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An Adaptive Spectrum-sensing Algorithm for Cognitive Radio Networks based on the Sample Covariance Matrix

Chhagan Charan* and Rajoo Pandey

Department of Electronics and Communication Engineering, National Institute of Technology, Kurukshetra - 136 119, India
**E-mail: chhagan.charan@nitkkr.ac.in*

ABSTRACT

A novel adaptive threshold spectrum sensing technique based on the covariance matrix of received signal samples is proposed. The adaptive threshold in terms of signal to noise ratio (SNR) and spectrum utilisation ratio of primary user is derived. It considers both the probability of detection and the probability false alarm to minimise the overall decision error probability. The energy- based spectrum sensing scheme shows high vulnerability under noise uncertainty and low SNR. The existing covariance-based spectrum sensing technique overcomes the noise uncertainty problem but its performance deteriorates under low SNR. The proposed covariance-based scheme effectively addresses the low SNR problem. The superior performance of this scheme over the existing covariance-based detection method is confirmed by the simulation results in terms of probability of detection, probability of error, and requirement of samples for reliable detection of spectrum.

Keywords: Cognitive radio; Spectrum sensing; Covariance matrices; Adaptive threshold; Signal to noise ratio

1. INTRODUCTION

The last decade has witnessed the everlasting growth of wireless communication technologies and high demand of capacity for wireless services leading to scarcity of frequency spectrum. In the present scenario, the radio frequency spectrum is under utilised, because of conventional fixed frequency band allocation policy[1]. The Cognitive radio (CR) is proposed to fully exploit the under utilised frequency spectrum. The key idea of cognitive radio is frequency reuse or spectrum sharing, which allows unlicensed secondary user (SU) to communicate over the frequency band assigned to licensed primary user (PU) when they are absent or not fully utilising the spectrum[2]. The SU requires ability to sense the frequency spectrum and adopt itself. Therefore, the unlicensed secondary user (SU) has to sense the presence of licenced user frequently in order to provide sufficient protection to PU[3].

The spectrum sensing is a fundamental component in CR networks. There are many schemes studied in the literature to perform the spectrum sensing for deciding the status (presence/absence) of primary user in the spectrum band, including energy-based detection (ED)[5,6,7], matched filter detection (MF)[8,9], cyclostationary feature detection[10,11], maximum eigen value detection method[12,13] and covariance-based method[14-19]. Among all these schemes, the ED-based spectrum sensing technique has gained a lot of attention in the literature because it is very easy to apply and does not need prior knowledge of the licensed PU. The ED-based method studied and analysed[5] and adaptive threshold for ED-based spectrum sensing scheme[6] tend to perform well under low SNR but are highly susceptible under noise uncertainty. However, as shown by studies[20], ED-based spectrum sensing schemes are vulnerable to noise uncertainty, thus, the performance severely deteriorates. In addition, these methods require an estimate of noise power, which, in practice, is very challenging to obtain[4,18]. Matched filter-based spectrum sensing technique[8,9] needs prior knowledge of licensed user, although it can detect even at low signal to noise ratio and under noise uncertainty environment. Cyclostationary feature-based detection requires the knowledge of the cyclic frequency of the PUs, which in some cases may not be realistic[10,11]. The covariance-based detection methods use the sample covariance matrix of the signal received by unlicensed user. As, the sample covariance matrix of noise and signal is different, this characteristic of covariance matrix has been used to detect the presence/absence of licensed user[14-16]. The threshold used in this method is a function of probability of false alarm and is fixed as it does not take into account other factors such as SNR, spectrum utilisation ratio of PU and signal correlation factor. Also, the performance of these methods deteriorates at low SNR[17-19].

In this paper, a new scheme is proposed which employs an adaptive threshold that minimises the decision error probability of spectrum sensing using the covariance matrix of received signal samples. The signal correlation information is also exploited to improve the performance of conventional covariance based spectrum sensing. The decision threshold is adaptive to SNR value and the spectrum utilisation ratio of PU.

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This makes the proposed method more suitable to overcome the noise uncertainty problem in comparison with ED-based spectrum sensing scheme. In addition, it addresses low SNR problem better than the existing covariance-based technique.

2. SPECTRUM SENSING PRELIMINARIES

This section briefly describes the spectrum-sensing model, conventional ED-based scheme and existing covariance-based spectrum sensing method.

2.1 Spectrum Sensing Model

The signal received by the unlicensed user can be expressed in the form of binary hypotheses[6,12,14] as:

$$H_0 : x(n) = \eta(n) \tag{1}$$

$$H_1 : x(n) = s(n) + \eta(n) \tag{2}$$

where $x(n)$ denotes the sample of signal received by SU, $s(n)$ is the sample of active radio signal of PU at the location of SU, and $\eta(n)$ is additive white Gaussian noise (AWGN) with mean 0 and variance σ_η^2. It is assumed that noise samples are independent and identically distributed (IID). The samples of active PU's signal $s(n)$ can be modelled as a Gaussian random process with σ_s^2 variance. Therefore, the signal to noise ratio (SNR)is $SNR = \sigma_s^2 / \sigma_\eta^2$.

The hypothesis H_0 represents that the primary user is absent whereas the hypothesis H_1 indicates that primary user is present. The performance of spectrum sensing can be measured in terms of probability of detection (P_d) i.e. an occupied spectrum-band is found to be busy and probability of false alarm (P_f) that an idle band is detected to be busy. To achieve the maximum protection to PU, P_d should be high, whereas P_f should be as small as possible to maximise the throughput of SU[20].

2.2 ED-based Spectrum Sensing

The test statistic used for ED-based spectrum sensing can be written as[5,6]:

$$T(x) = \frac{1}{N}\sum_{n=1}^{N} |x(n)|^2 \tag{3}$$

where N denotes the number of samples.

The test statistics $T(x)$ usually follows chi-square distribution and it can be approximated as Gaussian distribution according to the central limit theorem, when N is a large number. In conventional energy based spectrum sensing for a target value of probability of false alarm, the threshold γ can be expressed as described in[6] by:

$$\gamma = \sigma_\eta^2 \left(\sqrt{\frac{2}{N}} Q^{-1}(P_f) + 1 \right) \tag{4}$$

where P_f is probability of false alarm and the Q-function is described as:

$$Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

2.3 Covariance-based Spectrum Sensing Method

The statistical covariance matrix of received signal samples at SU can be defined as[14,15]:

$$R_x(N_s) = \begin{bmatrix} f(0) & f(1) & \cdots & f(L-1) \\ f^*(1) & f(0) & \cdots & f(L-2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ f^*(L-1) & f^*(L-2) & \cdots & f(0) \end{bmatrix} \tag{5}$$

where N_s represents the number of samples and $f(l)$ is computed as:

$$f(l) = \frac{1}{N_s}\sum_{m=0}^{N_s-1} x(m)x^*(m-l) \tag{6}$$

with $l = 0,1,...,L-1$; where denotes the smoothing factor.

In covariance absolute value (CAV) spectrum sensing method[14], the threshold (γ) is computed as the ratio of sum of absolute values of all elements to that of diagonal elements of matrix $R_x(N_s)$ and is shown to be:

$$\gamma = \frac{1 + (L-1)\sqrt{\frac{2}{N_s \pi}}}{1 - Q^{-1}(P_f)\sqrt{\frac{2}{N_s}}} \tag{7}$$

3. PROPOSED COVARIANCE-BASED ADAPTIVE THRESHOLD SPECTRUM SENSING SCHEME

The performance of the CR is well characterised by probability of detection (P_d) and probability of false alarm (P_f). The conventional spectrum sensing techniques are either based on constant false alarm rate (CFAR) or constant detection rate (CDR). If CR network is to be designed to guarantee PU's safety, the CDR method can be used and a high value of the target detection probability is chosen. The higher the target P_d, higher the protection to PU and lesser the interference by SU transmission. Whereas if the cognitive radio system is designed to maximise the throughput of the SU, the CFAR method is preferred and a very low value of probability of false alarm is kept as the target. The lower the value of target false alarm probability, the better chances of spectrum band utilised by the SU. In this paper, an algorithm based on adaptive threshold is proposed which takes into account both probability of detection (P_d) and probability of false alarm (P_f) to ensure that the interest of both PU and SU are taken care of.

As done by Zeng and Liang[14], the same test statistic $T(N_s)$ is used by the proposed method and is computed as the ratio of sum of absolute values of all elements to sum of absolute values of diagonal elements of statistical covariance matrix $R_x(N_s)$.

3.1 Thresholds for covariance-based CFAR and CDR methods

The auto-correlation matrix of the received signal at SU can be expressed as:

$$R_x(N_s) = R_s(N_s) + R_\eta(N_s) \tag{8}$$

where, $R_x(N_s)$, $R_s(N_s)$ and $R_\eta(N_s)$ denote the autocorrelation matrices of received signal at secondary user node, active radio signal transmitted by primary user, and noise, respectively.

As, the noise samples are assumed to be uncorrelated with zero mean and σ_η^2 variance, Eqn. (6) can also be expressed as:

$$R_x(N_s) = R_s(N_s) + \sigma_\eta^2 I_L \tag{9}$$

where I_L denotes the identity matrix of size $L \times L$.

If signal $s(n)$ is not present, $R_s(N_s) = 0$. Therefore, off-diagonal elements of $R_x(N_s)$ are all zeros. Whereas, if signal $s(n)$ is present and signal samples are correlated, $R_x(N_s)$ is not a diagonal matrix and off-diagonal elements of $R_x(N_s)$ have significant values.

To obtain the test statistic, we compute

$$T_1(N_s) = \frac{1}{L}\sum_{n=1}^{L}\sum_{m=1}^{L}\left|r_{nm}(N_s)\right| \tag{10}$$

and

$$T_2(N_s) = \frac{1}{L}\sum_{n=1}^{L}\left|r_{nn}(N_s)\right| \tag{11}$$

Here $r_{nm}(N_s)$ denotes the element of matrix $R_x(N_s)$ at the n^{th} row and m^{th} column. Then, if there is no signa l $T_1(N_s)/T_2(N_s) = 1$; and in the presence of signal, $T_1(N_s)/T_2(N_s) > 1$. Thus, the ratio $T_1(N_s)/T_2(N_s)$ forms the test statistic to detect the presence of signal.

If the cognitive radio system is designed to ensure the spectrum efficiency of the SU by using the CFAR method with certain target false alarm probability (P_f) under hypothesis H_0, it can be expressed as:

$$P_f = P(T_1(N_s) > \gamma_{P_f} T_2(N_s)) \tag{12}$$

For a given P_f, the associated threshold γ_{P_f} is derived in[18] as:

$$\gamma_{P_f} = \frac{1 + (L-1)\sqrt{\dfrac{2}{N_s \pi}}}{1 - Q^{-1}(P_f)\sqrt{\dfrac{2}{N_s}}} \tag{13}$$

If the main concern of spectrum sensing is to guarantee PU's safety against the interference of SU, the CDR method uses certain target detection probability which under hypothesis H_1, is expressed as:

$$P_d = P(T_1(N_s) > \gamma_{P_d} T_2(N_s)) \tag{14}$$

Zeng and Liang[14] has shown that for a very large N_s and a low SNR, probability of detection P_d can be expressed as:

$$P_d = 1 - Q\left(\frac{\dfrac{1}{\gamma_{P_d}} + \dfrac{\gamma_L SNR}{\gamma_{P_d}(SNR+1)} - 1}{\sqrt{2/N_s}}\right) \tag{15}$$

where $Y_L \triangleq \frac{2}{L}\sum_{l=1}^{L-1}(L-1)\left|\alpha_l\right|$, indicating the overall correlation strength among the consecutive L samples and α_l is the normalised correlation among the signal samples, given as: $\alpha_l = E\left[s(n)s(n-l)\right]/\sigma_s^2$.

From the expression (15), the threshold (γ_{P_d}) for a given value of P_d can be computed as:

$$\gamma_{P_d} = \frac{1 + \dfrac{\gamma_L SNR}{(SNR+1)}}{\sqrt{\dfrac{N_s}{2}}Q^{-1}(1-P_d) + 1} \tag{16}$$

Therefore, the threshold in CFAR and CDR based spectrum sensing scheme scan be obtained using Eqns. (13) and (16) as γ_{P_f} and γ_{P_d}, respectively.

3.2 Derivation of Proposed Adaptive Threshold

In this paper, the adaptive threshold for spectrum sensing is obtained through minimisation of probability of decision error (P_e) by taking into account of probability of detection (P_d) and false alarm probability (P_f).

For the minimisation of probability of decision error, we consider:

$$\min(P_e(\gamma)) = \min\left\{P(H_0)P_f + P(H_1)(1-P_d)\right\} \tag{17}$$

where $P(H_0)$ is the probability of absence of PU and $P(H_1)$ is the probability of spectrum being utilised by PU. The term $P(H_0)P_f$ in Eqn. (15) denotes the decision error probability when hypothesis H_1 is decided in place of H_0. Whereas the second term $P(H_1)(1-P_d)$ denotes the decision error probability when hypothesis H_0 is decided in place of H_1. Let the spectrum utilisation ratio of PU be i.e. $P(H_1) = \alpha$ and $P(H_0) = (1-\alpha)$. Hence, the objective function to find an adaptive threshold which minimises the total decision error probability, can be formulated as:

$$P_e(\gamma) = (1-\alpha)P_f + \alpha(1-P_d) \tag{18}$$

Substitution of P_f and P_d from Eqns. (11) and (13) in Eqn. (16) results into:

$$P_e(\gamma) = (1-\alpha)\left(1 - Q\left(\frac{\dfrac{1}{\gamma}\left(1 + (L-1)\sqrt{\dfrac{2}{N_s\pi}}\right) - 1}{\sqrt{\dfrac{2}{N_s}}}\right)\right) + \alpha Q\left(\frac{\dfrac{1}{\gamma} + \dfrac{\gamma_L SNR}{\gamma_L(SNR+1)} - 1}{\sqrt{2/N_s}}\right) \tag{19}$$

Let $U = \dfrac{\dfrac{1}{\gamma}\left(1 + (L-1)\sqrt{\dfrac{2}{N_s\pi}}\right) - 1}{\sqrt{\dfrac{2}{N_s}}}$ and $V = \dfrac{\dfrac{1}{\gamma} + \dfrac{\gamma_L SNR}{\gamma_L(SNR+1)} - 1}{\sqrt{2/N_s}}$

Then

$$P_e(\gamma) = \frac{(\alpha-1)}{\sqrt{\pi}}\int_{\frac{U}{\sqrt{2}}}^{\infty} e^{-z^2}dz + \frac{\alpha}{\sqrt{\pi}}\int_{\frac{V}{\sqrt{2}}}^{\infty} e^{-z^2}dz + 1 - \alpha \tag{20}$$

If the spectrum utilisation ratio is specified, then the probability of decision error $P_e(\gamma)$ becomes a convex function that varies with threshold γ.

To minimise the $P_e(\gamma)$, the derivative of $P_e(\gamma)$ is set equal to zero i.e. $\frac{\partial P_e(\gamma)}{\partial \gamma}$ which simplifies as:

$$\gamma^2 \ln\left[\frac{(1-\alpha)}{\alpha}\frac{1+(L-1)\sqrt{\frac{2}{N_s\pi}}}{1+\frac{\gamma_L SNR}{(SNR+1)}}\right]\frac{4}{N_s}$$

$$+\gamma\left[2(L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{2\gamma_L SNR}{(SNR+1)}\right]-\left[(L-1)^2\frac{2}{N_s\pi}+2(L-1)\sqrt{\frac{2}{N_s\pi}}\right.$$

$$\left.-\frac{\gamma_L^2 SNR^2}{(SNR+1)^2}-\frac{2\gamma_L SNR}{(SNR+1)}\right]=0 \tag{21}$$

It can be written as: $\gamma^2 A+\gamma B+C=0$ (22)

where $A=\ln\left[\frac{(1-\alpha)}{\alpha}\frac{1+(L-1)\sqrt{\frac{2}{N_s\pi}}}{1+\frac{\gamma_L SNR}{(SNR+1)}}\right]\frac{4}{N_s}$,

$$B=\left[2(L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{2\gamma_L SNR}{(SNR+1)}\right]$$

and $C=-\left[(L-1)^2\frac{2}{N_s\pi}+2(L-1)\sqrt{\frac{2}{N_s\pi}}\right.$

$$\left.-\frac{\gamma_L^2 SNR^2}{(SNR+1)^2}-\frac{2\gamma_L SNR}{(SNR+1)}\right] \tag{23}$$

let $a=\left[(L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{\gamma_L SNR}{(SNR+1)}\right]$ and

$$b=\left[(L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{\gamma_L SNR}{(SNR+1)}+2\right] \tag{24}$$

then $B=2a$ and $C=-ab$.

Now Eqn. (22) can be written as

$$\gamma^2 A+2a\gamma-ab=0 \tag{25}$$

The solution can be given as:

$$\gamma_1=\frac{a}{A}\left[-1+\sqrt{\left(1+\frac{Ab}{a}\right)}\right] \text{ and } \gamma_2=\frac{a}{A}\left[-1-\sqrt{\left(1+\frac{Ab}{a}\right)}\right]$$

As the decision threshold cannot be negative, γ_1 is chosen as the optimal decision threshold which by using Eqns. (23) and (24) is written as:

$$\gamma^*=\left[\frac{\left((L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{\gamma_L SNR}{(SNR+1)}\right)}{\ln\left[\frac{(1-\alpha)}{\alpha}\frac{1+(L-1)\sqrt{\frac{2}{N_s\pi}}}{1+\frac{\gamma_L SNR}{(SNR+1)}}\right]\frac{4}{N_s}}\right].$$

$$\left[-1+\left(1+\frac{\left((L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{\gamma_L SNR}{(SNR+1)}+2\right)}{\left((L-1)\sqrt{\frac{2}{N_s\pi}}-\frac{\gamma_L SNR}{(SNR+1)}+2\right)}\right.\right.$$

$$\left.\left..\ln\left[\frac{(1-\alpha)}{\alpha}\frac{1+(L-1)\sqrt{\frac{2}{N_s\pi}}}{1+\frac{\gamma_L SNR}{(SNR+1)}}\right]\frac{4}{N_s}\right)^{0.5}\right] \tag{26}$$

It can be noted that, under hypothesis H_1, the overall correlation strength among the consecutive L samples can be obtained from the off-diagonal elements of the sample covariance matrix $R_x(N_s)$. By using the test statistics $T_1(N_s)$ and $T_2(N_s)$ it can be written that:

$$\lim_{N_s\to\infty} E\left(T_1(N_s)-T_2(N_s)\right)=\frac{2\sigma_s^2}{L}\sum_{l=1}^{L-1}(L-1)\left|\alpha_l\right| \tag{27}$$

where α_l denotes the correlation among signal samples.

Now the overall correlation strength among the consecutive L samples is defined as:

$$\gamma_L=\frac{T_1(N_s)-T_2(N_s)}{SNR\sigma_\eta^2} \tag{28}$$

where N denotes the total number of samples including previous sensing intervals making $N >> N_s$.

3.3 Proposed Adaptive Covariance-based Detection Algorithm

Now the proposed Adaptive Covariance-based detection (ACD) scheme can be summarised as:

Step 1. Select the smoothing factor L and the number of samples (N_s).

Step 2. Obtain the sample covariance matrix of the signal received by SU using Eqn. (4).

Step 3. Compute the test statistics using Eqns. (10) and (11):

$$T_1(N_s)=\frac{1}{L}\sum_{n=1}^{L}\sum_{m=1}^{L}\left|r_{nm}(N_s)\right|$$

$$T_2(N_s)=\frac{1}{L}\sum_{n=1}^{L}\left|r_{nn}(N_s)\right|$$

where $r_{nm}(N_s)$ is the elements of the sample covariance matrix $R_s(N_s)$.

Step 4. Obtain the overall correlation strength (γ_L) among the consecutive L samples by using Eqn. (28), considering 30 sensing interval i.e. $N=30N_s$.

Step 5. Calculate the optimal decision threshold γ^* for specified spectrum utilisation ratio using Eqn. (26).

Step 6. Obtain the test statistic.

$$T(N_s)=\frac{T_1(N_s)}{T_2(N_s)} \tag{29}$$

Step 7. Take the sensing decision as:

$$D = \begin{cases} 1 & T(N_s) > \gamma^* \\ 0 & T(N_s) \le \gamma^* \end{cases} \tag{30}$$

where the decision $D = 0$ indicates that the PU is absent, whereas decision $D = 1$ means that primary user is active.

4. SIMULATION AND NUMERICAL RESULTS

The simulation results presented in this section demonstrate the performance of the Adaptive Covariance-based detection (ACD) approach. The existing covariance absolute value (CAV) based spectrum sensing scheme[14] and conventional ED-based spectrum sensing scheme[4] are also simulated to compare the performance.

In the simulation, the number of samples (N_s) in a sensing interval is kept 10000, the parameter L is chosen as 5 and spectrum utilisation ratio for PU is selected as 0.5. In order to obtain the P_d and P_e, 1000 independent runs of monte-carlo simulations were carried out and the averages are computed. Figure 1 shows the plot of probability of detection with SNR. From Fig. 1, it is observed that proposed adaptive covariance-based detection (ACD) method's performance is better than existing CAV method, especially, at low SNR, $SNR < -13dB$ .The proposed ACD technique attains higher P_d as 0.895 at $SNR = -16dB$ in comparison with CAV and ED which achieve the P_d as 0.138 and 0.685, respectively.

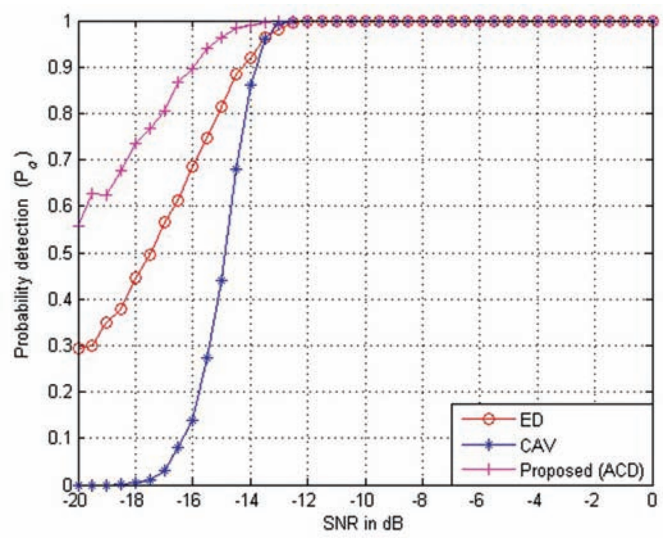

Figure 1. Plot of P_d Vs. SNR With 0dB noise uncertainty.

Figure 2 shows the variation of P_d with N_s number of samples at $SNR = -20dB$. From this Fig. 2, it is found that the proposed ACD spectrum sensing scheme provides a superior sensing performance than CAV and ED-based methods in terms of probability of detection. Typically, at $N_s = 80000$, the P_d obtained by the proposed scheme is 0.958, whereas the respective for probabilities of CAV and ED are 0.36 and 0.781 only. Thus, it can be observed from Figs. 1 and 2 that ED-based spectrum sensing performs better than the existing CAV-based scheme in absence of noise uncertainty.

The variation of threshold value with SNR is as shown in Fig. 3 for $N_s = 10000$. From Fig. 3, it is observed that the

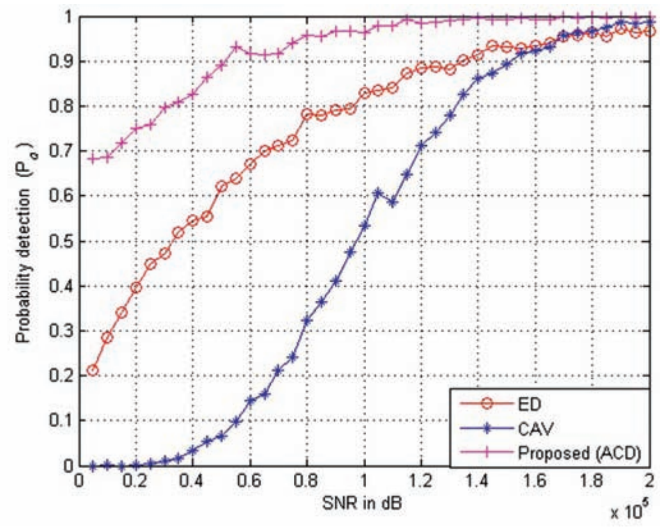

Figure 2. Plot of P_d *vs* N_s with noise uncertainty of 0 dB.

threshold in adaptive covariance-based algorithm (ACD) varies with SNR so as to minimise the decision error probability whereas the threshold of existing CAV method is constant.

Figure 3 capture the variation of threshold at low SNR. It is noted that as the utilisation ratio of PU increases, the probability of PU's presence also increases requiring smaller value of adaptive threshold to achieve the reliable spectrum detection, and vice versa. This is not possible in a fixed threshold-based scheme such as CAV method.

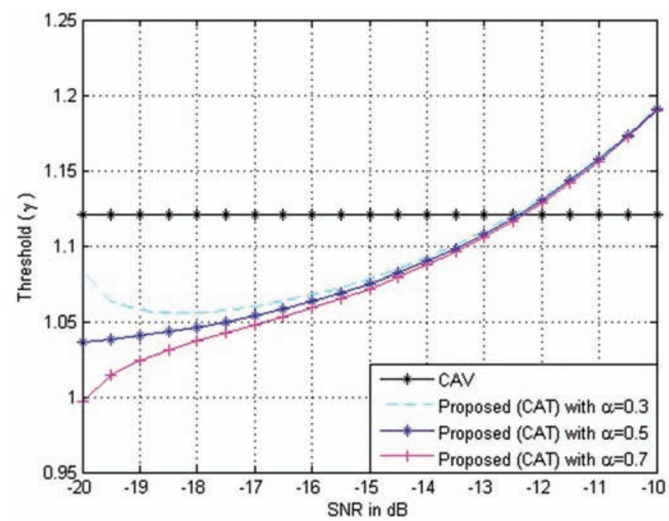

Figure 3. Threshold value (γ_e) *vs* SNR, magnified Fig. 3.

In practice, some uncertainty is involved in the estimation of noise[17]. For example, an estimated noise power $\partial_\eta^2 = \alpha\sigma_\eta^2$, will produce the uncertainty factor[13,17,18] as:

$$B = \max\{\log_{10}\alpha\}dB \tag{31}$$

where, the parameter is assumed to be uniformly distributed in an interval of $[-B, B]$. In literature, it is assumed that the noise uncertainty factor of 1dB to 2dB is normally observed in practice[18].

The variation of P_d with SNR under noise uncertainty of 2dB is as shown in Fig. 4. From Fig. 4, it can be observed that at high SNR $(SNR > -15dB)$ the performance of ED-based spectrum sensing scheme degrades most in comparison with other methods under noise uncertainty. However, at low SNR $(SNR < -15dB)$ the P_d remains almost constant $(P_d \approx 0.5)$. Therefore, it is observed that under noise uncertainty covariance-based method outperforms ED-based spectrum sensing scheme. It is observed that the method presented here attains higher P_d as 0.896 at $SNR = -16dB$ in comparison with CAV-based and ED-based schemes which have the probability of detection as 0.168 and 0.505, respectively.

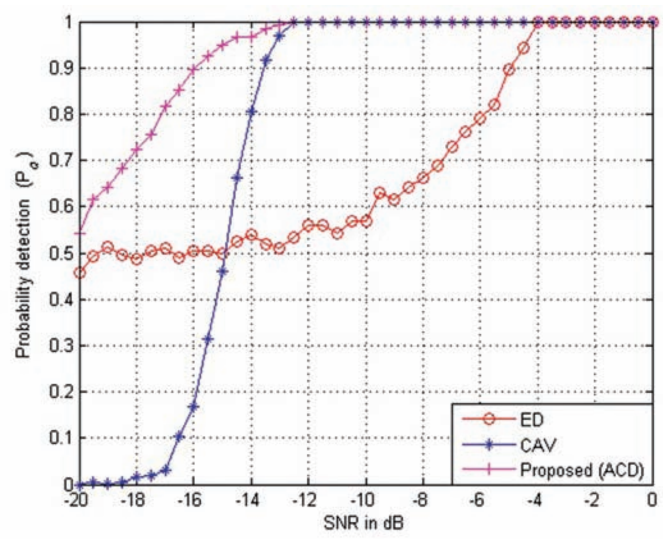

Figure 4. Plot of P_d vs SNR with 2 dB noise uncertainty.

Figure 5 shows the variation of P_d with number of samples for the case of noise uncertainty of 2dB at $SNR = -20dB$. Hence, it can be noticed that the presented scheme provides a superior sensing performance than CAV and ED-based techniques in terms of P_d. Typically, when the number of samples $N_s = 90000$, the P_d attained by the presented scheme is 0.883, whereas CAV and ED, achieve of 0.456 and 0.496 only. It is also observed that in ED-based scheme remains almost constant $(P_d \approx 0.5)$. Therefore, performance of the ED-based method highly degrades under noise uncertainty.

The variation of probability of error (P_e) with SNR for noise uncertainty of 2dB is as shown in Fig. 6. Figure 6 shows that reveals that the proposed method has less probability of error than CAV method. For example, the resulting in the P_e present scheme at $SNR = -15dB$ is 0.026, whereas P_e is 0.2695 and 0.4905 for existing CAV and ED-based methods.

In general, the proposed adaptive covariance-based spectrum detection (ACD) scheme performs significantly better than the existing covariance-based method (CAV) at low SNR and its performance is much better than ED-based spectrum sensing even under noise uncertainty. This is due to the ability of the proposed algorithm to effectively utilise the information available in terms of spectrum utilisation ratio of PU and SNR.

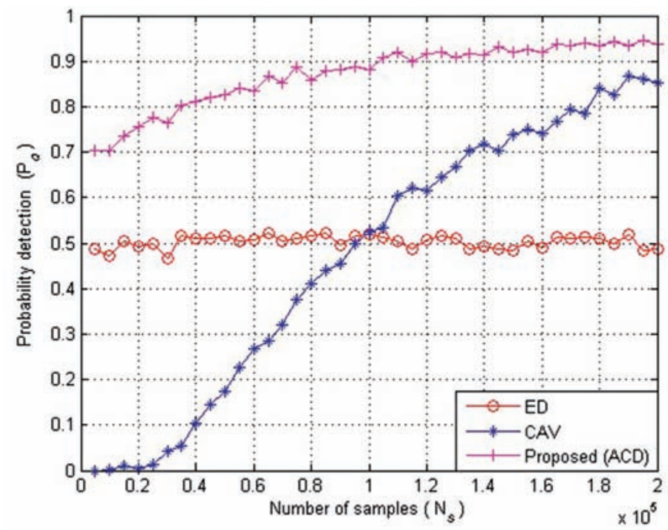

Figure 5. Plot of P_d vs N_s with noise uncertainty of 2 dB.

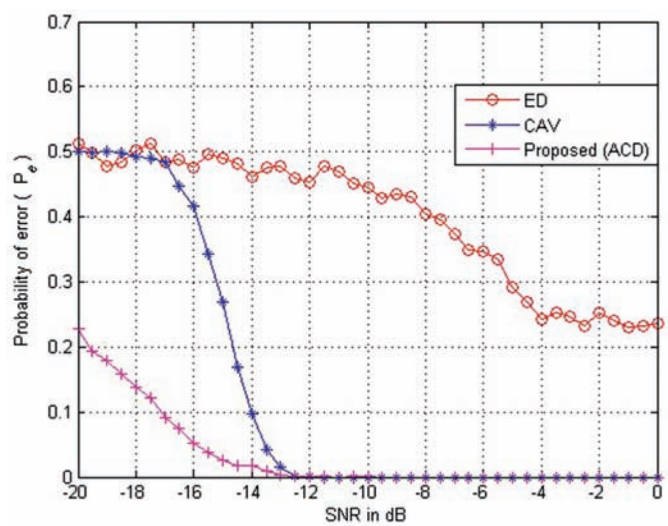

Figure 6. P_e vs SNR with 2 dB noise uncertainty.

5. CONCLUSIONS

In this study, a scheme for spectrum sensing using adaptive threshold based on covariance matrix of received signal samples at SU is proposed. The adaptive threshold minimises the total decision error probability of spectrum sensing. The closed form expression of adaptive threshold based on statistical covariance matrix of received signal is derived. The proposed adaptive threshold changes with spectrum utilisation ratio of PU's and SNR, maintaining a low value of probability of overall decision errors. The proposed scheme has much better performance under low SNR conditions compared to ED-based and existing CAV-based spectrum sensing and needs fewer samples. The analysis presented in the paper is supported by the simulation results.

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CONTRIBUTORS

Mr Chhagan Charan received his BTech (Electr. Comm. Engg.) and MTech (Microwave and Optical Comm. Engg.) from Delhi Technological University (DTU), Delhi. He is working as an Assistant Professor in the Department of Electronics and Communication Engineering, National Institute of Technology Kurukshetra, India. His research interests include: Cognitive radio, digital communication, microwave engineering, wireless sensor networks, and next generation of communication systems.
In the current study, he investigated the problem of minimization of probability of decision error. The optimisation problem formulation in Eqn. (19) and analytical solution is obtained in Eqn. (26). The extensive simulation has been performed.

Dr Rajoo Pandey received his BE from Government Engineering College, Jabalpur, and MTech from NIT-Kurukshetra (formerly REC Kurukshetra). He received his PhD from Indian Institute of Technology, Roorkee, in 2001. He is presently working as a Professor in the Department of Electronics and Communication Engineering, National Institute of Technology Kurukshetra, India. His research interests include: Signal processing, communication systems, digital signal processing, image processing and cognitive radio etc.
In the current study, he proposed to use the utilisation factor and correlation factor of licensed user for spectrum sensing. He thoroughly re-checked paper.