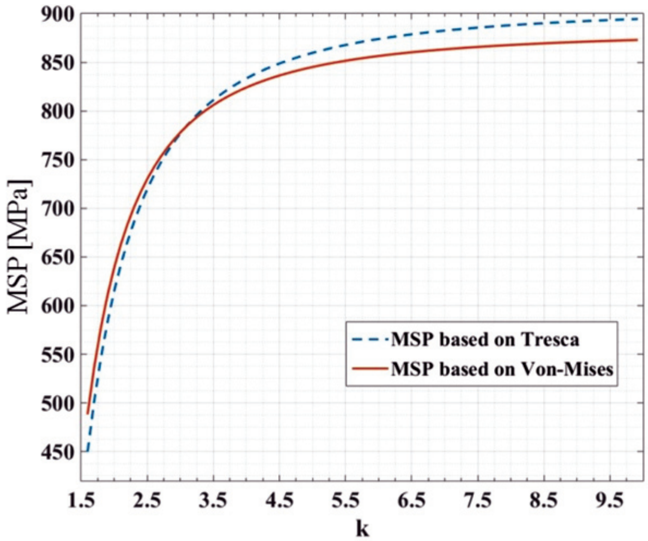

Figure 4. Comparison of MSP.

However, it is also clear that for lower values of k, von Mises gives higher MSP as compared to Tresca criterion. For the same working pressure, MSP with von Mises is increased by approximately 8 per cent with k equal to 1.6. For optimisation of barrel weight von Mises criterion must be used as it is more accurate for ductile materials and also gives higher MSP. If the barrel is assumed to be cylindrical with uniform thickness throughout, then, for the same MSP, the thickness of gun barrel can be lesser with von Mises criterion as compared to Tresca criterion. For the parameters of a gun barrel mentioned above, the barrel weight per unit length based on von Mises will be 0.285 kg/mm as compared to 0.34 kg/mm with Tresca criterion. This amounts to 16 per cent overall weight reduction.

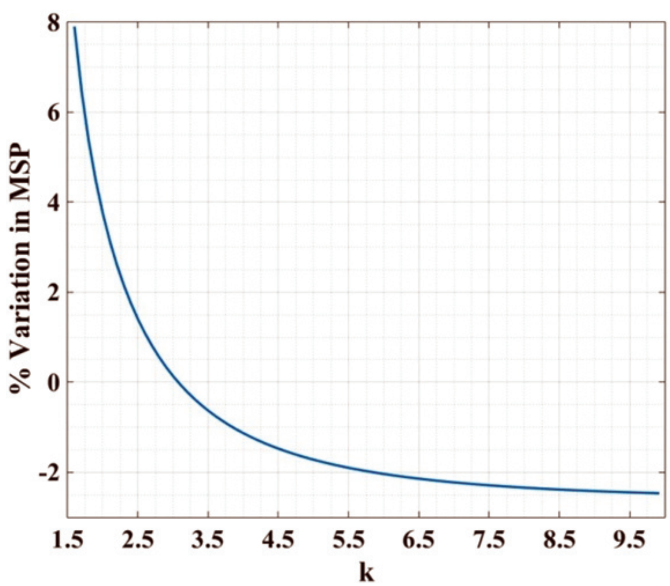

Figure 5. Percentage variation in MSP based on von Mises w.r.t. Tresca criterion.

5. CONCLUSIONS

The comparison between Tresca and von Mises yield criteria has been presented for the design of an autofrettaged gun barrel. The comparison included the effect of yield criteria on optimum autofrettage pressure, radius and equivalent stress as well as overall stress distribution on an autofrettaged gun barrel. The methodology for evaluation of maximum safe pressure (MSP) based on von Mises criterion has also been included. The case study included clear emphases on the conservativeness of Tresca yield criterion. Whereas, von Mises yield criterion, being more realistic, is proved to be complementary for optimum design of the autofrettaged gun barrel. However, Tresca criterion has been the first choice of a gun barrel designer due to its simplicity. On the other hand, reduction in the barrel weight calls for using more appropriate yield criterion i.e. von Mises. There is possibility of reduction in barrel weight by upto approximately 16 per cent.

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CONTRIBUTORS

Mr Amit Bhetiwal, received his BE (Mechanical engineering) from VNIT, Nagpur, and MSc (Gun System Design) from Defence Academy College of Management and Technology, Cranfield University. Presently working as scientist 'D' in ARDE. He has more than 10 years of experience in the area of rocket launcher and gun system development. Currently he is working on the development of Advanced towed artillery gun system.

Mr Sunil S Kashikar, is a graduated in Mechanical engineering, Post graduate in Industrial Engineering and completed his MSc (Gun System Design) from Defence Academy College of Management and Technology, Cranfield University. Presently working as scientist 'E' in ARDE. He has more than 15 years of experience in the area of weapon system development. Currently he is working on the development of advanced towed artillery gun system.

Mr Haribhau Markale, is a BTech (Mechanical Engineering) and MTech (Mechanical Engineering) from IIT Kanpur. Presently working as scientist 'G' in ARDE. He has experience in the area of sighting system and related electro-mechanical system and gun system development. Currently he is working on the development of advanced towed artillery gun system.

Dr S.V. Gade, received his MTec (Mechanical Engineering) from IIT Mumbai. Presently working as Scientist 'G' and Project Director of project 'Advanced towed artillery gun system' in ARDE. He has experience in the area of rocket launcher system development and in weapon system development including small arms and ammunition.