Defence Science Journal, Vol. 67, No. 3, May 2017, pp. 233-239, DOI : 10.14429/dsj.67.10289

Numerical Investigation of Rotating Lid-driven Cubical Cavity Flow

Jitendra Kumar* and N.R. Panchapakesan

Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai – 600 036, India
**Email: jitendra@jetmail.ada.gov.in*

ABSTRACT

The present work numerically investigates the flow field in a cubical cavity driven by a lid rotating about an axis passing through its geometric center. Behaviour of core flow and secondary vortical structures are presented. Grid-free critical Reynolds number at which flow turns oscillatory is estimated to be 1606. This differs significantly from the linear lid-driven cubical cavity as well as circular lid-driven cylindrical cavity flows which have been reported to attain unsteadiness at higher Reynolds numbers. A stationary vortex bubble similar to rotating lid-driven cylindrical cavity flow has been observed to be present in the flow.

Keywords: Lid-driven cavity flow; Critical Reynolds number; Unsteady flow

1. INTRODUCTION

The lid-driven flow in cubical cavity is a complex, three dimensional phenomena but, well posed benchmark standard problem. Despite simplicity of formulation it allows for study of various complex flow features such as, steady and unsteady vortical structures, transition physics turbulence, etc. Study of cavity flow has helped in better understanding of the processes involved in fluid mixing, short-dwell coaters, melt-spinning, etc.

One of the most exhaustive documentation of physics behind lid-driven cavity flows and the research work carried out in driven cavity flows was by Shankar[2], *et al.* Ghia[2]*, et al.* has reported benchmark 2D results. Sorenson[3], *et al.* published detailed experimental findings of critical Re and dominant frequency mode mapping experiments for flow in cylindrical cavity driven by rotating lid for $1 \leq h \leq 3.5$. They have reported an excellent match with published numerical results. Chiang[4], *et al.* reported over-prediction of velocities by 2-D computations compared to 3-D computations, as a result of ignoring wall effects. Feldman[5], *et al.* computed the critical *Re* number to be 1914 for a linear lid-driven flow in a cubical cavity. Liberzon[6], *et al.* reported and Leriche[7], *et al.* confirmed the transition region to be between $1700 < Re < 1970$ using particle image velocimetry (PIV) on lid-driven cubical cavity and have shown good qualitative agreement with computation. Extensive studies have been done for cubical[8] and cylindrical cavity flows, however flow in a cubical cavity driven by rotating lid has not yet been investigated. The work reported here attempts a systematic study of flow behaviour in steady and oscillatory flow regimes and estimation of critical Reynolds number for such a flow.

Received : 25 July 2016, Revised : 22 March 2017
Accepted : 30 March 2017, Online published : 24 April 2017

2. METHODOLOGY

The computation work for this work has been carried out using widely used open-source CFD tool, OpenFOAM. OpenFOAM is essentially a collection of C++ function libraries which are used to create executable applications categorised as solvers (e.g. icoFoam, pisoFoam, etc) and utilities (e.g. foamCalc, Vorticity, etc).

2.1 Governing Equations and Flow Scales

Following equations govern incompressible Newtonian flow in cubical cavity driven by constant angular velocity rotating lid:

Continuity Equation

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

Momentum Equation

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{2}$$

The setup for the study (as shown in Fig. 1) comprises of a cubical cavity of length *L* (length scale) and lid rotating at constant positive angular velocity *w* (angular velocity scale).

The scales to non-dimensionalise maximum linear velocity U_{max}, time *t*, frequency *f* for dominant mode of oscillation and pressure *p* are derived from *L* and *w* using appropriate equations;

$$U_{\max} = \frac{wL}{\sqrt{2}}, t = \frac{U_{\max}}{L} = \frac{w}{\sqrt{2}}, f = \frac{w}{2\pi}, p = \rho U_{\max}^2 \tag{3}$$

The Reynolds number for the flow is defined by;

$$\mathrm{Re} = \frac{wR^2}{\upsilon} \, where, R = \frac{L}{2} \tag{4}$$

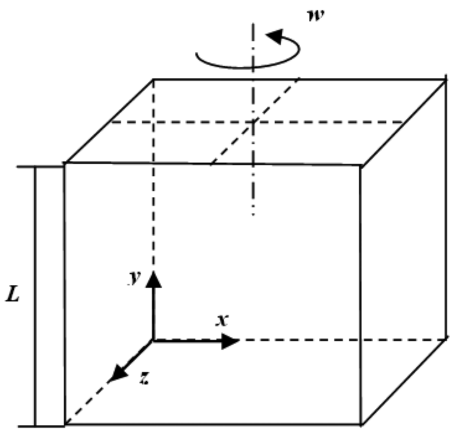


Figure 1. Cubical cavity setup and co-ordinate system.

This definition is adopted since the flow under study is similar to the flow in a cylindrical cavity driven by a rotating lid with aspect ratio of $h/R = 2$.

2.2 Pre-processing, Solver Schemes, Solution Control and Post-processing

By default OpenFOAM defines a mesh of arbitrary polyhedral cells in 3-D, bounded by arbitrary polygonal faces. A mesh with this general structure is known in OpenFOAM as a polyMesh. The blockMesh utility used for mesh generation in this work has capability to create parametric meshes by compiling the polyMesh structure. The mesh is generated from a dictionary file named blockMeshDict located in the constant polyMesh directory of a case. In blockMeshDict file patches of faces are defined and appropriate patch types keywords are assigned to them. The blockMesh utility collects faces based on patch type assignments and among other files also generates a file containing list of boundary faces. The boundary conditions are assigned to these patches using field variable data in time directories. The five walls of the cavity i.e. four sides and one bottom wall serve as stationary boundary on which no-slip boundary condition is applied. The top lid which drives the fluid also has to satisfy the no-slip boundary condition. Initial values of boundary conditions have been set in initial time directory for velocity and pressure fields. The velocity for lid has been set using 'rotatingWall' boundary condition. Pressure field is set at zero gradients at all walls and lid.

A transient solver for incompressible laminar Newtonian fluid, icoFoam has been used for this work. In this solver, each of the term in the governing equation is replaced by a numerical scheme and are discretised to convert the mathematical model into a set of algebraic equations, which are solved iteratively as follows:

- Time derivative has been discretised using first order, bounded, implicit Euler scheme
- Interpolations are done using central difference linear scheme. Interpolation is required for divergence and gradient terms
- Divergence terms in flux and velocity has been discretised by Gauss linear (second order, unbounded) scheme
- Gradient pressure term is discretised using Gauss linear scheme
- Surface normal gradients are discretised using orthogonal scheme
- Gauss linear orthogonal scheme has been used to discretise the Laplacian term.

The solver tolerance level for pressure and velocity is set at 1e-06 to achieve good accuracy. The solver relative tolerance limits the relative improvement from initial to final solution. As this work requires transient resolution of flow, the solver relative tolerance has been set to zero to force the solution to converge to the solver tolerance in each time step.

The pressure and velocity field data generated by solving the flow has been processed using built-in utilities from OpenFOAM (Vorticity, foamCalc, etc) to calculate vorticity, field magnitudes, etc. The generated datasets have been visualised in three dimensional space using paraFoam utility (provided by OpenFOAM, which is based on Paraview) and Tecplot. A Matlab code has been written to automate the process of identifying the most dominant mode of frequency (non-decaying) using FFT. The temporal variation of velocity data and frequency content along with decay rate has been analysed using that Matlab code. The decay rate values are then linearly extrapolated from sub-critical *Re*-regime to estimate critical *Re* using Richardson extrapolation technique.

2.3 Richardson Extrapolation Technique

In numerical analysis, Richardson extrapolation is a widely used sequence acceleration technique. It improves the rate of convergence and being generic in nature has a very wide range of applicability.

Assume a quantity Q^* is to be approximated by a method $Q(h)$, where h is an independent parameter to control $Q(h)$ and kh is another value of same parameter. Then the method can be written as:

$$Q(h) = Q^* + O(h^n) = Q^* + Ch^n + O(h^{n+1}) where, h \to 0 \tag{5}$$

Also,

$$Q(kh) = Q^* + O(kh^n) = Q^* + Ck^n h^n + O(h^{n+1}) where, kh \to 0 \tag{6}$$

Here n is the order of error estimate and C is coefficient in first expansion term. Richardson proposed a method such that,

$$Q_R(h,k) = \frac{k^n Q(h) - Q(kh)}{k^n - 1} \tag{7}$$

On substituting the expressions for $Q(h)$ and $Q(kh)$ expansions and then on simplification previous equation reduces to

$$Q_R(h,k) = Q^* + O(h^{n+1}) \tag{8}$$

The method $Q_R(h,k)$ is termed as the Richardson extrapolation of $Q(h)$ and has a higher order error estimate of $O(h^{n+1})$ compared to method $Q(h)$.

This technique has been implemented in this work to estimate critical Reynolds number. The critical *Re* has been estimated by extrapolating decay rates to zero using RET, for *Re* case pair at same grid size. However, it is necessary to estimate grid-free critical Reynolds number[1]. This has been done by running same *Re* case pair at a higher grid density and extrapolating the inverse of grid size to zero using RET.

2.4 Validation and Grid-independence Study

For this study official version 2.2.0 of OpenFOAM was used. There are numerous validation studies available[9,10,11]. However, some validation cases have been chosen relevant to the study i.e. lid-driven flow in two-dimensional and three-dimensional cavities for which benchmark results are available in literature. The numerical scheme and solution control methods have also been verified by comparing the computational results against published data. An excellent match between OpenFOAM results and published results has been achieved.

In absence of any previous work for this flow, it is important to show that above a certain resolution, results are invariant with grid size. Grid-independence study has been carried out in both flow regimes: steady and oscillatory. Numerical simulations were carried out for grid-sizes of 1, 2 and 4 million cells for two Reynolds numbers in each regime of flow. The velocity profiles at mid plane and a plane near lid has been analysed and the variation has been found to be within 1 per cent tolerance. This has given the necessary confidence to use 1 million cells for all the cases.

3. RESULTS AND DISCUSSION

This section deals with the flow and vortical structure behaviour in various *Re* regimes, and estimation of critical *Re* for the cubical cavity driven by lid rotating about an axis passing through its geometric center. The flow features and estimation of various parameters have all been explained in comparison to rotating lid-driven cylindrical cavity, as the flow in absence of corners of cube should behave as such.

3.1 Flow Behaviour in Low Re Regime

The flow very close to center behaves in a manner similar to cylindrical cavity flow but, the presence of four vertical corners generate the corner flows as well as influence the core flow.

Instead of showing axi-symmetry it has been observed that flow has four prominent zones and exhibits 4-fold rotational symmetry. The presence of corners has resulted in recirculation zones even at a low Reynolds number of 35, as shown in Fig. 2. The lid being a no slip wall rotates the adjacent fluid layer at lid angular velocity (counter clockwise or positive angular velocity about y-axis). This layer transfers its momentum to adjacent fluid layer due to shear and so on, thus a counter clockwise flow is set in motion. The fluid particles gain forward and tangential velocity as they are pushed outward towards wall, but due to no slip condition at side and upper wall, simultaneously start moving downward. This results in a spiraling downward trajectory as shown in Fig. 2. Once the flow reaches the bottom wall, it is fed back to central core flow which moves up in a spiral or twisting motion to the top lid. This cycle of flow continues in steady state. This analysis has been backed by the vertical velocity contours as shown in Fig. 2. A central positive U_y contour and four corner zones

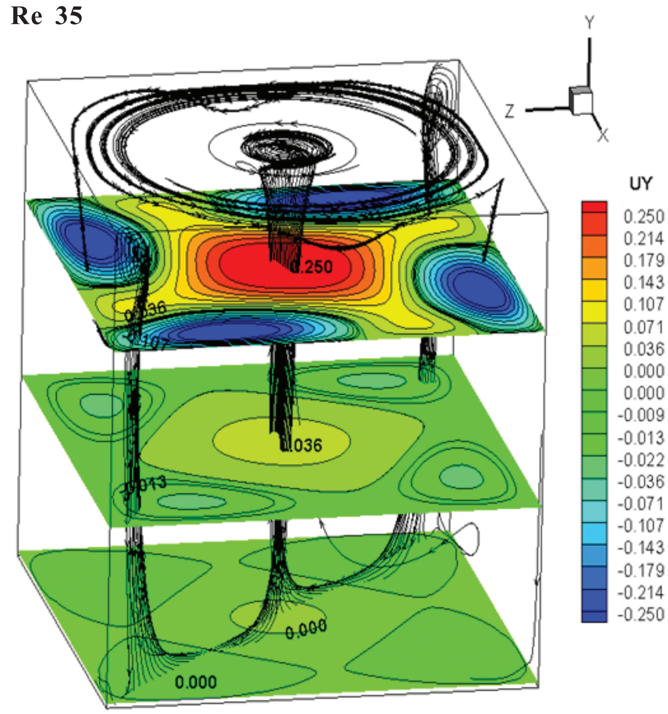


Figure 2. Non-dimensional U_y contours and bottom-wall stream traces, *Re* = 35.

of negative contours confirm the upward movement of central core and zonal downward feeding of fluid to central core.

An interesting feature in the flow is the recirculation zones existing at all the corners (only two corner structures have been shown). They are fed fluid from the streams close to side walls. These downward spiraling streams lose momentum due to wall shear and closest streams stick to wall, but fluid stream a little farther from the wall avoid sticking and eventually get in the region of favourable U_y gradient and gain upward momentum. These streams have been traced to contribute to the corner vortex. At corner due to proximity of two side walls the flow re-circulates and eventually due to negative velocity gradient is pushed down and then to the central core.

Similar behaviour has been observed for other *Re* cases in low speed regime, as shown in Fig. 3. Another interesting feature to note is a wavy ring-like streamline structure around the central core, which are circulating flow streams affected by alternating positive and negative U_y gradients.

3.2 Estimation of Critical Re

To arrive at a near critical region of *Re* a technique similar to binary search has been used. One low (*Re* = 350) and one relatively high (*Re* = 2100) cases have been run. Temporal statistics has been generated and it is observed that low *Re* case has shown velocity fluctuations of the order of numerical error, whereas high *Re* flow has shown large velocity amplitudes. Therefore a location with max standard deviation in x-axis velocity (U_x) has been identified as control point for high *Re* case, as shown in Fig. 4.

Both cases have been analysed for fluctuations in velocity at the control point over a long period of time, as shown in Fig. 5. It has been observed that at low *Re* a steady flow is

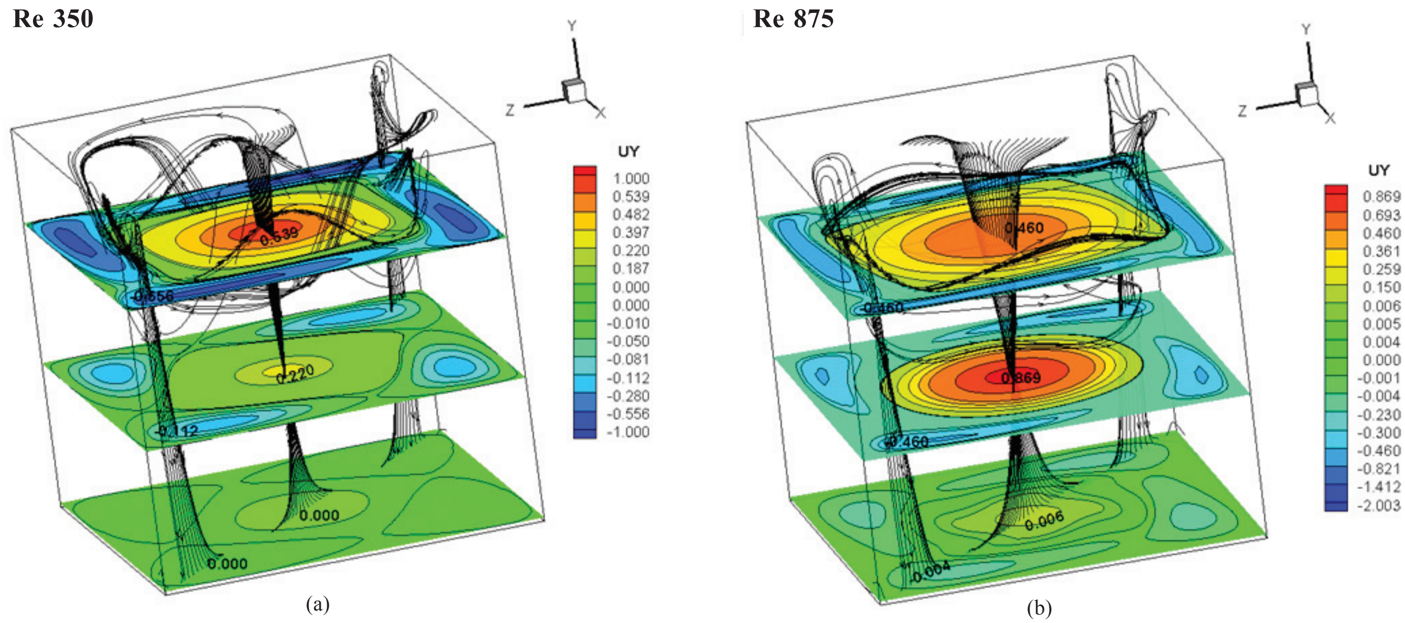


Figure 3. Non-dimensional U_y contours and stream traces; (a) *Re* = 350, (b) *Re* = 875.

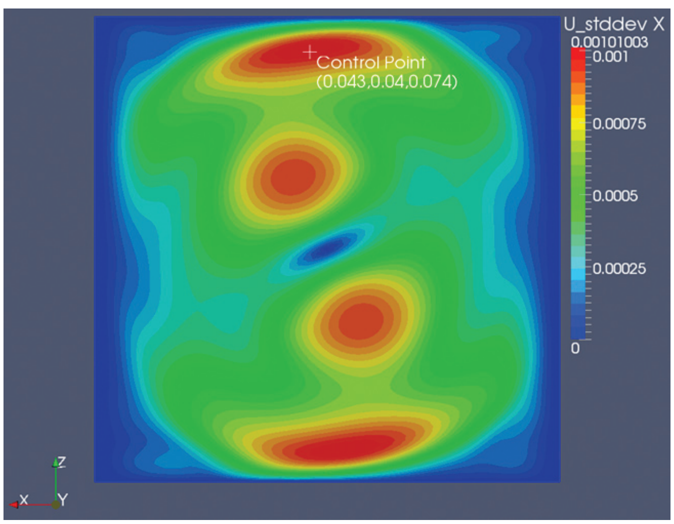


Figure 4. U_x Std. Deviation Plot for *Re* = 2100 and control point location.

reached whereas, flow behaviour shows fully developed oscillations at high *Re*. Therefore, control point for all further analysis has been identified as (*x, y, z*) = (0.043, 0.04, 0.074).

It is seen from temporal velocity profile that initial low frequency disturbances die down by around 400 s in the flow. Whereas at high *Re* there is a dominant disturbance which does not decay to zero and flow remains oscillatory, marking onset of super-critical *Re* regime. Based on this result a narrower region of *Re* between 1400 and 1610 has been identified as sub-critical *Re* regime. These cases have been run for additional 250 s of flow time at Nyquist sampling rate of 2 Hz.

Frequency response analysis of all identified sub-critical *Re* cases have been carried out using developed matlab code (Fig. 6). Similar analysis has been carried out for other 1 and 4 million cells cases to estimate grid-free critical Reynolds number. Frequency (ω) of dominant mode of oscillation has been found to be consistent at 0.66 (non-dimensional).

Two separate pairs of cases have been used for extrapolation and the estimated values of critical *Re* has been found to be in very close range (Table 1).

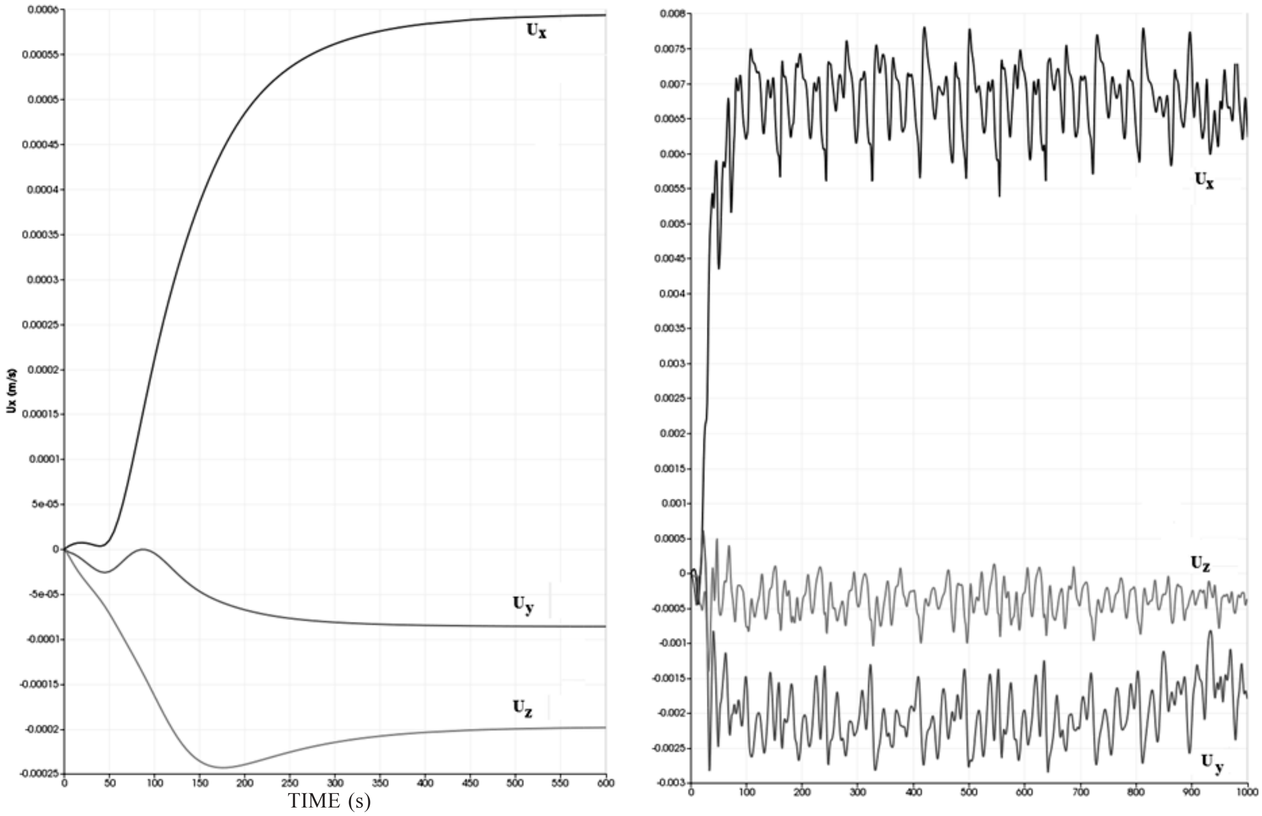
The grid independent critical *Re* has been estimated using results for 1 and 4 mil grid sizes and has been reported in Table 2.

Table 1. Estimation of grid-dependent critical Reynolds number

Grid resolution			Critical Re
100^3	Re = 1470	Re = 1505	
	σ = -5.397e-03	σ = -4.548e-03	1692
	ω = 0.66	ω = 0.66	
100^3	Re = 1575	Re = 1610	
	σ = -2.406e-03	σ = -1.575e-03	1676
	ω = 0.66	ω = 0.66	

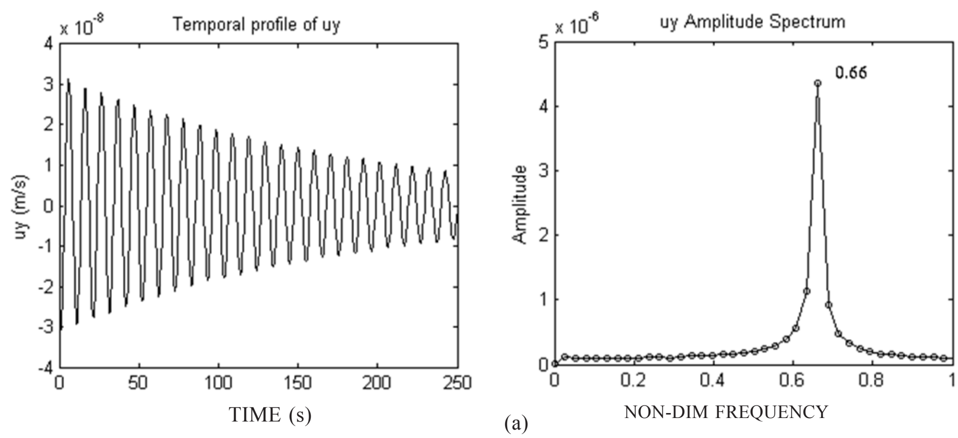
Table 2. Estimation of grid-free critical Reynolds number

Grid resolution			Grid dependent critical *Re*	Grid-free critical *Re*
100^3	*Re* = 1470	*Re* = 1505		
	σ = -5.397e-03	σ = -4.548e-03	1692	
	ω = 0.66	ω = 0.66		1606
160^3	*Re* = 1470	*Re* = 1505		
	σ = -5.630e-03	σ = -4.470e-03	1640	
	ω = 0.66	ω = 0.66		

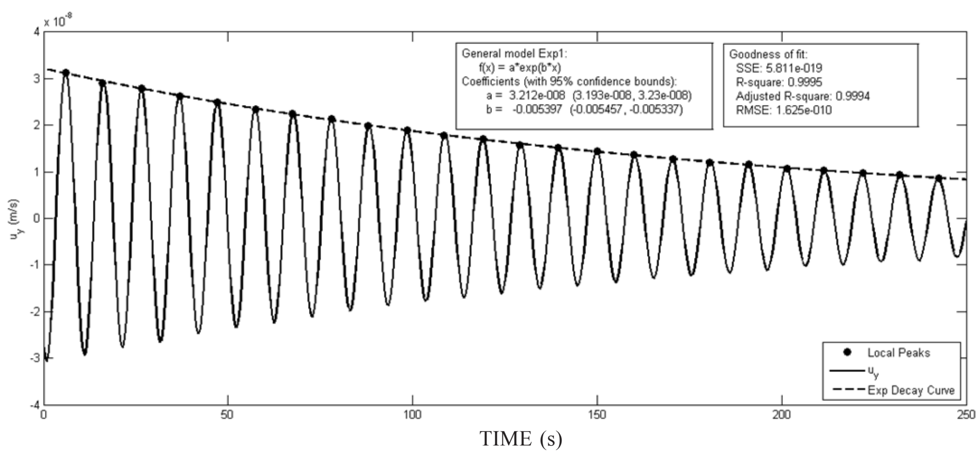


(a) (b)

Figure 5. Velocity profiles (U_x, U_y, U_z) at control point for; (a) *Re* = 350 (steady) and, (b) *Re* = 2100 (oscillatory).



(a)



(b)

Figure 6. *Re*=1470, 1 mil cells – (a) Frequency response, (b) Exponential decay rate (σ).

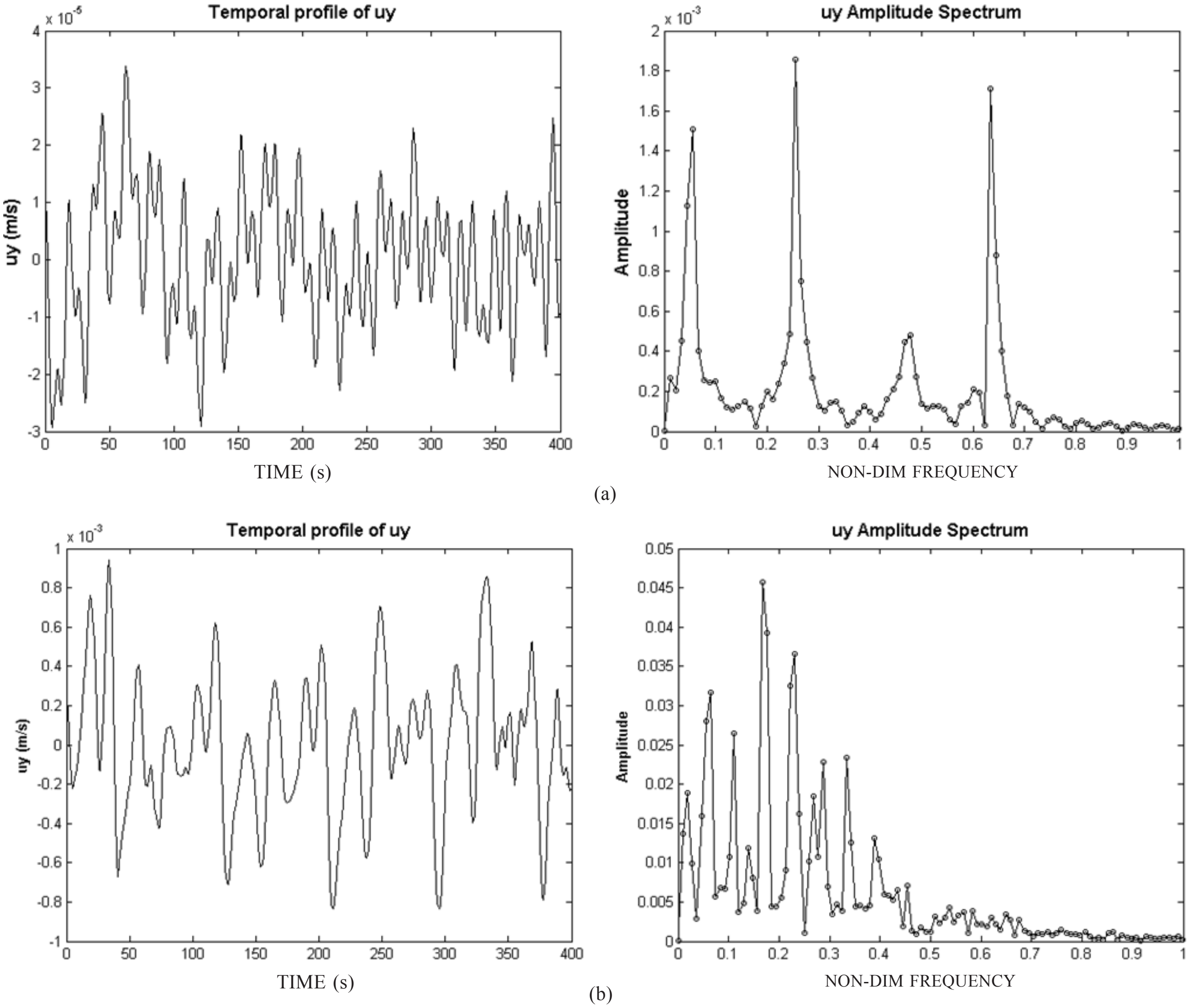


Figure 7. Frequency response of velocity fluctuation, uy – (a) Re = 1750, and (b) Re = 2100.

3.3 Flow Behaviour in Super-critical Re Regime

Flow in super-critical regime is oscillatory in nature with theoretically zero decay rate of amplitude of velocity fluctuations and velocity oscillations dominated by multimode frequencies, as shown in Fig. 7.

From frequency response, it is evident that frequencies are roughly equally spaced. The frequencies have been observed to be 0.05, 0.25, 0.47 and 0.66 (non-dimensional) approximately. This suggests that the modes are harmonic in nature. As Reynolds number is increased to 2100, frequency response tends towards continuous spectrum, i.e. turbulent flow feature, as shown in Fig. 7. Flow has reached a fully developed oscillatory regime and may lead to turbulent behaviour on increasing operating Reynolds number. The study in turbulent region is out of scope for this work.

The overall flow in oscillatory regime has been observed to behave in similar manner as steady flow, except that the central core stream-traces coming from bottom wall terminate before reaching the lid (Fig. 8). The U_y contour at constant $y = -0.13$ (non-dimensional) plane shows a central core moving upwards, whereas constant $y = 0.25$ plane (non-dimensional) shows a

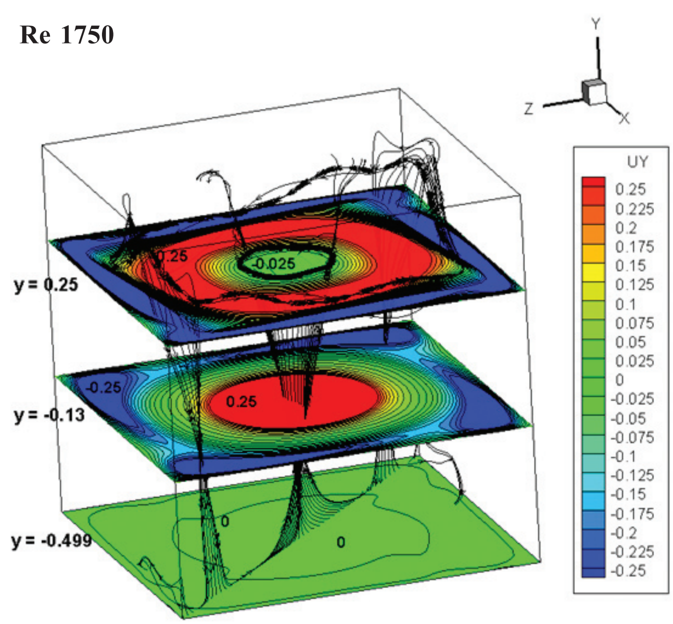


Figure 8. Non-dimensional U_y contours and stream traces at Re = 1750.

stagnant or stationary region at core and upwards moving flow around it. This observation suggests the existence of stationary vortex bubble similar to as in rotating lid-driven cylinder and coincides with onset of non-decaying oscillations[8]. This vortex bubble formation and development has not been investigated in this work, but will be covered in a subsequent paper.

4. CONCLUSIONS

Overall flow in the cavity under study seems similar in nature to cylindrical cavity flow, but closer investigations have revealed significant differences. Though, an upward moving central core flow is established by fluid being fed from outer region, four corners of the cube divide the flow in four distinct zones. The outer flow influences shape of the central core region and 4-fold rotational symmetry in flow is observed. The corners also influence flow behaviour with respect to increasing Reynolds number and oscillations set in earlier than rotating cylindrical cavity or linear lid-driven cubical cavity flows.

A rotating cylindrical cavity of aspect ratio 2 has been reported to reach oscillatory state of flow at Re = 2600 [3] and linear lid-driven cubical cavity has been reported at Re = 1914 [5]. However, the flow under study differs significantly and is observed to reach oscillatory flow state at even lower Re = 1606. The frequency of oscillation has been estimated to be 0.66 (non-dimensional).

Investigations in oscillatory regime has shown existence of a stationary vortex bubble, which coincides with onset of unsteadiness in flow.

REFERENCES

1. Shankar, P.N. & Deshpande, M.D. Fluid mechanics in the driven cavity. *Annu. Rev. Fluid Mech.*, 2000, **32**(1), 93-135.
   doi: 10.1146/annurev.fluid.32.1.93
2. Ghia, U.; Ghia, K.N. & Shin, C.T. High-Re solutions for incompressible flow using the Navier-Stokes equations and a Multigrid method. *J. Comp. Phy.*, 1982, **48**(3), 387-411.
   doi: 10.1016/0021-9991(82)90058-4
3. Sorenson, J.N.; Naumov, I. & Mikkelsen, R. Experimental investigation of three-dimensional flow instabilities in a rotating lid-driven cylinder. *Exp. Fluids*, 2006, **41**(3), 425-440.
   doi: 10.1007/s00348-006-0170-5
4. Chiang, T.P.; Sheu, W.H. & Hwang, R.R. Effect of Reynolds number on the eddy structure in a lid-driven cavity. *Int. J. Numer. Meth. Fluids*, 1998, **26**(5), 557-579.
   doi: 10.1002/(sici)1097-0363(19980315)26:5<557::aid-fld638>3.0.co;2-r
5. Feldman, Yu. & Gelfgat, A. Yu. Oscillatory instability of a three-dimensional lid-driven flow in a cube. *Phy. Fluid*, 2010, **22**(9), 093602.
   doi: 10.1063/1.3487476
6. Liberzon, A.; Feldman, Yu. & Gelfgat, A. Yu. Experimental observation of the steady-oscillatory transition in a cubic lid-driven cavity. *J. Comp. Phy.*, 2011, **23**(8), 084106.
   doi: 10.1063/1.3625412
7. Leriche, E.; Loiseau, J. Ch. & Robinet, J. Ch. Global stability and transition to intermittent chaos in the cubical lid-driven cavity flow problem. *In* WSPC Proceedings, February 2015, pp. 94-102.
   doi: 10.1142/9789814635165_0007
8. Hendrik, C.K. & Stefan, A. Stability of the steady three-dimensional lid-driven flow in a cube and the supercritical flow dynamics. *Phys. Fluid*, 2014, **26**(2), 024104.
   doi : 10.1063/1.4864264
9. Wuthrich, B. Simulation and validation of compressible flow in nozzle geometries and validation of OpenFOAM for this application. Institute of Fluid Dynamics, ETH Zurich, 2007. (PhD Thesis).
10. Hillier, B.; Schram, M. & Maltby, T. OpenFOAM validation for high Reynolds number flow. Capstone Project, The University of Melbourne, Identifier CP-AOO-123, 2012.
11. Sudharsan, N.M.; Jambekhar, V.A. & Babu, V. A validation study of OpenFOAM using the supersonic flow in a mixed compression intake. *In* Proceedings of the Institution of Mechanical Engineers; 2009, **224**(6), 673-679.
    doi: 10.1243/09544100JAERO651

CONTRIBUTORS

Mr. Jitendra Kumar obtained his BTech (Mechanical Engg.) from IIT Roorkee, Roorkee, in 2003 and MTech (Aerospace) from IIT Madras, Chennai. Currently working as Scientist D at Aeronautical Development Agency, Bengaluru. His research interests includes : Aircraft aerodynamics (steady and unsteady turbulent flows) and performance.
Contribution in the current study, he has estimated the grid-independent critical Reynolds number at which flow turns oscillatory. The literature survey, simulation and analysis work has been carried out.

Dr N. R. Panchapakesan obtained his PhD (Aerospace Engg.) from Cornell University, New York, in 1991. He is currently working as Associate Professor in department of Aerospace engineering, IIT Madras, Chennai. His area of expertise include: Fluid mechanics, stability and transition of fluid flows and turbulence. His current research interests include turbulent jets, mixing layers and confined cavity flows.
Contribution in the current study, he has provided able guidance through definition of the problem, work content and constant technical reviews.