Defence Science Journal, Vol. 68, No. 4, July 2018, pp. 417-422, DOI : 10.14429/dsj.68.10595

Similarity Analysis of Projectile Penetration into Concrete

Meng Huang, Zhuo-Cheng Ou, Yi Tong*, Zhuo-Ping Duan, and Feng-Lei Huang

State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, China

**E-mail: tongyi@bit.edu.cn*

ABSTRACT

A dimensionless model for the depth of penetration (DOP) of a projectile penetrating into a concrete target, based on the similarity theory involving intermediate asymptotics, complete similarity, and incomplete similarity is presented. The calculated numerical results are in good agreement with previous experimental data, including two sets of full-scale and twenty-four sets of sub-scale penetration of non-deformable projectiles into concrete targets. Moreover, compared with several empirical and semi-empirical DOP models, the new model is applicable within a relatively broader range, including the penetration of both sub-scale and full-scale projectiles. For the limitations of the validity, dimensionless parameters $\Pi_3 = \phi_t/\phi$ larger than 12, $\Pi_4 = (\phi^3 f_c)/(Mv_0^2)$ smaller than 0.1, and the initial impact velocity of the projectile less than about 900 m/s to 1000 m/s are necessary for the model.

Keyword: Similarity theory; Intermediate asymptotics; Complete similarity; Incomplete similarity; Non-deformable projectile; Concrete target

NOMENCLATURE

v_0	Initial impact velocity
M	Projectile mass
E	Modulus of elasticity
μ	Poisson ratio
ϕ	Projectile shank diameter
ϕ_{in}	Inner cavity diameter
ϕ_t	Target diameter
l	Projectile shank length
l_{in}	Inner cavity shank length
V	Projectile volume
V_{in}	Inner cavity volume
ρ_p	Projectile density
ρ_t	Target density
H	Target thickness
P	Depth of penetration
S	Empirical coefficient
A	Projectile shank cross sectional area
ψ	Caliber-radius-head
f_c	Concrete unconfined compressive strength
F and F_i	Some unknown function (i= 1, 2, 3, …)
α, α_i, β_i and γ	Some undetermined constants (i= 1, 2, 3, …)

1. INTRODUCTION

Projectile penetration into the concrete targets has received much attention in the field of weapon design and civil engineering for a long time[1]. Research on such a problem has also been conducted over many years[2,3], which includes mainly three approache, i.e., experimental study, theoretical analysis and numerical simulation. Among those, the experimental study on developing empirical models for engineering applications has played a very important role[4-7], because of the difficulties in both the theoretical analysis and the numerical simulation due to the complexity of the dynamic constitutive behaviours of materials. Over the past decades, many empirical or semi-empirical DOP models for the projectile penetration into various materials have been opened[8-12]. Young[13] proposed an empirical DOP model based on a number of experimental data for the penetration of full-scale projectiles at sub ordnance and ordnance velocities into quasi-brittle materials such as concrete, rock and soil; Forrestal and his colleagues[14-16] also presented an semi-empirical DOP formula for an ogival-nosed projectile penetrating into concrete targets, based on the spherical cavity-expansion theory and their experimental data of sub-scale projectiles. Li & Chen[17] developed again a semi-empirical DOP formula available for the penetration of sub-scale projectiles by using dimensional analysis, in which an impact function and a geometry function are introduced for the first time. In addition, there are still some other empirical DOP models[2,5,18]. However, calculated results from all these existing models are usually not so consistent with each other, which makes it difficult to estimate their practical engineering applications. Meanwhile, there seems not yet a relatively general DOP formula applicable for both the sub-scale and the full-scale projectiles. So that a relatively general DOP model is expected urgently.

This study therefore focuses on developing a more general DOP model for projectile penetration into concrete, and based on the similarity theory involving intermediate asymptotics, complete similarity, and incomplete similarity, a new dimensionless DOP model is proposed.

2. SIMILARITY ANALYSIS

For a projectile penetrating into a concrete target, its DOP P depends on the initial impact velocity v_0 and several other

Received : 31 August 2016, Revised : 12 April 2018
Accepted : 07 May 2018, Online published : 25 June 2018

physical and geometric properties of both the projectile and the target, which can be expressed qualitatively as

$$P = f\left(v_0, M, E, \mu, \phi, l, V, V_{in}, H, \phi_t, f_c, \rho_t, E_t, \mu_t\right) \quad (1)$$

where M, ϕ, l, V, E and μ are the mass, shank diameter, length, volume, modulus of elasticity, and Poisson ratio of the projectile, respectively; V_{in} is the volume of the inner cavity of the projectile, which may be filled with the shaped charge; H, ϕ_t, f_c, ρ_t, E_t, and μ_t are the thickness, diameter, unconfined compressive strength, bulk density, Young's modulus and Poisson's ratio of the concrete target, respectively.

For penetration of non-deformable projectiles at sub ordnance or ordnance velocity into concrete targets, it is found experimentally that the influence of both modulus of elasticity and Poisson ratio of the projectile[19] as well as of the target[20,21] on the DOP can be neglected. In addition, most DOP models[10] appear independent on ρ_t. For the penetration into concrete targets, the density of the concrete varies over a narrow range, and the density of the projectile ρ_p could be determined by its mass M and volume V, which is why both ρ_t and ρ_p are left out of consideration in this analysis. Additionally, the omitted parameters will prove to be reasonable by the calculation in Section 3. As a result, one can re-write Eqn. (1) into

$$P = f\left(v_0, M, \phi, V, V_{in}, l, H, \phi_t, f_c\right) \quad (2)$$

In the MLT (mass, length, time) class of systems of units, the dimension of each quantity involved in Eqn. (2) is, respectively,

$$\begin{aligned} &[P] = [\phi] = [l] = [H] = [\phi_t] = L, \quad [v_0] = \frac{L}{T} \\ &[M] = M, \quad [V] = [V_{in}] = L^3, \quad [f_c] = \frac{M}{T^2 L} \end{aligned} \quad (3)$$

It is straight forward to verify that the dimensions of v_0, M and ϕ are independent. Taking v_0, M and ϕ as the governing parameters, one has

$$\begin{aligned} &[P] = [l] = [H] = [\phi_t] = [\phi] \\ &[V] = [V_{in}] = [\phi]^3, \; [f_c] = [M]\,[v_0]^2 / [\phi]^3 \end{aligned} \quad (4)$$

By the Buckingham Pi theorem, Eqn. (4) can be written into the following dimensionless form

$$\Pi = F\left(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6\right) \quad (5)$$

where F is an arbitrary function, and

$$\begin{aligned} &\Pi = \frac{P}{\phi}, \quad \Pi_1 = \frac{l}{\phi}, \quad \Pi_2 = \frac{H}{\phi}, \quad \Pi_3 = \frac{\phi_t}{\phi} \\ &\Pi_4 = \frac{\phi^3 f_c}{M v_0^2}, \quad \Pi_5 = \frac{V}{\phi^3}, \quad \Pi_6 = \frac{V_{in}}{\phi^3} \end{aligned} \quad (6)$$

Substituting Eqn. (6) into Eqn. (5), there is

$$\frac{P}{\phi} = F\left(\frac{l}{\phi}, \frac{H}{\phi}, \frac{\phi_t}{\phi}, \frac{\phi^3 f_c}{M v_0^2}, \frac{V}{\phi^3}, \frac{V_{in}}{\phi^3}\right) \quad (7)$$

There exists now six dimensionless arguments in Eqn. (5) or Eqn. (7), which can be simplified further by using intermediate asymptotics, complete similarity, and incomplete similarity[22,23].

Firstly, consider the parameter Π_2. When the other dimensionless parameters remain constant, the DOP will be a non-zero finite value as $H \to \infty$, i.e., the limits of Π are finite and non-zero at $\Pi_2 \to \infty$. That is, for a semi-infinite target with sufficiently large Π_2, there is complete similarity or similarity of the first kind in the parameter Π_2. Therefore, for a semi-infinite target, the parameter Π_2 in Eqn. (5) can be excluded, and hence Eqn. (5) becomes

$$\Pi = \alpha_1 F_1\left(\Pi_1, \Pi_3, \Pi_4, \Pi_5, \Pi_6\right) \quad (8)$$

where α_1 is a constant coefficient and F_1 is another arbitrary function different from the arbitrary function F.

Next, according to Fig. 3 in the work by Frew & Forrestal[24], *et al.* there exists negligible change in the DOP as ϕ_t/ϕ increases from 12 up to 24, which makes it reasonable to infer that Π tends to a non-zero finite value as $\Pi_3 \to \infty$. In other words, for sufficiently large Π_3, there exits complete similarity in the parameter Π_3. Similar to the aforementioned procedure, the dimensionless argument Π_3 in Eqn. (8) can also be excluded, and hence Eqn. (8) reaches to

$$\Pi = \alpha_1 \alpha_2 F_2\left(\Pi_1, \Pi_4, \Pi_5, \Pi_6\right) \quad (9)$$

where α_2 is also a constant coefficient and F_2 is an arbitrary function different from both F and F_1.

Thirdly, when the length of the projectile tends to infinite, i.e., $l \to \infty$, or $\Pi_1 \to \infty$, the bulk density of the projectile ρ_p will tend to infinitesimal, and thus the DOP should also tend to infinitesimal, that is, $\Pi \to 0$. Therefore, for sufficiently large Π_1, there is the first type of incomplete similarity in the parameter Π_1, and then Eqn. (9) can be further written in the following simplified form

$$\Pi = \alpha_1 \alpha_2 \Pi_1^{\beta_1} F_3\left(\Pi_4, \Pi_5, \Pi_6\right) \quad (10)$$

where β_1 is an undetermined negative constant exponent and F_3 is an arbitrary function.

Finally, for the parameter Π_4, it is straightforward to understand that $P \to \infty$ when $f_c \to 0$ (which implies that the resistance of projectile penetration vanishes), i.e., $\Pi \to \infty$ when $\Pi_4 \to 0$. Thus, for sufficiently small Π_4, there is the first type of incomplete similarity in the parameter Π_4, and Eqn. (10) becomes

$$\Pi = \alpha_1 \alpha_2 \Pi_1^{\beta_1} \Pi_4^{\beta_2} F_4\left(\Pi_5, \Pi_6\right) \quad (11)$$

where β_2 is another undetermined negative constant exponent and F_4 is an arbitrary function.

Moreover, when $\Pi_5 \to \infty$, Π does not converge to any limit. Actually, we have here the second type of incomplete similarity as $\Pi_5 \to \infty$. Then, Eqn. (11) could be rewritten in the following form

$$\Pi = \alpha_1 \alpha_2 \Pi_1^{\beta_1} \cdot \Pi_4^{\beta_2} \cdot \Pi_5^{\beta_3} \cdot F_5\left(\Pi_6 / \Pi_5\right) \quad (12)$$

where β_3 is an undetermined constant exponent, and F_5 is an arbitrary function, which will be determined by using a numerical fitting approach. To this end, Eqn. (12) is assumed as in the following form

$$\Pi = \Pi_1^{\beta_1} \cdot \Pi_4^{\beta_2} \cdot \Pi_5^{\beta_3} \cdot \left[\alpha\left(\Pi_6 / \Pi_5\right)^{\beta_4} + \gamma\right] \quad (13)$$

where α and γ are constant coefficients, β_4 are undetermined constant exponents.

To determine the values of the six constants α, β_1, β_2, β_3, β_4 and γ in the new penetration model, Eqn. (13), the numerical fitting technique together with six sets (two full-scale and four previous sub-scale) of experimental data[14] of non-deformable projectile penetration into concrete target has been developed, which bases on the data analysis software of Origin. For the numerical fitting, begin with fitting the data of Π_1, and Π_4 for Π, obtaining

$$\beta_1 = -0.53, \quad \beta_2 = -0.67 \tag{14}$$

Then, fitting the data of Π_5, and Π_6 for $\Pi/(\Pi_1^{\beta_1}\Pi_4^{\beta_2})$ yields

$$\alpha = 0.08, \ \beta_3 = 1.94, \ \beta_4 = 1.37, \ \gamma = 0.03 \tag{15}$$

wherein data of the necessary parameters are listed in Table 1, l_{in} and ϕ_{in} are the shank length and diameter of the inner cavity of the projectile, the two full-scale ones are denoted as Set 1 and Set 2, and the sub-scale ones are denoted as Set 3− Set 6[14], respectively. The shank volume of the inner cavity of the projectile V_{in} could be approximately evaluated by $V_{in}=\pi l_{in}\phi_{in}^{\ 2}/4$. Substituting Eqns. (14) and (15) into Eqn. (13) reaches the new DOP empirical formula as follows

$$\Pi = \Pi_1^{-0.53}\Pi_4^{-0.67}\Pi_5^{1.94}\left[0.08\cdot(\Pi_6/\Pi_5)^{1.37}+0.03\right] \tag{16}$$

or, substituting Eqn. (6) into Eqn. (16) reads in the form of the ratio of the DOP to the diameter of the projectile, as

$$\frac{P}{\phi} = \left[0.03+0.08\cdot(V_{in}/V)^{1.37}\right]V^{1.94}l^{-0.53}\phi^{-7.3}\left(Mv_0^2/f_c\right)^{0.67} \tag{17}$$

Note that, the influence of the shaped charge on DOP could be approximately quantified by filling it into the inner cavity of the projectile V_{in}.

Necessarily, the new model should satisfy all conditions for the application of similarity analysis. For the first type of incomplete similarity in the parameter Π_1, complete similarity or similarity of the first kind in the parameter Π_2 and Π_3, and the second type of incomplete similarity in the parameter Π_5, parameters Π_1, Π_2, Π_3, and Π_5 should be sufficiently large. For example, according to Barenblatt[22,23], Π should converge sufficiently rapidly to a limit as Π_1, Π_2, Π_3, and Π_5 tend to infinity, that is, for $\Pi_1 = l/\phi$, $\Pi_2 = H/\phi$, $\Pi_3 = \phi_t/\phi$, and $\Pi_5 = V/\phi^3$ larger than 10, Π will assume values sufficiently close to that limit. Generally, to eliminate the boundary effect on the penetration, the thickness of the target H is limited to be semi-infinite, while according to Frew[22], Π_3 should be larger than 12. For the first type of incomplete similarity in the parameter Π_4 should be sufficiently small. For example, Barenblatt considers $\Pi_4 = (\phi^3 f_c)/(Mv_0^{\ 2})$ should be smaller than 0.1[22,23], which proves to be reasonable by the calculation from Section 3.

Table 1. Parameters in the experiments

Sets	v_0 (m/s)	M (kg)	ϕ (mm)	ϕ_{in} (mm)	l (mm)	l_{in} (mm)	f_c (MPa)	P_e (m)
1	737	101	153	106	812	812	30	4.57
2	835	81	140	100	849	849	30	5.1
3	499	0.912	26.9	10.1	206.8	201.7	33.5	0.48
4	584	0.898	26.9	10.1	206.8	201.7	91	0.384
5	371	0.064	12.92	6.35	67.5	50.8	14	0.127
6	345	0.064	12.92	6.35	63.1	50.8	14	0.111

3. NUMERICAL RESULTS

To certify the universality of the new empirical DOP formula, Eqn. (16) or Eqn. (17), another twenty sets of previous sub-scale experimental data (Set 7− Set 26) for the DOP[14,25] have also been calculated. Denote the relative-error between the calculated results of the new DOP formula P and the experimental data P_e by $R = (P - P_e)/P_e$. The numerical results of the relative-errors under different impact velocities are as listed in Table 2. It can be seen that for most experimental data except for a few individual data, predictions of Eqn. (16) or Eqn. (17) are in good agreement with the experimental data, which remain the relative-errors generally less than 20 per cent. In general, the experimental data are difficult to be fitted into just one curve because of their scattering. Therefore, the range of relative error of this model is proposed as about 20 per cent, when taken the total data results into account.

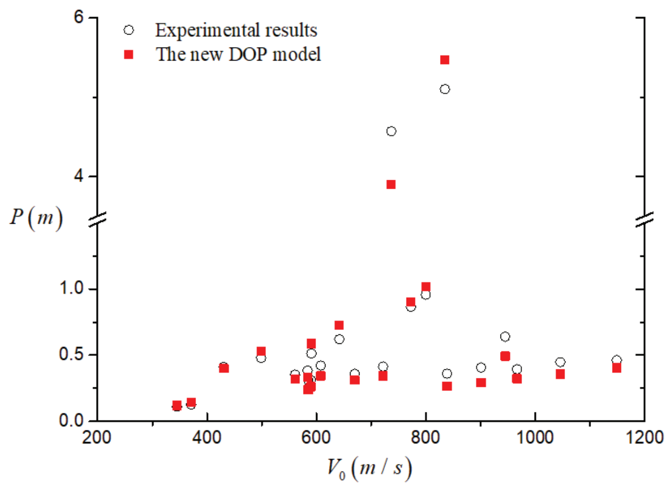

Figure 1. Comparison between the calculated numerical results and the experimental data.

For clarity, both the calculated results from the new DOP formula and the experimental data are as shown in Fig. 1, which also displays the agreement.

As comparison with the new DOP model, five well-known empirical formulas for the DOP are discussed briefly in the following.

Based on the extensive wartime penetration tests and previous research, Young[13] developed empirically the Young/Sandia penetration equation (Young's formula) for concrete targets, as

$$P = \begin{cases} 0.008SN(M/A)^{0.7}\ln(1+2.15v_0^2 10^{-4}), & v < 61\text{m/s} \\ 0.000018SN(M/A)^{0.7}(v_0 - 30.5), & v > 61\text{m/s} \end{cases} \tag{18}$$

where the projectile has cross sectional area of the shank A, caliber-radius-head ψ, and nose performance coefficient N, which takes the form $N = 0.18\,(\psi - 0.25)^{0.5} + 0.56$, and the target has the index of penetrability S, which is an empirical coefficient and usually equal to 0.9. Equation (18) may not be applicable when M < 5 kg. As shown in Table 2 and Fig. 2, in the full-scale experiments, Young's formula predicts acceptable DOP compared with the experimental data. However, Young's formula is not applicable for

Table 2. The relative-error R between the experimental data and predictions

Formula sets	M (kg)	v_0 (m/s)	Eqn. (19) (%)	Young (%)	Forrestal (%)	Li-Chen (%)	ACE (%)	BRL (%)
1	101	737	−14.7	−16.9	−18.3	−32.3	−17.7	2.6
2	81	835	7.2	−21.4	−13.9	−27.7	−16.8	2.1
3	0.912	499	10.8	0.4	2.7	−15.4	−15.9	13.5
4	0.898	584	−13.4	46.3	11.2	−13.3	−19.6	4.5
5	0.064	371	10.8	−13.5	−4.1	−19.6	−15.7	17.5
6	0.064	345	6.7	−2.0	−3.0	−22.2	−12.9	22.1
7	0.901	591	14.4	11.2	4.6	−1.1	−8.8	20.1
8	0.901	773	4.4	−12.7	5.4	−0.9	−15.6	7.3
9	0.904	800	6.2	−18.0	9.5	−0.5	−15.2	7.4
10	0.907	431	−2.8	−0.3	−19.9	−31.1	−25.6	1.7
11	0.905	642	16.8	0.8	12.7	2.2	−8.2	19.8
12	0.907	561	−9.8	53.8	10.8	−11.8	−16.4	9.1
13	0.898	584	−13.4	46.3	7.5	−13.3	−19.6	4.5
14	0.908	608	−18.6	40.2	2.1	−15.3	−23.2	−0.7
15	0.064	371	10.8	--	−3.7	-19.6	−15.7	17.5
16	0.064	590	−16.0	--	−12.5	−17.4	−33.3	−11.3
17	0.064	670	−13.4	--	−5.5	−9.7	−30.2	−8.7
18	0.064	722	−17.0	--	−7.1	−10.8	−32.5	−12.6
19	0.064	945	−23.0	--	−7.0	−10.5	−35.1	−19.1
20	0.064	345	6.7	--	−2.6	−22.2	−12.9	22.1
21	0.064	585	−23.0	--	−13.8	−16.1	−34.1	−12.3
22	0.0685	839	−26.4	−14.8	4.0	3.6	−31.3	−13.4
23	0.0685	901	−28.4	−18.9	3.5	3.9	−32.6	−15.7
24	0.069	967	−18.2	−9.3	20.9	22.0	−22.2	−3.7
25	0.0688	1046	−20.3	−13.7	20.2	21.9	−23.4	−6.2
26	0.0684	1149	−12.7	−8.3	34.2	36.7	−15.1	2.4

Note: The symbol -- denotes empty values.

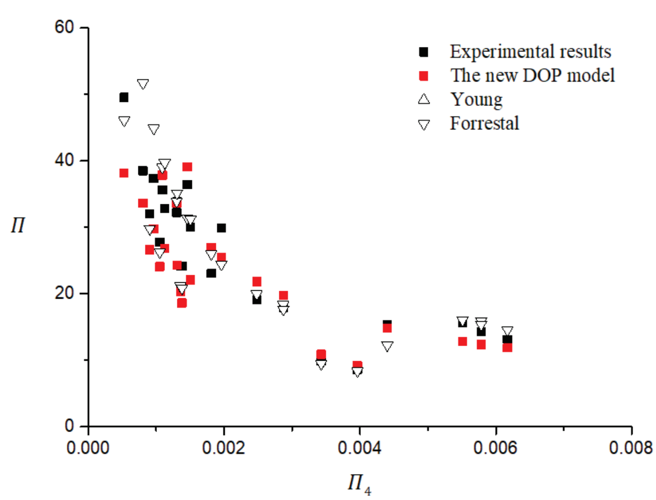

Figure 2. Comparison of $\Pi(\Pi_4)$ relationship between the experimental data and the calculated results (the new DOP model, Young formula, and Forrestal formula).

penetration of sub-scale projectiles.

Based on their experimental data and dynamic cavity expansion theory, Forrestal and his colleagues have presented another semi-empirical formula[14,15] (Forrestal formula) for an ogival-nosed projectile penetrating into concrete targets in the following form

$$P = \begin{cases} \dfrac{v_0}{\sqrt{c/M}}, & P < 2\phi \\ \dfrac{M}{2A\rho_t N}\ln\left(1+\dfrac{N\rho_t v_1^2}{Sf_c}\right)+2\phi, & P > 2\phi \end{cases} \tag{19}$$

where S is an empirical constant, and for ogive-nose projectiles.

$$N = \frac{8\psi - 1}{24\psi^2}, \quad v_1^2 = \frac{Mv_0^2 - 2A\phi Sf_c}{M + 2A\phi N\rho_t},$$

$$c = \frac{M(v_0^2 - v_1^2)}{4\phi^2} \tag{20}$$

However, it is necessary to determine the empirical constant S in advance. Moreover, the Forrestal formula seemed more applicable for the sub-scale experiments under the impact velocity v_0 varying from 300 m/s to 900 m/s. As shown in Table 2 and Fig. 2, for v_0> 900 m/s or full-scale projectiles, significant discrepancies are introduced, which shows why the Forrestal formula is not so applicable in the high-velocity penetration or that with full-scale projectiles.

Li and Chen[16] have further improved the Forrestal formula by presenting a semi-empirical formula (Li-Chen formula), in the form of

$$\frac{P}{\phi} = \begin{cases} \sqrt{\dfrac{\left(1+(k\pi/4N)\right)}{\left(1+(I/N)\right)}\dfrac{4k}{\pi}I}, & \dfrac{P}{\phi} \le k \\ \dfrac{2}{\pi}N\ln\left[\dfrac{1+(I/N)}{1+(k\pi/4N)}\right]+k, & \dfrac{P}{\phi} > k \end{cases} \tag{21}$$

where k= 2, and

$$I = \frac{1}{S}\left(\frac{Mv_0^2}{\phi^3 f_c}\right), \qquad N = \frac{1}{N^*}\left(\frac{M}{\rho_t \phi^3}\right) \tag{22}$$

For ogive-nose projectiles, $N^* = (8\psi - 1)/(24\psi^2)$. However, although it shows that experimental data on shallow, medium and deep penetration in a broad range of concrete strength, impact velocity and projectile geometry can be uniquely represented by the two dimensionless numbers I and N, this formula is applicable in the range basically similar to that of the Forrestal formula.

Based on the experimental data from Ordnance Department of the US Army and the Ballistic Research Laboratory prior to

1943, the Army corps of engineers developed an ACE formula[5] for the DOP, as

$$\frac{P}{\phi}=\frac{3.5\times10^{-4}}{\sqrt{f_c}}\left(\frac{M}{\phi^3}\right)\phi^{0.215}v_0^{1.5}+0.5 \quad (23)$$

As shown in Table 2 and Fig. 3, however, there are some larger relative errors in the calculated results comparing with the sub-scale experimental data. For example, for v_0 varying from 300 m/s to 800 m/s, the relative error scan reach up to approximately more than −30 per cent. In other words, the ACE formula seems applicable only for the penetration experiments of full-scale projectiles.

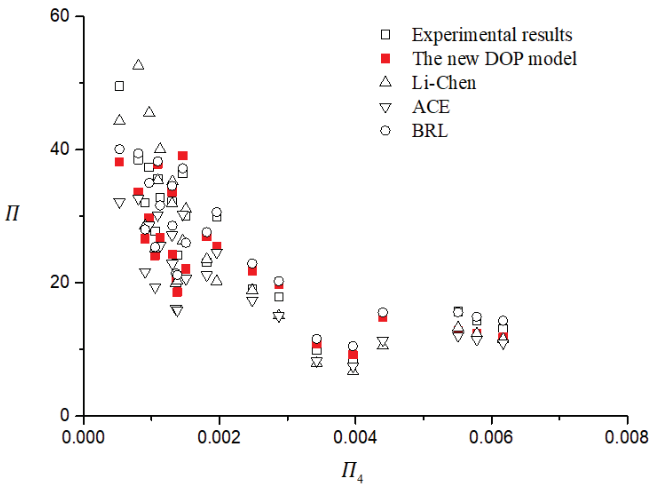

Figure 3. Comparison of $\Pi(\Pi_4)$ relationship between the experimental data and the calculated results (the new DOP model, Li-Chen formula, ACE formula, and BRL formula).

The Ballistic Research Laboratory have developed formula[2,5,18] (BRL formula) in 1941 to calculate the DOP in concrete impacted by a hard projectile, which takes the form of

$$\frac{P}{\phi}=\frac{1.33\times10^{-3}}{\sqrt{f_c}}\left(\frac{M}{\phi^3}\right)\phi^{0.2}v_0^{1.33} \quad (24)$$

As shown in Table 2 and Fig. 3, the calculated results of the BRL formula is in good agreement with the full-scale experimental data. While, in the sub-scale experiments with v_0 less than 400 m/s, the relative-error of the calculated results with the experimental data are approximately 25 per cent.

Compared with the DOP models mentioned-above, the new DOP model presented in this paper appears applicable in the penetration experiments with both the sub-scale and the full-scale projectiles. However, the new model mainly deals with the non-deformable projectile penetration. When the impact velocity of the projectile exceeds 1000m/s, mass abrasion of the projectile, the compressibility of the target medium, and dynamic properties through constitutive relationships become influential, the new model may not keep applicable.

4. CONCLUSIONS

A new dimensionless DOP model of a non-deformable projectile penetrating into concrete targets at normal impact is developed by similarity analysis involving intermediate asymptotic, complete similarity, and incomplete similarity. Comparing with the well-known empirical or semi-empirical DOP formulas such as the Young, Forrestal, Li-Chen, ACE, and BRL formulas, the newly developed DOP formula will be applicable within a broader range, including the penetration of both full-scale and sub-scale projectiles.

REFERENCES

1. Roy, P.K.; Rama, R.K. & Patkar, M.R. Penetration dynamics of earth penetration warhead into composite target media. *Def. Sci. J.*, 1987, **37**, 347-360. doi: 10.14429/dsj.37.5922
2. Sliter, G.E. Assessment of empirical concrete impact formulas. *ASCE J. Struct. Div.*, 1980, **106**(ST5), 1023-1045.
3. Barr, P. Guidelines for the design and assessment of concrete structures subjected to impact. Report, UK Atomic Energy Authority, Safety and Reliability Directorate, HMSO, London, 1990.
4. Adeli, H. & Amin, A.M. Local effects of impactors on concrete structures. *Nucl. Eng. Design*, 1985, **88**, 183-203. doi:10.1016/0029-5493(85)90165-7
5. Marvin, E.B. & Werner, G. The mechanics of penetration of projectile into targets. *Int. J. Eng. Sci.*, 1978, **16**, 1-99. doi: 10.1016/0020-7225(78)90002-2
6. Yankelevsky, D.Z. Local response of concrete slabs to low velocity missile impact. *Int. J. Impact Eng.*, 1997, **19**, 331-343. doi: 10.1016/S0734-743X(96)00041-3
7. Kennedy, R.P. A review of procedures for the analysis and design of concrete structures to resist missile impact effects. *Nucl. Eng. Design*, 1976, **37**, 183-203. doi: 10.1016/0029-5493(76)90015-7
8. Gabi, Ben-Dor. Ballistic impact: recent advances in analytical modeling of plate penetration dynamics– A review. *ASME J. Appl. Mech.*, 2005, **58**, 355-371. doi: 10.1115/1.2048626
9. Teland, J.A. A review of empirical equations for missile impact effects on concrete. FFI/RAPPORT-7/05856, Norwegian Defence Research Establishment, 2007.
10. Li, Q.M.; Reid, S.R.; Wen, H.M. & Telford, A.R. Local impact effects of hard missiles on concrete targets. *Int. J. Impact Eng.*, 2005, **32**, 224-284. doi: 10.1016/j.ijimpeng.2005.04.005
11. Rama Chandra Murthy, A.; Palani, G.S. & Nagesh, R. Iyer impact analysis of concrete structural components. *Def. Sci. J.*, 2010, **60**, 307-319. doi: 10.14429/dsj.60.358
12. Kosteski, L.E.; Riera, J.D.; Iturrioz, I.; Singh, R.K. & Kant, T. Assessment of empirical formulas for prediction of the effects of projectile impact on concrete structures. *Fatigue Fracture Eng. Mat. Str.*, 2015, **38**, 948-959. doi: 10.1111/ffe.12285
13. Young, C.W. Penetration Equations. SAND97-2426, Sandia National Laboratories, 1997.
14. Forrestal, M.J.; Altman, B.S.; Cargile, J.D. & Hanchak,

S.J. An empirical equation for penetration depth of ogive-nose projectiles into concrete targets. *Int. J. Impact Eng.*, 1994, **15**, 395-405.
doi: 10.1016/0734-743x(94)80024-4
15. Forrestal, M.J.; Frew, D.J.; Hickerson, J.P. & Rohwer, T.A. Penetration of concrete targets with deceleration-time measurements. *Int. J. Impact Eng.*, 2003, **28**, 479-497.
doi: 10.1016/S0734-743x(02)00108-2
16. Frew, D.J.; Hanchak, S.J.; Green, M.L. &Forrestal, M.J. Penetration of concrete targets with ogive-nose steel rods. *Int. J. Impact Eng.*, 1998, **21**, 489-497.
doi: 10.1016/S0734-743x(98)00008-6
17. Li, Q.M. & Chen, X.W. Dimensionless formulae for penetration depth of concrete target impacted by a non-deformable projectile. *Int. J. Impact Eng.*, 2003, **28**, 93-116.
doi: 10.1016/S0734-743x(02)00037-4
18. Beth, R.A. Penetration of projectiles in concrete. PPAB Interim Report No. 3, November 1941.
19. Rosenberg, Z. & Dekel, E. The penetration of rigid long rods– revisited. *Int. J. Impact. Eng.*, 2009, **36**, 551-564.
doi: 10.1016/j.ijimpeng.2008.06.001
20. Forrestal, M.J. & Luk, V.K. Dynamic spherical cavity expansion in a compressible elastic–plastic solid. *ASMEJ. Appl. Mech.* 1988, **55**, 275-279.
doi: 10.1115/1.3173672
21. Luk, V.K.; Forrestal, M.J. & Amos, D.E. Dynamics spherical cavity expansion of strain-hardening materials. *ASMEJ. Appl. Mech.*, 1991, **58**, 1-6.
doi: 10.1115/1.2897150
22. Barenblatt, G.I. Scaling, self-similarity, and intermediate Asymptotics. Cambridge University Press, Cambridge, 1996. ISBN: 0521435226
23. Barenblatt, G.I. Scaling. Cambridge University Press, Cambridge, 2003. ISBN: 0521826578
24. Frew, D.J.; Forrestal, M.J. & Cargile, J.D. The effect of concrete target diameter on projectile deceleration and penetration depth. *Int. J. Impact Eng.*, 2006, **32**, 1584-1594.
doi: 10.1016/j.ijmpeng.2005.01.012
25. Guo, L.; He, Y.; Zhang, X. F.; He, Y. & Qiao, L. Thermal effect on mass loss of projectile during penetration into concrete: Experimental and numerical study. *In* 28th International Symposium on Ballistics, 2014, 958-967.

ACKNOWLEDGEMENTS

This work was supported by The National Natural Science Foundation of China under Grant 11772056 and also The National Natural Science Foundation of China under Grant 11521062.

CONTRIBUTORS

Mr Meng Huang obtained BSc (Engineering Mechanics) from Taiyuan Univesity of Science and Technology, in 2009 and currently pursuing his PhD from Beijing Institute of Technology, P.R. China. His research areas are impact dynamics of solids and dynamic fracture mechanics.
In the current study, he has provided the initial idea, developed the model and prepared the manuscript.

Dr Zhuo-Cheng Ou obtained his PhD from (Solid Mechanics) from Xi'an Jiaotong University, China, in 2003. Currently working as a Professor in the State Key laboratory of Explosion Science and Technology, Beijing Institute of Technology, P.R. China. His research areas include impact dynamics of solids, dynamic fracture mechanics and fractal fracture mechanics.
In the current study, he has guided for developing the model and preparing the manuscript.

Dr Yi Tong obtained his PhD from Beijing Institute of Technology, China, in 2000. Currently he is a Associate Professor at State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, P.R. China. His research areas are explosion and its application.
In the current study, he has planned and developed the numerical calculation.

Dr Zhuo-Ping Duan obtained his PhD from Beijing Institute of Technology, China, in 1994. Presently, he is a Professor at State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, P.R. China. His research areas are explosion damage technology and its application.
In the current study, he has planned and developed the experiments.

Dr Feng-Lei Huang obtained his PhD from Beijing Institute of Technology, P.R. China, in 1992. Presently, he is a Professor at State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, P.R. China. His research areas are explosion dynamics, explosion damage and explosion protection.
In the current study, he has planned and developed the experiments.