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# Constrained Target Clustering for Military Targeting Process

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**ABSTRACT**

Constrained target clustering (CTC) is proposed to support the targeting decision-making in the network centric warfare environment. When area targets are detected by sensors, it is required to decide the points at which a missile or bomb is aimed to achieve operational goals. CTC can determine the optimal numbers and positions of aiming points by transforming the targeting problem into clustering-based optimisation problems. The CTC formulations include objective functions and constraints in consideration of area targets, protected objects, target-level background information, lethal radius, and required damage rate. The numerical example shows how to apply the CTC formulation given a sample data set. In order to compare the effects of different constraints, the demonstration explores from an unconstraint problem to constrained problems by adding constraints. The results show that CTC can effectively decide the aiming points with consideration of both targets and capabilities of friendly weapons, and serve as a targeting decision support system in the network centric warfare environment.

**Keywords:** Target clustering; Constrained k-means clustering; Targeting

## 1. INTRODUCTION

Sensor networks are widely used in both military and non-military applications for utilising real-time information. In non-military applications, companies such as Caterpillar (PRODUCT Link™) and John Deere (JDLink™) are providing telematics systems to gather and analyse machinery's operational data[1]. The collected information can be used for fleet location tracking, utilisation monitoring, and health prognostics. In military applications, traditional platform centric warfare has been shifted to network centric warfare[2] with the support of sensor networks. This networked battlefield environment allows the visualisation of friendly, adversarial, and neutral objects. When targets are detected by sensors, a decision support system for targeting is required instead of relying on heuristics. This paper proposes a model to serve as a part of the targeting decision support system with other available models.

### 1.1 Targeting Process

According to US army techniques publication 3-60[3], a target is defined as 'an entity or object that performs a function for the adversary considered for possible engagement or other actions'. Also, targeting is 'the process of selecting and prioritising targets and matching the appropriate response to them considering operational requirements and capabilities'. Targeting follows the four functions as a loop: decide, detect, deliver, and assess.

The decide function represents the issuing of a plan from all the necessary information and guidance. The detect function gathers information based on the plan and the deliver function attacks targets. Finally, the assess function evaluates the effects and repeat the cycle if necessary. The focus area in this study is from the detect function to the deliver function.

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### 1.2 Related Studies

Two studies related to the focus area are identified. The first one is data fusion[4-6]. Data fusion is the study of integrating data from sensors. The well-known data fusion model is from the Joint Director's of Laboratories (JDL). The JDL model[4] has five levels. Level 0 is associated with pre-processing activities of sensor data. Level 1 is concerned with object refinement to estimate an entity's state. Level 2 is about situation refinement to describe the relations among entities and events. Level 3 is concerned with threat refinement to infer enemy threats. Level 4 is about process refinement which monitors the whole data fusion process and manages resources.

Das[5] proposed target classification and aggregation as Level 1 1/2 Fusion to improve situation and threat assessments (Levels 2&3 Fusion). Target classification analyses sensor data and classifies observations to real-world objects (e.g., aircraft, armoured vehicle, and missile launcher). Naive Bayesian classifier, Rule-Based Expert Systems, Dempster-Shafer Theory, and Fuzzy Logic were illustrated for target classification. Target aggregation identifies groups of related objects which can be represented as units comprising a number of subcategories (e.g., platoons, companies and squadrons) or a specific situation (e.g., ambush and retreat). Clustering techniques were developed for spatiotemporal cluster patterns in a dynamic operational environment.

The second related study is weapon target assignment (WTA)[7-10] (also known as target allocation and target problem). The WTA problem seeks to find the optimal allocation of friendly weapons to targets for minimising the expected survival value or maximising the expected damage of targets[10]. Depending on whether a single stage or multiple stages are considered, the WTA problem can be static or dynamic. Since the WTA problem cannot be solved by any exact methods, various heuristic methods were proposed[10]. There are many factors to vary the WTA problem such as missions (attacking or protecting), types and capabilities of targets and weapons, combat conditions, strategies, etc.

While both data fusion and WTA can support the focus area (i.e., from detect to deliver function), they have different objectives. Data fusion puts more emphasis on processing of targeting data and improving situation awareness. WTA focuses on the optimal allocation of friendly resources to processed targets.

### 1.3 Contribution

The proposed model, constrained target clustering (CTC), is related to both data fusion and WTA though it has a different purpose. CTC aims at determining the optimal numbers and positions of aiming points (points at which a missile is aimed in order to strike targets) when targets can be area targets. It can be viewed as target aggregation (e.g., targets, no-damage objects, and geographically separated targets) with the consideration of friendly resources (e.g., lethal radius and damage rate). CTC also proposes a new approach for targeting, clustering-based optimisation, while data fusion applies target aggregation and classification, and WTA uses matching-based optimisation.

## 2. PROPOSED APPROACH: CONSTRAINED TARGET CLUSTERING

### 2.1 Problem Description

The targeting process is the application domain of this study, especially from the detect function to the deliver function. Let there be $m$ targets, $t=(t_1, t_2, \ldots, t_m)$, from sensor networks (detect function) in the battlefield. Based on the locations of the targets, a distance can be defined between any two targets in Cartesian coordinates. If $x_i$ and $y_i$ represent the $x$ and $y$ locations of a target $t$, the distance (Euclidean distance or $L^2$ norm) between any two targets, $t_i$ and $t_j$, is defined as

$$d(t_i, t_j) = \| t_i - t_j \|^2 = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{1}$$

If targets can be only perceived as single targets (i.e., the position of a target is identical with the position of aiming points), WTA can be conducted to allocate one or more friendly resources (weapons) to all or some of targets. If targets can be viewed as area targets (or a group of targets) and a lethal radius of a weapon is considered, the positions of aiming points can be a decision variable. The numbers of aiming points also can be a decision variable for the efficiency of the deliver function. Moreover, in this situation, there are additional things to be considered such as friendly units nearby targets and geographical conditions. Constrained target clustering (CTC) is proposed to deal with these unaddressed issues and Fig. 1 shows where CTC can be fitted in the focus area.

### 2.2. Constrained Target Clustering

Given a set of detected targets ($t=(t_1 : t_m)$) in $R^2$ and a lethal radius of friendly weapons (a damage rate within the lethal radius is same and this will be relaxed later in this paper), the objective function is to cover more targets in terms of the Euclidean distance. The lethal radius is the distance from point of burst at which a missile or bomb can destroy a target. Note that circular error probability (CEP) is not considered in the model (e.g., GPS guided, laser guided missiles or bombs). The decision variables are the minimum numbers ($k^*$) and positions of aiming points (i.e., cluster centres $c_1, c_2, \ldots, c_{k^*}$). The objective function can be mathematically defined as

$$\min f_0 = \sum_{i=1}^{k} \sum_{t \in c_i} \| t - c_i \|^2 \tag{2}$$

where $t=(t_1, t_2, \ldots, t_m)$ is a set of targets, $C_i=(C_1, C_2, \ldots, C_k)$ is a set of clusters with a fixed number $k$, and $c_i$ is the centroid of cluster $C_i$. This can be interpreted that the determined aiming point should be adjacent to targets as possible in each cluster. Note that if k=m (numbers of aiming points = numbers of targets), targets are only perceived as single targets and WTA can be followed.

The constraint for a lethal radius can be defined as

$$\| t - c_i \|^2 \le R_l \tag{3}$$

where $t \in C_i$, $i=1, \ldots, k$, and $R_1$ is the lethal radius, which enforces targets to be within the lethal radius.

If there are no constraints, the popular and traditional clustering algorithm, k-means clustering[11], can be directly used for the objective function in Eqn. (2). The k-means clustering algorithm partitions a set of objects into k clusters while minimising Eqn. (2). The algorithm starts from the randomly selected k centroids. Based on the distance between

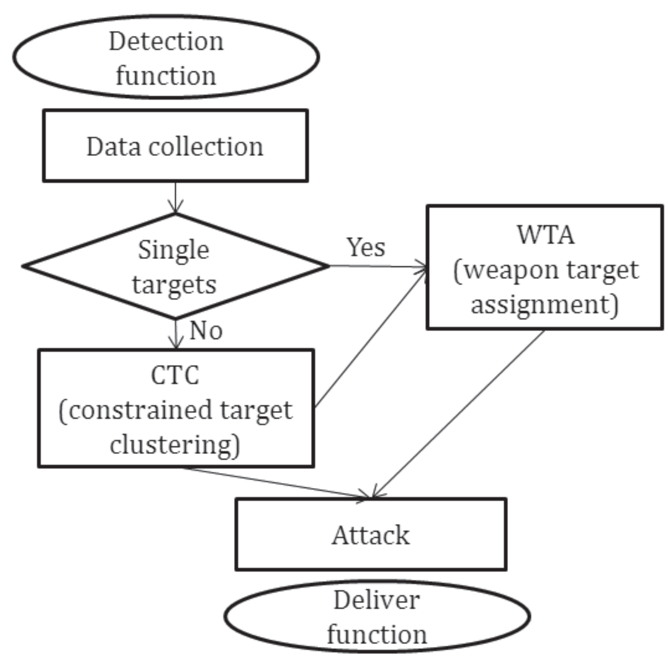


**Figure 1. CTC and WTA within the focus area.**

the centroids and objects, k clusters can be formed. After new centroids are calculated as the mean value of the locations of objects in each cluster, new k clusters can be defined. This process iterates until new centroids can be found, which reduces the value of Eqn. (2).

Since the traditional k-means clustering algorithm does not provide any mechanisms to incorporate background information (i.e., constraints), some researchers proposed that prior knowledge can improve the performance of clustering. Constrained clustering (also known as clustering with constraints) allows the user to integrate prior knowledge so that the clustering process can be guided. Wagstaff[12-13], *et al.* introduced pairwise must-link and cannot-link constraints which are based on instance-level background information. Must-link constraints impose that two objects must be in the same cluster while cannot-link constraints indicate that two objects cannot be in the same cluster. Zhu[14], *et al.* introduced a different type of constraints, size constraints. Size constraints enforce clusters are of the same size.

CTC proposed by this paper transforms the targeting problem in Section 2.1 into clustering problems with an additional objective function and constraints. The original objective function in Eqn. (2) can be used with the lethal radius constraint in Eqn. (3) since the damage rate is assumed to be the same within the lethal radius so that targets covered by the lethal radius can be destroyed. If the damage rate can be modelled by distance and expressed as percentage, the goal is to maximise the total damage rate

$$\max f_1 = \sum_{i=1}^{k} \sum_{t \in c_i} D(\| t - c_i \|^2) \tag{4}$$

where $D(\ )$ is the function of damage rate by distance between blast points and targets. The damage rate function can be linearly or non-linearly decreased as the distance between a blast point and a target is increased. If the damage rate is linear, minimising $f_0$ in Eqn. (2) or maximising $f_1$ in Eqn. (4) without constraints will generate the same optimal solutions.

For additional constraints, first, there can be objects that should not be damaged (no-damage objects). The objects (e.g., friendly units, civilians, etc.) should be outside of the lethal radius of friendly weapons (or plus more safe distance).

$$\| s - c_i \|^2 > R_l \tag{5}$$

where $i=1, \ldots, k$, and $s=(s_1, s_2, \ldots, s_n)$ is a set of objects that should not be damaged.

Second, due to a variety of terrains, there can be objects that should not be in the same cluster. If this prior knowledge is available, two objects can be forced to be in the different cluster. When targets $t_i$ and $t_j$ cannot be in the same cluster $C$ and $I$ is an index function as follows

$$I = \begin{cases} 1, & \text{if target t is in C} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

then, following constraint can be added,

$$I_i \times I_j = 0 \tag{7}$$

Overall, the CTC formulation can be summarised as follows and the details have been introduced in this section.

$$\begin{aligned} &\max \ \sum_{i=1}^{k} \sum_{t \in c_i} D(\| t - c_i \|^2) \\ &\text{s.t.} \quad \| t - c_i \|^2 \le R_l \\ &\qquad\ \ \| s - c_i \|^2 > R_l \\ &\qquad\ \ I_i \times I_j = 0 \end{aligned} \tag{8}$$

The problem can be solved iteratively similar to the k-means clustering algorithm while restricting the constraints. If the constraints cannot be satisfied, the clustering process will be stopped without partitioning. Finally, the minimal number of aiming points can be decided by increasing the number of clusters from one when there are constraints that should be satisfied.

## 3. NUMERICAL EXAMPLE

### 3.1 Background

In this section, CTC is demonstrated with a sample data set. The data set is given in Fig. 2. There are a total of 20 targets (blue diamond) with the *x* and *y* locations. Note that for simplicity, units will be omitted (e.g., meter, kilometer, etc.) Furthermore, targets and weapons are assumed to be homogeneous. When the 20 targets are detected by sensors, the optimal numbers and positions of aiming points for GPS guided missiles are required to destroy the targets. Test cases include unconstrained clustering (original k-means clustering), clustering with constraints (lethal radius, no-damage objects, target-level background information), and clustering with the damage rate objective function.

The CTC formulations in Sect. 2 were modelled and implemented in Excel. For the initialisation of the k-means algorithm, the seed centroids were designed to be selected randomly among the targets. Since the k-means algorithm converges to local minimiser, multiple seed values (10 different values) were used to get the solutions.

### 3.2 Results

Figure 3 shows the result with the objective function $f_0$ in Eqn. (2) using the Generalised Reduced Gradient (GRG) algorithm in Excel. The values of the objective function (also known as within cluster sum of squared errors) are 84.49, 53.62, and 30.72 for k=1, 2, and 3 (Fig. 3 (a) - 3(c)). Note that as the number of clusters (k) is increased, more targets can be covered with a less distance deviation.

When the lethal radius constraint is considered as follows with the objective function $f_0$,

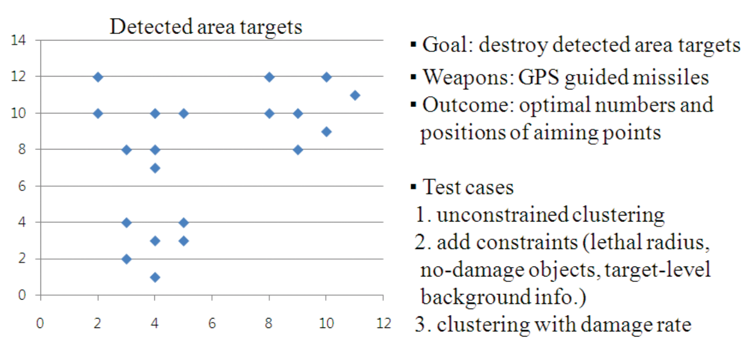


**Figure 2. Numerical test scenario.**

$$\| t - c_i \|^2 \leq 3 \tag{9}$$

Figure 4 (a) shows the result using the GRG algorithm in Excel. The minimum number of aiming points (k*) is three (i.e., k=1, 2 are infeasible) and one target (2, 12) that was out of the lethal radius (Fig. 3 (c)) is now deviated within the lethal radius (from 3.3 to 2.99). In this case, the value of the objective function is 30.8. Note that the value is increased by adding the constraint (30.72 without the constraint (Fig. 3 (c)).

Next, when there are objects that should not be damaged (no-damage objects) in addition to the previous case (Eqn. (2) + Eqn. (9)), following constraint can be added,

$$\| s - c_i \|^2 > 3.5 \tag{10}$$

Figure 4 (b) shows the result using the GRG algorithm in Excel. It can be seen that the distances between the no-damage object and the centroids are over 3.5. In this case, the value of the objective function is 33.93.

When the information is available that two objects (target no. 3 (4, 7) and 7 (4, 8)) should be in the different cluster (e.g., hills between the targets) in addition to the previous case (Eqn. (2) + Eqn. (9)), following constraint can be added,

$$I_3 \times I_7 = 0 \tag{11}$$

Since the problem becomes non-smooth with the constraint in Eqn. (11), the Evolutionary algorithm in Excel is used and Fig. 4(c) shows the result. Without the constraint in Eqn. (11), target $t_3$ and $t_7$ were in the same cluster (2nd cluster in Fig. 4(a)) but now they are forced to be in the different cluster (target no. 3 is in the 1st cluster).

When the damage rate can be modelled by distance, the

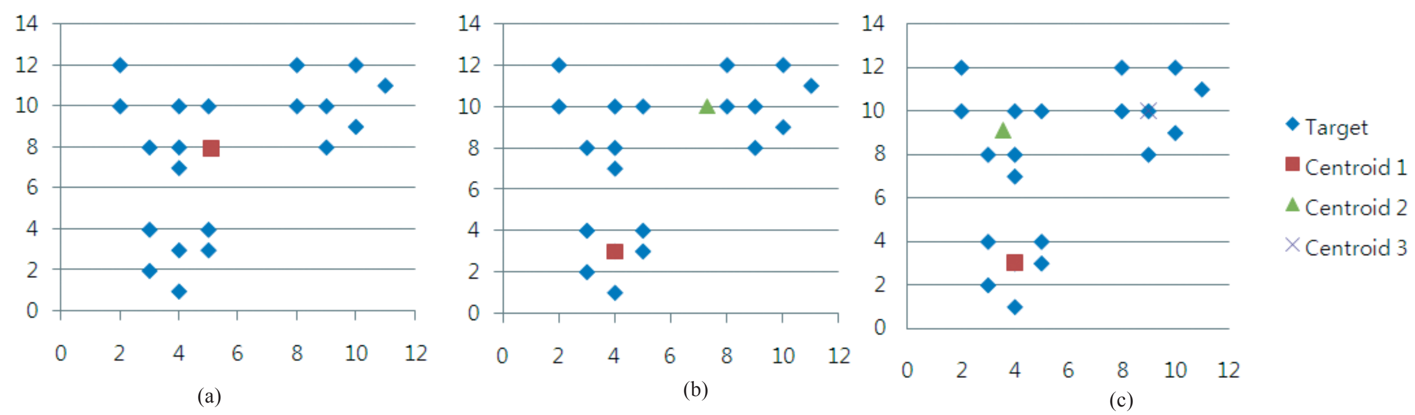


**Figure 3. Clustering results with cluster numbers k=1 (a), 2 (b), and 3 (b) without constraints.**

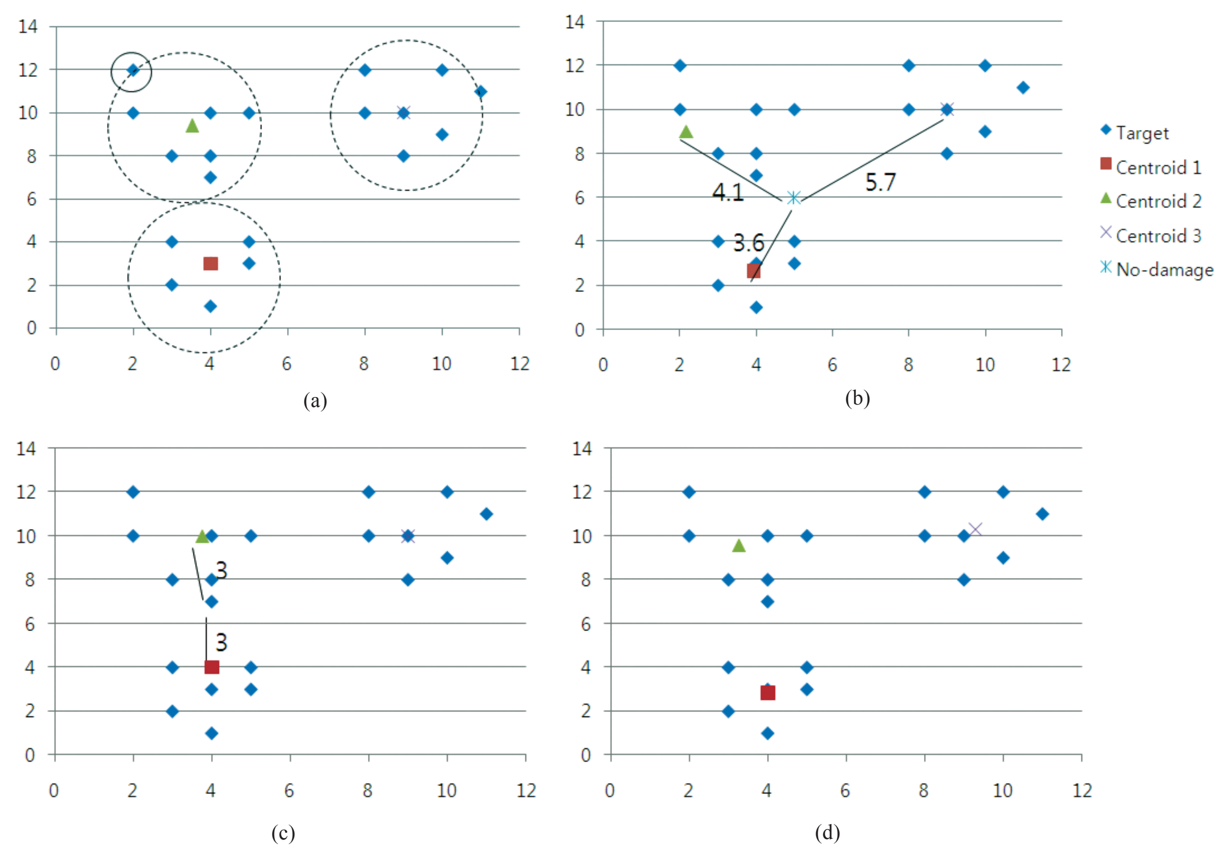


**Figure 4. Clustering results with different objectives and constraints.**

new objective function can be as follows

$$\max f_1 = \sum_{i=1}^{k} \sum_{t \in c_i} 100 - (0.9 \times \| t - c_i \|^2) \qquad (12)$$

The objective function indicates that when targets are identical to centroids, the value will be 2000 per cent (i.e., add 100 per cent damage for 20 targets). Otherwise, the value is non-linearly decreased by distance between targets and centroids. Similar to the lethal radius constraint, the required damage rate for each target can be added as follows

$$100 - (0.9 \times \| t - c_i \|^2) \geq 95 \qquad (13)$$

Figure 4 (d) shows the result using the GRG algorithm in Excel with the value of 1953 per cent. Constraints in Eqns. (10) and (11) can be also added for this case.

## 4. DISCUSSION

In Section 3, CTC was demonstrated with a sample data set. The data set was intentionally simplified to enhance the visual understanding. The demonstration showed the step-by-step implementation, i.e., from an unconstrained problem to constrained problems by adding different constraints so that the effects of different constraints could be compared. Finally, the damage rate was used as an objective function for CTC with a required damage rate for each target, which made a problem more realistic.

As discussed in Section 2.2, if k=m (aiming points are identical to the positions of targets), WTA can be conducted to allocate various friendly weapons directly. CTC deals with the case when k<m, which has not discussed before.

However, this paper only considers homogeneous targets and friendly weapons. To extend this, more constraints can be used but the complexity of the problem will be increased accordingly. In Fig. 1, it was discussed that CTC could be considered as a preprocessing for WTA (i.e., WTA can be conducted with the result of CTC instead of raw target data).

Furthermore, CTC is based on the k-means algorithm and the algorithm's performance can be greatly affected by the initial cluster centres. The k-means++ algorithm[15] is proposed to find good starting centres based on the k-means algorithm. Different clustering algorithms also can be utilised other than the k-means algorithm. For example, X-means algorithm[16] incorporates model selection based on the Bayesian information criterion; expected maximisation algorithm[17] is a generalisation of maximum likelihood estimation when there are unobserved latent variables (assignments of observed values to clusters); Fuzzy clustering[18] is based on Fuzzy Set theory; density-based spatial clustering of applications with noise (DBSCAN)[19] is based on density; etc.

## 5. CONCLUSIONS

The constrained target clustering (CTC) formulation is developed in this paper, to support the military targeting process. When there are area targets, CTC can determine the optimal numbers and positions of aiming points by transforming the targeting problem into clustering-based optimisation problems. The within cluster sum of squared errors and damage rate are used for objective functions. The lethal radius, no-damage objects, target-level background information, and required damage rate are modelled as constraints. The numerical example shows the sample results of the CTC formulation over a sample data set. The results indicate that CTC can effectively decide the aiming points with consideration of both targets and capabilities of friendly weapons.

In the future, various large-scale data sets can be tested with the CTC formulation. An efficient modeling technique can be researched for heterogeneous targets and friendly resources. Finally, an overall framework to combine data fusion, WTA, and CTC will be beneficial for supporting targeting decision-making.

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