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Nonlinear Free Vibration Analysis of Laminated Carbon/Epoxy Curved Panels

C.K. Hirwani[#], T.R. Mahapatra[@], S.K. Panda[#,*], S.S. Sahoo[#], V.K. Singh[#], and B.K. Patle[!]

[#]*Department of Mechanical Engineering, National Institute of Technology, Rourkela – 769 008, India*
[@]*School of Mechanical Engineering, KIIT University, Bhubaneswar - 751 024, India*
[!]*Department of Mechanical Engineering, RSR, Rungta College of Engineering and Technology, Bhilai – 490 024, India*
[*]*E-mail: pandask@nitrkl.ac.in*

ABSTRACT

Nonlinear frequency responses of the laminated carbon/epoxy composite curved shell panels have been investigated numerically and validated with in-house experimentation. The nonlinear responses have been computed numerically via customised computer code developed in MATLAB environment with the help of current mathematical model in conjunction with the direct iterative method. The mathematical model of the layered composite structure derived using various shear deformable kinematic models (two higher-order theories) in association with Green-Lagrange nonlinear strains. The current model includes all the nonlinear higher-order strain terms in the formulation to achieve generality. Further, the modal test has been conducted experimentally to evaluate the desired frequency values and are extracted via the transformed signals using fast Fourier transform technique. In addition, the results are computed using the simulation model developed in commercial finite element package (ANSYS) via batch input technique. Finally, numerical examples are solved for different geometrical configurations and discussed the effects of other design parameters (thickness ratio, curvature ratio and constraint condition) on the fundamental linear and nonlinear frequency responses in details.

Keywords: Single/doubly curved panels; Nonlinear FEM; Experimental vibration; Modal analysis

1. INTRODUCTION

The recent advances in design and manufacturing technologies have greatly enhanced the use of advanced fibre reinforced laminated composite structures in weight sensitive and high-performance engineering applications. Increasing usage of laminated composite shells in mechanical, civil, aircraft, aerospace, automotive, biomedical, nuclear, petro-chemical and marine engineering have created the necessity of the analysis of their responses precisely. It is well known that, these structures are very often subjected to large amplitude vibration during their service life. Hence, it is important to define the state variables with exact mathematical modelling incorporating geometrical nonlinearity, which is a not possible using linear strain displacement relation. It is because; the total deformation of the laminated structures is significantly larger than the linear strains. In addition, the laminated composites are prone to fail under shear and the effect of shear deformation, depending on geometrical and material properties, plays a significant role in determining the global characteristics of the structure. To obtain the exact response with less computational effort, the nonlinear vibration behaviour of laminated structures has been investigated numerically using mathematical models based on various classical; shear deformation and refined theories[1]. Furthermore, the finite element method (FEM) has been proved to be a versatile tool for the analysis of complex laminated structures/structural components.

We note a considerable amount of articles are already published on the geometrically nonlinear free vibration analysis of the laminated composite flat/curved panel structure using the FEM. In addition, many experimental studies related to the vibration analysis of the fabric composite structures are also available[2-8]. Also, the past studies indicates time to time regarding the suitability of the higher-order shear deformation theory (HSDT) for the analysis purpose of the laminated structures due to the accurate approximations of the transverse shear stress and strains as well as the shear correction factor[9] can be avoided and it will reduce in reduction of error accumulation in the final responses. Reddy and Liu[10] developed first time a general laminated composite shell panel model in the framework of the HSDT mid-plane kinematics to investigate the bending and vibration responses of cylindrical/spherical shells. Reddy[11] presented a mathematical model of laminated composite plates based on the HSDT and von-Karman strain that predicts the deflections, stresses and frequencies more accurately when compared to the first-order shear deformation theory (FSDT) and classical plate theory (CPT). Shin[12] investigated the large amplitude vibration behaviour of laminated composite doubly curved shells using von-Karman type geometrical nonlinearity in the framework of the FSDT kinematics. Liu and Huang[13] reported the nonlinear free vibration responses

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of the laminated plate structure by taking the nonlinear kinematics via von-Karman sense in the framework of the FSDT under unlike temperature load. Further, the analytical solutions of the free vibration responses of the laminated composite and sandwich plate structure are investigated using the HSDT kinematic model by Kant and Swaminathan[14]. Kerur and Ghosh[15] reported the numerical solutions of the geometrically nonlinear transient responses of the laminated composite plate using FEM technique via developing a model using the FSDT kinematics and von-Karman nonlinear strain. Kishore[16], *et al*. reported the nonlinear static deflection behaviour of the smart composite plate structure using von-Karman type of geometrical nonlinear strain including the third-order shear deformation theory (TSDT) to define the mid-plane deformation. Naidu and Sinha[17] developed a mathematical model based on the FSDT kinematic model and Green-Lagrange nonlinear strains to investigate the free vibration responses of the composite shell panels under the hygrothermal environments using FEM. Nanda and Bandyopadhyay[18] presented the nonlinear free vibration frequency responses of the laminated composite cylindrical shell panel structure using the numerical model and the model is developed using the FSDT kinematics and von-Karman type of geometrical nonlinear strain. The free vibration responses of the laminated composite plate is analysed by Ngo-Cong[19], *et al*. in the framework of the FSDT mid-plane kinematics. Further, a C^0 FE model based in conjunction with the HSDT mid-plane theory has been developed by Pradyumna and Bandyopadhyay[20] to predict the static and dynamic deflection values of the laminated composite shell. Similarly, a general higher-order equivalent single layer theory is proposed by Tornabene[21], *et al*. to analyse the free vibration behaviour of the doubly curved laminated composite shell panels. The nonlinear FEM steps utilised to compute the large amplitude free vibration frequency responses of the doubly curved laminated composite shell panels using the HSDT kinematics model by Singh and Panda[22] and Mahapatra[23], *et al*. including the effect of the environmental effect. Also, a large volume of the research articles on the dynamic behaviour of the laminated composite plate and shell structure reported using variational asymptotic method (VAM)[24-31]. Studies indicate that the VAM approach is capable of analysing the geometrical nonlinearity including geometries effect with adequate accuracy.

Their view clearly indicates that many numerical/analytical/experimental studies have already been reported on the free vibration behaviour of the laminated composite structures using the FSDT/HSDT kinematics in conjunction with FEM. However, we note that no study has been reported yet on the geometrically nonlinear free vibration responses of the laminated flat/curved panel structure numerically using Green-Lagrange geometrical nonlinear strain in the framework two higher-order shear deformation theories as well as the experimental validation. The primary objective of this article is to establish the inevitability of the higher-order kinematic models for the analysis of nonlinear frequency responses. In order to do so, the laminated structural responses are obtained numerically using the different mid-plane kinematics in conjunction with Green-Lagrange nonlinearity. Also, the responses are computed via simulation model with the help of ANSYS parametric design language (APDL) code and experimentation. Firstly, the numerical results are computed for the linear cases and compared with available published literature, simulation (ANSYS) and experimental values. Based on the validity of linear frequencies the model is extended for the nonlinear case and compared with those available published results. In addition, for the validation study, the composite material properties are also obtained experimentally using simple uniaxial tensile test. Finally, the nonlinear frequency responses of the laminated composite single and doubly curved shell panels (cylindrical, elliptical and hyperboloid) have been computed numerically with the help of presently proposed higher-order nonlinear models to bring out the effect of the geometrical parameters (thickness ratios and curvature ratio) and support conditions on the nonlinear free vibration responses.

2. MATHEMATICAL FORMULATION

Figure 1 shows the single/doubly curved laminated composite shallow shell panels (cylindrical/hyperboloid/ellipsoid/plate/spherical) of length a, width b and composed of a finite number of orthotropic layers of uniform thickness h, considered for the present analysis. The principal radii of curvatures of the shallow shell panel are, R_x and R_y (twist radius of curvature $R_{xy} = \infty$) along the x and the y directions, respectively. The different kinematic models adopted in the present investigation are as follows:

Model-1 $(\varepsilon_{zz} \neq 0)$

Firstly, the kinematic model of the laminated panel is developed using the cubic variation of in-plane displacement and the linear variation through the thickness as[32]:

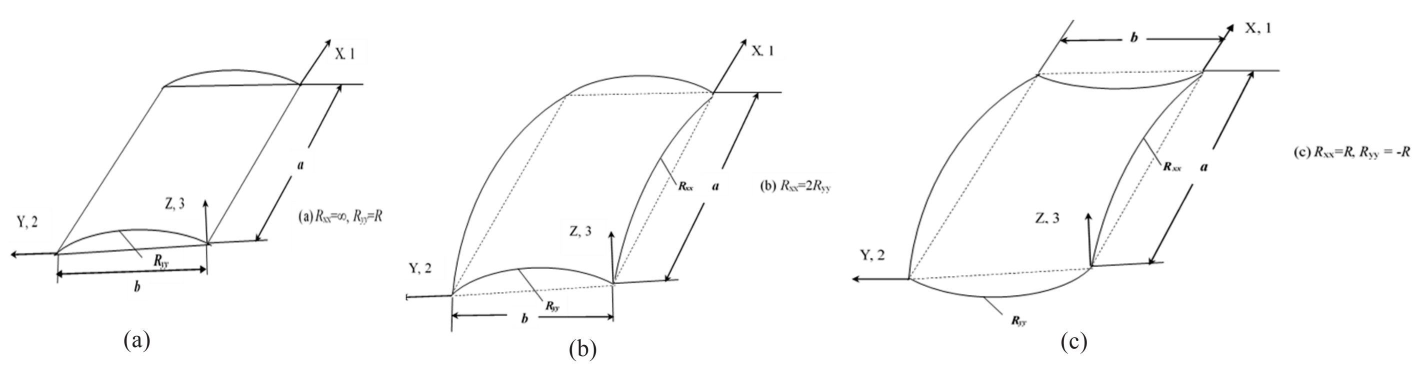


Figure 1. Laminated composite curved panels (a) Cylindrical (b) Ellipsoid (c) Hyperboloid.

$$\begin{aligned} u^{(k)}(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ &\quad + z^2\phi_x(x,y,t) + z^3\lambda_x(x,y,t) \\ v^{(k)}(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ &\quad + z^2\phi_y(x,y,,t) + z^3\lambda_y(x,y,t) \\ w^{(k)}(x,y,z,t) &= w_0(x,y,t) + z\theta_z(x,y,t) \end{aligned} \tag{1}$$

Model-2 $(\varepsilon_{zz} = 0)$

Further, the laminated composite panel model is developed mathematically based on the higher-order kinematics using the displacement field as in Reddy[33]:

$$\begin{aligned} u^{(k)}(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ &\quad + z^2\phi_x(x,y,t) + z^3\lambda_x(x,y,t) \\ v^{(k)}(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ &\quad + z^2\phi_y(x,y,t) + z^3\lambda_y(x,y,t) \\ w^{(k)}(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2}$$

Model-3

To develop a simulation model in ANSYS using APDL code, Shell 281 element has been chosen from ANSYS element library for the discretisation purpose. This shell element is suitable for analysing thin to moderately thick shell structures as it has more (eight) nodes than other shell elements. This element assumes a displacement field of six degrees of freedom (DOF) based on the FSDT:

$$\begin{aligned} u^{(k)}(x,y,z,t) &= u_0(x,y) + z\theta_x(x,y) \\ v^{(k)}(x,y,z,t) &= v_0(x,y) + z\theta_y(x,y) \\ w^{(k)}(x,y,z,t) &= w_0(x,y) + z\theta_z(x,y) \end{aligned} \tag{3}$$

where $u^{(k)}$, $v^{(k)}$ and $w^{(k)}$ are the displacements of a point along the (x, y, and z) directions, respectively on any k^{th} layer. u_0, v_0 and w_0 are the mid-plane displacements at time t; θ_x and θ_y are the rotations of the normal to the mid-surface ($z = 0$) about y and x-axes, respectively; θ_z is the extension of transverse normal along the thickness direction. The functions $\phi_x, \phi_y, \lambda_x$ and λ_y are the higher-order terms in the Taylor's series expansion and represents higher-order transverse cross sectional deformation modes.

The nonlinear Green–Lagrange strain–displacement relation for any general material continuum is expressed as[17]:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ w_{,z} \\ v_{,z} + w_{,y} \\ u_{,z} + w_{,x} \\ u_{,y} + v_{,x} \end{Bmatrix} + 1/2 \begin{bmatrix} (u_{,x})^2 + (v_{,x})^2 + (w_{,x})^2 \\ (u_{,y})^2 + (v_{,y})^2 + (w_{,y})^2 \\ (u_{,z})^2 + (v_{,z})^2 + (w_{,z})^2 \\ 2\{(u_{,z})(u_{,y}) + (v_{,z})(v_{,y}) + (w_{,z})(w_{,y})\} \\ 2\{(u_{,z})(u_{,x}) + (v_{,z})(v_{,x}) + (w_{,z})(w_{,x})\} \\ 2\{(u_{,x})(u_{,y}) + (v_{,x})(v_{,y}) + (w_{,x})(w_{,y})\} \end{bmatrix} \tag{4}$$

The total strain vector $\{\varepsilon\}$ is further modified as the linear $\{\varepsilon_L\}$ and nonlinear $\{\varepsilon_{NL}\}$ strains and expressed as:

$$\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\} \tag{5}$$

In view of Eqns. (1) and (4), the in-plane strain displacement relations for laminated curved panel using Model-1 is given by

$$\begin{aligned} \{\varepsilon_L\} + \{\varepsilon_{NL}\} &= \begin{Bmatrix} \varepsilon_1^{l_0} \\ \varepsilon_2^{l_0} \\ \varepsilon_3^{l_0} \\ \varepsilon_4^{l_0} \\ \varepsilon_5^{l_0} \\ \varepsilon_6^{l_0} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} \varepsilon_1^{nl_0} \\ \varepsilon_2^{nl_0} \\ \varepsilon_3^{nl_0} \\ 2\varepsilon_4^{nl_0} \\ 2\varepsilon_5^{nl_0} \\ 2\varepsilon_6^{nl_0} \end{Bmatrix} + \\ & z\left[\begin{Bmatrix} k_1^{l_1} \\ k_2^{l_1} \\ 0 \\ k_4^{l_1} \\ k_5^{l_1} \\ k_6^{l_1} \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} k_1^{nl_1} \\ k_2^{nl_1} \\ k_3^{nl_1} \\ 2k_4^{nl_1} \\ 2k_5^{nl_1} \\ 2k_6^{nl_1} \end{Bmatrix}\right] + z^2\left[\begin{Bmatrix} k_1^{l_2} \\ k_2^{l_2} \\ 0 \\ k_4^{l_2} \\ k_5^{l_2} \\ k_6^{l_2} \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} k_1^{nl_2} \\ k_2^{nl_2} \\ k_3^{nl_2} \\ 2k_4^{nl_2} \\ 2k_5^{nl_2} \\ 2k_6^{nl_2} \end{Bmatrix}\right] \\ & + z^3\left[\begin{Bmatrix} k_1^{l_3} \\ k_2^{l_3} \\ 0 \\ k_4^{l_3} \\ k_5^{l_3} \\ k_6^{l_3} \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} k_1^{nl_3} \\ k_2^{nl_3} \\ k_3^{nl_3} \\ 2k_4^{nl_3} \\ 2k_5^{nl_3} \\ 2k_6^{nl_3} \end{Bmatrix}\right] + z^4\frac{1}{2}\begin{Bmatrix} k_1^{nl_4} \\ k_2^{nl_4} \\ k_3^{nl_4} \\ k_4^{nl_4} \\ k_5^{nl_4} \\ k_6^{nl_4} \end{Bmatrix} + z^5\frac{1}{2}\begin{Bmatrix} k_1^{nl_5} \\ k_2^{nl_5} \\ k_3^{nl_5} \\ 2k_4^{nl_5} \\ 2k_5^{nl_5} \\ 2k_6^{nl_5} \end{Bmatrix} \\ & + z^6\frac{1}{2}\begin{Bmatrix} k_1^{nl_6} \\ k_2^{nl_6} \\ 0 \\ 0 \\ 0 \\ 2k_6^{nl_6} \end{Bmatrix} \end{aligned} \tag{6}$$

Now the above strain–displacement relation can be rearranged in matrix form as follows:

$$\{\varepsilon_L\} + \{\varepsilon_{NL}\} = \left[T^L\right]\{\overline{\varepsilon_L}\} + \frac{1}{2}\left[T^{NL}\right]\{\overline{\varepsilon_{NL}}\} \tag{7}$$

where $\{\overline{\varepsilon_L}\} = \{\varepsilon_1^{l_0}\,\varepsilon_2^{l_0}\,\varepsilon_3^{l_0}\,\varepsilon_4^{l_0}\,\varepsilon_5^{l_0}\,\varepsilon_6^{l_0}\,k_1^{l_1}\,k_2^{l_1}\,k_4^{l_1}\,k_5^{l_1}\,k_6^{l_1}\,k_1^{l_2}\,k_2^{l_2}\,k_4^{l_2}\,k_5^{l_2}$ $k_6^{l_2}\,k_1^{l_3}\,k_2^{l_3}\,k_4^{l_3}\,k_5^{l_3}\,k_6^{l_3}\}^T$ and $\{\overline{\varepsilon_{NL}}\} = \begin{Bmatrix} \varepsilon_1^{nl_0} & \varepsilon_2^{nl_0} & \varepsilon_3^{nl_0} & \varepsilon_4^{nl_0} \\ k_2^{nl_3} & k_3^{nl_3} & k_4^{nl_3} & k_5^{nl_3} \end{Bmatrix}$

$\begin{matrix} \varepsilon_5^{nl_0} & \varepsilon_6^{nl_0} & k_1^{nl_1} & k_2^{nl_1} & k_3^{nl_1} & k_4^{nl_1} & k_5^{nl_1} & k_6^{nl_1} & k_1^{nl_2} & k_2^{nl_2} \\ k_6^{nl_3} & k_1^{nl_4} & k_2^{nl_4} & k_3^{nl_4} & k_4^{nl_4} & k_5^{nl_4} & k_6^{nl_4} & k_1^{nl_5} & k_2^{nl_5} & k_4^{nl_5} \end{matrix}$

$\left.\begin{matrix} k_3^{nl_2} & k_4^{nl_2} & k_5^{nl_2} & k_6^{nl_2} & k_1^{nl_3} \\ k_5^{nl_5} & k_6^{nl_5} & k_1^{nl_6} & k_2^{nl_6} & k_6^{nl_6} \end{matrix}\right\}$ are the mid-plane linear and nonlinear strain terms. Similarly, $\left[T^L\right]$ and $\left[T^{NL}\right]$ are the linear and the nonlinear thickness coordinate matrices. The terms containing superscripts l_0, l_1, l_2-l_3 in $\{\varepsilon_L\}$ and nl_0, nl_1, nl_{2-6} in $\{\varepsilon_{NL}\}$ are the membrane, curvature and higher-order strain terms, respectively.

Now the linear $\{\varepsilon_L\}$ and the nonlinear $\{\varepsilon_{NL}\}$ mid-plane strain vectors are modified again in the following form:

$$\left.\begin{aligned}\{\varepsilon_L\} &= [B_L]\{d\} \\ \{\varepsilon_{NL}\} &= \tfrac{1}{2}[B_{NL}]\{d\}\end{aligned}\right\} \tag{8}$$

or $\{\varepsilon_{NL}\} = \tfrac{1}{2}[B_{NL}]\{d\} = \tfrac{1}{2}[A][G]\{d\}$

where $[B_L]$ and $[B_{NL}]$ are linear and nonlinear differential operator matrices and $\{d\}$ is the displacement field vector. The details of $[A]$ and $[G]$ matrices are discussed by Singh and Panda[22].

The stress-strain relationship for the laminated shell panel can be expressed as:

$$\{\sigma\} = [\bar{Q}]\{\varepsilon\} \tag{9}$$

where $\{\sigma\}$, $[\bar{Q}]$ and $\{\varepsilon\}$ are the stress vector, transformed reduced stiffness matrix and the strain vectors, respectively.

The kinetic energy for N number of orthotropic layered composite shell panel will be

$$V = \frac{1}{2}\int_A \left(\sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} \{\dot{\delta}\}^T [f]^T \rho^k [f] \{\dot{\delta}\} dz \right) dA$$

$$= \frac{1}{2}\int_A \{\dot{\delta}\}^T [m] \{\dot{\delta}\} dA \tag{10}$$

where $[m] = \sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} [f]^T \rho^k [f] dz$ is elemental inertia matrix.

Now the strain energy expression is computed using the following steps

$$U = \tfrac{1}{2}\int_V \{\varepsilon\}^T \{\sigma\} dV \tag{11}$$

Now Eqn. (11) can be rewritten by substituting strains and stresses from Eqns. (7) and (9) as:

$$U = \frac{1}{2}\int_A \begin{pmatrix} \{\bar{\varepsilon}_l\}^T [D_1]\{\bar{\varepsilon}_l\} + \{\bar{\varepsilon}_l\}^T [D_2]\{\bar{\varepsilon}_{nl}\} \\ +\{\bar{\varepsilon}_{nl}\}^T [D_3]\{\bar{\varepsilon}_l\} + \{\bar{\varepsilon}_{nl}\}^T [D_4]\{\bar{\varepsilon}_{nl}\} \end{pmatrix} dA \tag{12}$$

where

$$[D_2] = \int_{-h/2}^{+h/2} [T^l]^T [\bar{Q}][T^{nl}] dz, \quad [D_1] = \int_{-h/2}^{+h/2} [T^l]^T [\bar{Q}][T^l] dz,$$

$$[D_3] = \int_{-h/2}^{+h/2} [T^{nl}]^T [\bar{Q}][T^l] dz \text{ and } [D_4] = \int_{-h/2}^{+h/2} [T^{nl}]^T [\bar{Q}][T^{nl}] dz. \tag{13}$$

The presently developed nonlinear HSDT model is discretised using the displacement based FE formulation steps. The mathematical models described previously are discretised by engaging a nine nodded isoparametric Lagrangian with ten (Model-1) and nine (Model-2) degrees of freedoms (DOF) per node, respectively. The corresponding state space displacement field vectors $\{d\}$ at any point on the mid-surface for any model is expressed as following:

$$d = \sum_{i=1}^{n} N_i(x,y) d_i \tag{14}$$

where $\{d_i\} = \{u_{0_i}\ v_{0_i}\ w_{0_i}\ \theta_{x_i}\ \theta_{y_i}\ \theta_{zi}\ \phi_{xi}\ \phi_{yi}\ \lambda_{xi}\ \lambda_{yi}\}^T$, $\{d_i\} = \{u_{0_i}\ v_{0_i}\ w_{0_i}\ \theta_{x_i}\ \theta_{y_i}\ \phi_{xi}\ \phi_{yi}\ \lambda_{xi}\ \lambda_{xi}\}^T$ and $\{d_i\} = \{u_{0_i}\ v_{0_i}\ w_{0_i}\ \theta_{x_i}\ \theta_{y_i}\ \theta_{zi}\}^T$ are the nodal displacement vectors for the i^{th} node for Model-1, Model-2, and Model-3, respectively and $[N_i]$ is the shape function matrix for any i^{th} node. The details of the displacement vector and the shape functions can be seen in Cook[34], *et al*.

The final form of the governing motion equation of the vibrated laminated curved shell panel is derived by solving the Lagrangian functional via Hamilton's principle and expressed as:

$$\delta\int_{t_1}^{t_2} (V - U) dt = 0 \tag{15}$$

$$[M]\{\ddot{\delta}\} + \left([K]_L + \frac{1}{2}[KN1] + \frac{1}{3}[KN2]\right)\{\delta\} = 0 \tag{16}$$

where

$$\{\delta\} = \{u_0\ \ v_0\ \ w_0\ \ \theta_x\ \ \theta_y\ \ \theta_z\ \ \phi_x\ \ \phi_y\ \ \lambda_x\ \ \lambda_y\}^T$$

is the displacement vector, $[M]$ and $[K]_L$ are the global mass matrix and global linear stiffness matrix, respectively. $[KN1]$ and $[KN2]$ are the linear and quadratic nonlinear mixed stiffness matrices that depend on the displacement vector linearly and quadratically, respectively. Further, Eqn (16) has been solved to obtain the desired nonlinear frequency values iteratively via the direct iterative method[35] and the detailed steps of implementation are discussed in the following steps.

(i) Initialisation of the elemental stiffness and the mass matrices using the common approach as in FEM.
(ii) Evaluation of linear and nonlinear global stiffness matrices and corresponding mass matrix following the assembling procedure with the help of the elemental matrices.
(iii) Further, linear frequency responses are obtained in the first step by applying the required constraint and dropping the inappropriate nonlinear terms.
(iv) The steps extended further i.e., extracting the eigen values and corresponding eigenvectors using the common eigen value extraction algorithm. The eigenvectors are scaled up to introduce the nonlinearity with the help of amplitude ratio (W_{max}/h, where W_{max} is the maximum central deflection and h is the total thickness of panel).
(v) The nonlinear stiffness matrices are updated at each steps by taking the linear values as the initial input.
(vi) Finally, the iteration steps are carried out till the convergence criteria ($\varpi_i - \varpi_{(i-1)} \leq 10^{-3}$) is achieved and print the final frequency values.

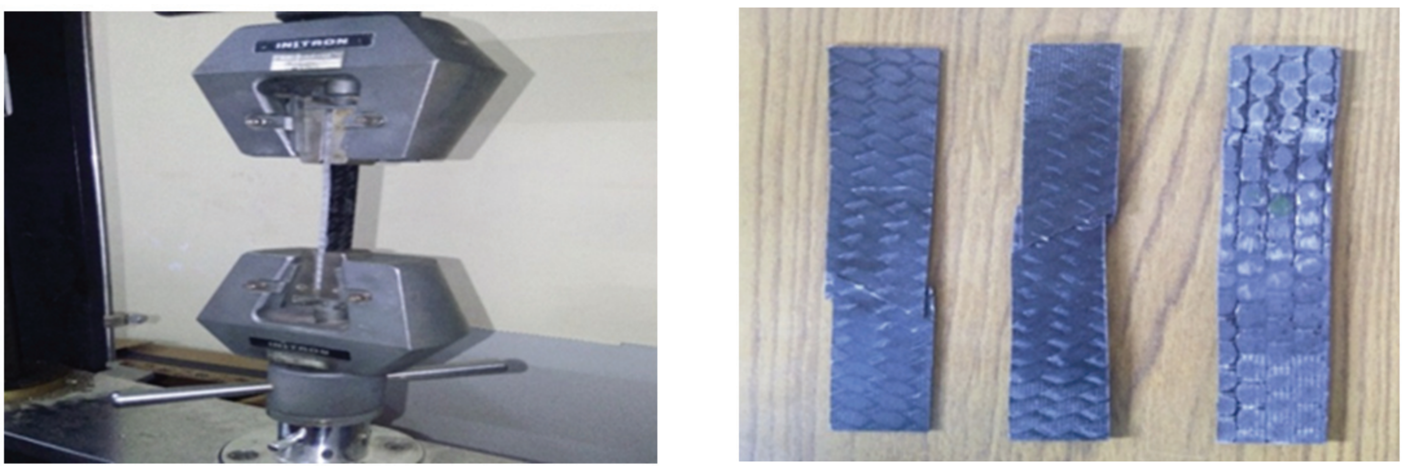
3. EXPERIMENTAL VIBRATION ANALYSIS

As a very first step, the material properties of Carbon/Epoxy laminated composite plates have been evaluated experimentally. For the experimentation purpose, one cross-ply [$(0°/90°)_s$] and two angle-ply [$(\pm45°)_1$ and $(\pm45°)_s$] Carbon/Epoxy laminated plate specimens are prepared for the uniaxial tensile test as per the ASTM standard (D 3039). The mechanical properties are obtained using universal testing machine (UTM, INSTRON 1195) at National Institute of Technology (NIT), Rourkela, India, by setting the loading rate as 1 mm/min. The UTM and the deformed specimen configurations are as shown in Figs. 2(a) and 2(b), respectively. The experimentally obtained composite properties are presented in Table 1. Further, the modal test has been conducted to obtain the free vibration responses of Carbon/Epoxy laminated composite specimens experimentally using a lab-scale experimental set-up developed at NIT as shown in Fig. 3. The laminated plate specimen (6) is fixed with the help of a fixture (4) subjected to an initial excitation with the help of an impact hammer (5) (SN 33452, National Instruments) and the mechanical responses are sensed by a uniaxial accelerometer (7) (SN LW161939, National Instruments), mounted at the center. The sensed mechanical signal has been transformed into electric signal by the uniaxial accelerometer and is fed to the cDAQ-9178 (2) (National Instruments) through the BNC cable (1). The corresponding electric signal in the form of acceleration in time domain is obtained on the output window (3) through the graphical programming language LabView14.0. The frequency responses are extracted by transformed signals in the frequency domain using the in-built fast Fourier transform (FFT) function in the lab view and the responses are presented in Fig. 4.

Initially, the first five modes of natural frequencies are computed using the proposed higher-order models as well as the simulation model for the comparison purpose including the

Table 1. Experimentally evaluated Carbon/Epoxy material properties

Properties	$[\pm45°]_1$	$[\pm45°]_s$	$[0°/90°]_s$
No. of layers	2	4	4
Young's modulus x direction (E_l)	6.695 GPa	6.469 GPa	12.34 GPa
Young's modulus y direction (E_t)	6.314 GPa	5.626 GPa	10.45 GPa
Young's modulus z direction (E_z)	6.314GPa	5.626 GPa	10.45 GPa
Shear modulus (G_{lt})	2.7 GPa	2.05 GPa	6.45 GPa
Shear modulus (G_{tz})	1.35 GPa	1.025 GPa	3.225 GPa
Shear modulus (G_{lz})	2.7 GPa	2.05 GPa	6.45 GPa
Density (ρ)	1388 Kgm^{-3}	1388 Kgm^{-3}	1388Kgm^{-3}



(a) (b)

Figure 2. Experimental setup for tensile testing (a) universal testing machine (UTM INSTRON-1195) (b) angle-ply/cross-ply Carbon/Epoxy specimen.

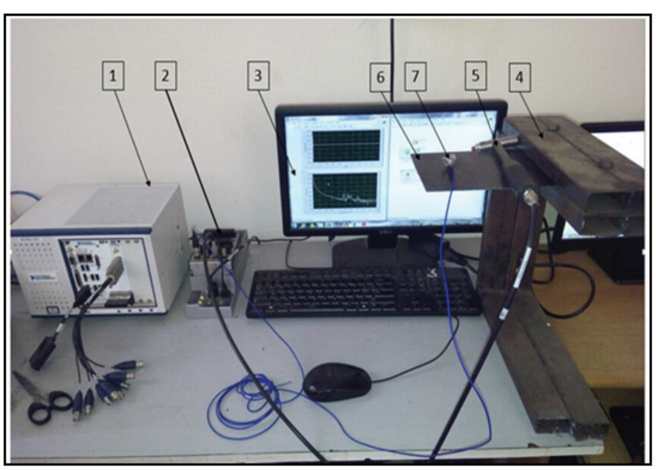


Figure 3. Modal testing of cross-ply Carbon/Epoxy plate specimen of experimental setup.

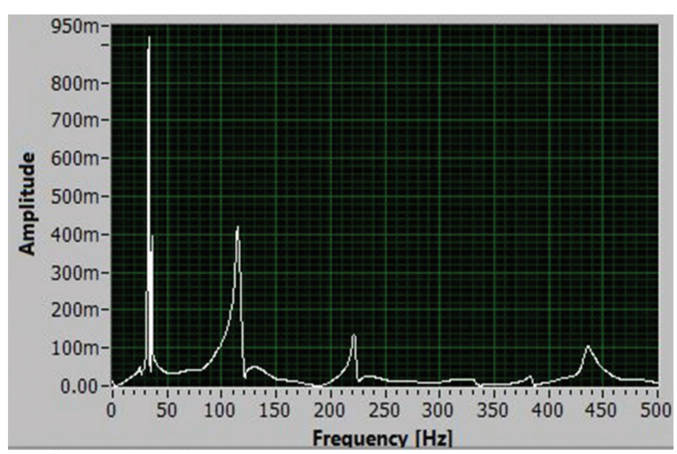


Figure 4. Experimental natural frequency of cross ply Carbon/Epoxy laminate.

experimental values. The frequency responses are computed using the experimental composite properties as provided in Table 1. The comparison of frequency values are presented in Tables 2(a) - 2(b) for different lay-up sequence, the number of layer and support conditions. The results clearly indicate that the responses obtained using the present HSDT models (Model-1 and Model-2) are close to the experimental values in comparison to the FSDT model (Model-3). The comparison clearly indicates the inevitability of the presently adopted HSDT mid-plane kinematics for the analysis of the laminated structures/structural components. In addition, the values of the various mode of frequencies including the amplitude obtained in the present experiment are as shown in Fig. 4. It is interesting to note that the current experimental graph of the frequency responses are showing the high amplitude for the first mode and decreases subsequently with the increase in mode numbers. It is important to mention that when the amplitude of the frequency values is too high as in the present case then the linear analysis may not hold or the linear solutions are not appropriate. The necessary steps of the conversion from nonlinear to the linear including the definition regarding the linear and nonlinear frequency parameter discussed by Dym[36]. Based on the present experimental values (Fig. 4), it is clear that the amplitude of the frequency is more than 10 times of the thickness ratio values i.e., a/h = 75, (a = 150 mm and h = 2 mm) whereas the amplitude is nearly 900 mm. Hence the present linear analysis unable to predict the desired solution because the structural response becomes nonlinear in nature[37]. To show the accuracy of the proposed numerical models, the frequency responses are computed using the presently developed higher-order models and compared with available numerical studies in the further sub-sections.

Table 2. (a) Fundamental natural frequency (ϖ_l) of cantilever carbon/ Epoxy laminated composite plate

Lamination Scheme	Mode	Experimental Results	Model-1 (HSDT)	Model-2 (HSDT)	Model-3 (FSDT)
	1	88	91.36	97.31531	90.403
	2	180	169.8	174.1709	169.34
$[\pm 45°]_1$	3	377	380.1	409.3941	364.75
	4	437	438.4	468.2937	435.7
	5	484	475.5	501.2964	463.28
	1	62.5	63.96	60.07	59.77
	2	152	153.23	151.4042	149.52
$[\pm 45°]_s$	3	378	383.40	371.47	362.75
	4	485	494.80	477.88	470.59
	5	561	562.36	549.98	536.25

Table 2. (b) Fundamental frequency (ϖ_l) SFSF supported carbon/ Epoxy composite plate

Lamination Scheme	Mode	Experimental Results	Model-1 (HSDT)	Model-2 (HSDT)	Model-3 (FSDT)
	1	33.5	35.45	32.78	32.58
	2	81.5	81.73	79.88	78.933
$[\pm 45°]_1$	3	210	217.93	207.38	200.67
	4	249	262.14	257.44	253.05
	5	302	308.84	296.73	287.93
	1	62.5	63.96	60.07	59.77
	2	152	153.23	151.4042	149.52
$[\pm 45°]_s$	3	378	383.40	371.47	362.75
	4	485	494.80	477.88	470.59
	5	561	562.36	549.98	536.25
	1	88	88.27	86.70	87.31
	2	242	242.87	238.89	239.47
$[0°/90°]_s$	3	518	547.33	517.25	537.17
	4	718	757.90	717.63	683.26
	5	846	859.44	833.34	834.71

4. THEORETICAL INVESTIGATION

After the experimental validation of the frequency values the proposed numerical higher-order models, the study has been extended to study the nonlinear responses further. The free vibration responses of the laminated composite flat/curved panels are computed using the present nonlinear FE code in MATLAB environment. The material properties of the Carbon/Epoxy composite lamina used for the comparison purpose and computation of new results are taken as M1 and M2, respectively and provided in Table 3. The support conditions adopted in the present investigation are presented in Table 4. Unless stated otherwise square (a/b = 1) simply supported symmetric cross-ply $[(0°/90°)_s]$ laminated composite curved panels having a/h = 100, R/a = 10 and M2 material properties are considered for the computation purpose in the new examples. The non-dimensional frequency is obtained using the formulae $(\varpi) = \omega b^2 \left\{\rho / \left(E_2 h^2\right)\right\}^{1/2}$.

Firstly, the convergence behaviour of the nondimensional fundamental frequencies using the present higher-order models (Model-1 and Model-2) have been established by solving different kinds of numerical examples. The responses are computed for the square symmetric cross-ply $(0°/90°)_s$ cylindrical shell panel under different support conditions (SSSS, CCCC and SCSC) and presented in Fig. 5. It is observed from the present solutions that the responses are following good convergence rate with mesh refinement. Based on the convergence study, a (6×6) mesh has been utilised for the computation of the responses throughout the analysis. Subsequently, the proposed models are validated by comparing the present linear and nonlinear frequency parameters with the available published results. The natural frequencies (Hz) of the square simply supported $(0°/90°)_2$ cross-ply and angle-ply $(45°/-45°)_2$ laminated composite spherical shell panel (R/a = 5 and 10) are evaluated using the each of the models and compared with Parhi[38], *et al.* and presented in Table 5. Further, the fundamental frequencies of the square sixteen-layer laminated composite spherical shell (R/a = R/b = 5) panel is computed for two constraint conditions (SSSS and CCCC) including the comparison (Panda[39], *et al.*)

Table 3. Composite material properties considered for numerical analysis

Properties	Material-1[17] [M1]	Material-2[33] [M2]
Longitudinal modulus (E_1)	40 MPa	138.6 Gpa
Transverse modulus (E_2)	1.0 MPa	8.27 Gpa
In-plane Poisson's ratio (v_{12})	0.25	0.26
Transverse Poisson's ratio (v_{13})	0.25	0.26
Transverse Poisson's ratio (v_{23})	0.25	0.26
In-plane Shear modulus (G_{12})	0.6 MPa	4.12 Gpa
Transverse Shear modulus (G_{23})	0.5 MPa	$0.6E_2$
Transverse Shear modulus (G_{13})	0.6 MPa	$0.6E_2$

Table 4. Details of Support conditions

CCCC	$u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = \phi_x = \phi_y = \lambda_x = \lambda_y$ at $x = 0, a$ and $y = 0, b$
SSSS	$v_0 = w_0 = \theta_z = \phi_y == \lambda_y = 0$ at $x = 0, a$; $u_0 = w_0 = \theta_z = \phi_x = \lambda_x = 0$ at $y = 0, b$
SCSC	$v_0 = w_0 = \theta_z = \phi_y = \lambda_y = 0$ at $x = 0, a$; $u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = \phi_x = \phi_y = \lambda_x = \lambda_y = 0$ at $y = 0, b$
HHHH	$u_0 = v_0 = w_0 = \theta_z = \phi_y = \lambda_y = 0$ at $x = 0, a$; $u_0 = v_0 = w_0 = \theta_z = \phi_x = \lambda_x = 0$ at $y = 0,b$

tabulated in Table 6. Similarly, the comparison study for the nondimensional nonlinear frequency responses is obtained using the geometrical parameters and the material properties are same[13,17] and presented in Fig. 6. The Fig. 6 indicates that the present frequency values obtained via higher-order models predicted smaller values than the references. It is because of the fact the results obtained in the references using the FSDT kinematics which makes the model stiffer and overestimates the frequencies. In addition, the nonlinear strains are modelled mathematically using von-Karman and Green-Lagrange strain-displacement relations. In both the references, the nonlinear strain kinematics are unrealistic in nature because of the non-consideration of exact kinematics in the first case whereas the strains up to second order for the latter case. Hence, the results obtained using the presently developed higher-order models within the expected line.

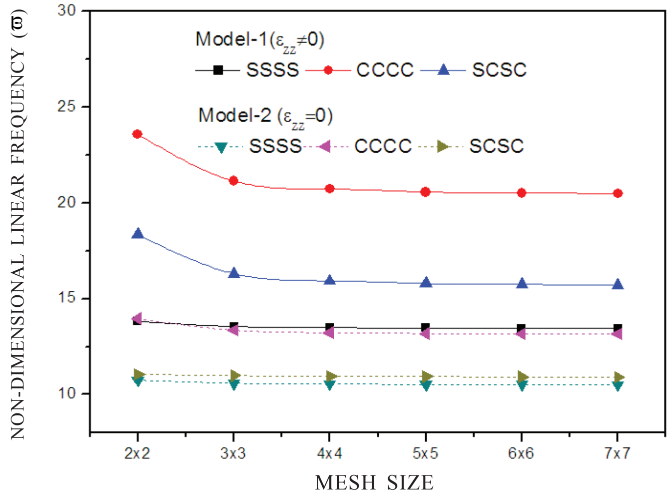


Figure 5. Convergence behaviour of fundamental natural frequency of a simply supported square symmetric cross ply $(0°/90°)_s$ cylindrical shell panels.

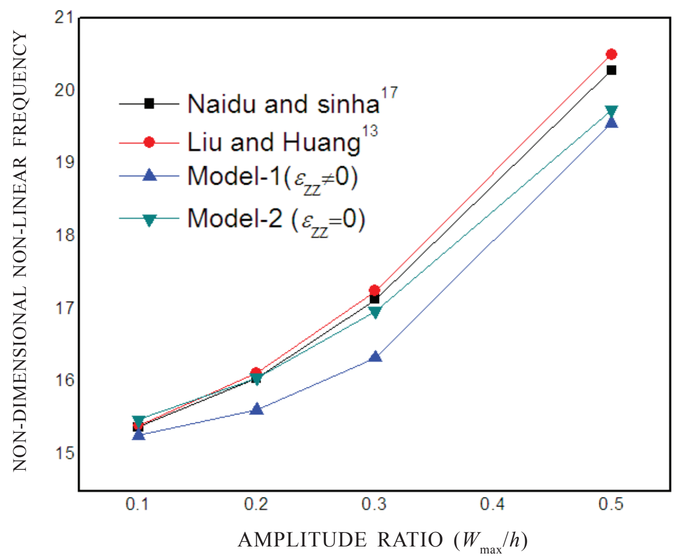


Figure 6. Comparison of nonlinear frequency of a square simply supported symmetric $(0°/90°)_s$ laminated composite plate for (a/b=1, a/h=10).

The theoretical and subsequent experimental validation study demonstrated the efficacy and accuracy of the proposed higher-order models. Now, few more examples have been solved for carbon/epoxy laminated composite curved panels using the present models to examine the effect of different

Table 5. Comparison study of the natural requency (Hz) of square simply supported laminated composite spherical shells panel

Lamination scheme	Parhi[38], *et al.*		Model-1		Model-2		Model-3	
	R/a=5	R/a=10	R/a=5	R/a=10	R/a=5	R/a=10	R/a=5	R/a=10
$(0°/90°)_2$	202.02	129.20	202.30	129.95	202.02	129.49	202.43	129.12
$(45°/-45°)_2$	347.52	301.13	393.81	265.38	392.85	265.10	384.75	264.85

Table 6. Comparison study of the fundamental frequency of square sixteen-layer laminated composite spherical shell (R/a=R/b=5) panel

Lamination scheme	Panda[39], *et al.*		Model-1		Model-2		Model-3	
	SSSS	CCCC	SSSS	CCCC	SSSS	CCCC	SSSS	CCCC
$(0°)_{16}$	458.24	821.8	602.32	1102.0	470.6	858.10	471.69	846.85

geometrical parameters and the support conditions on the nonlinear free vibration behaviour.

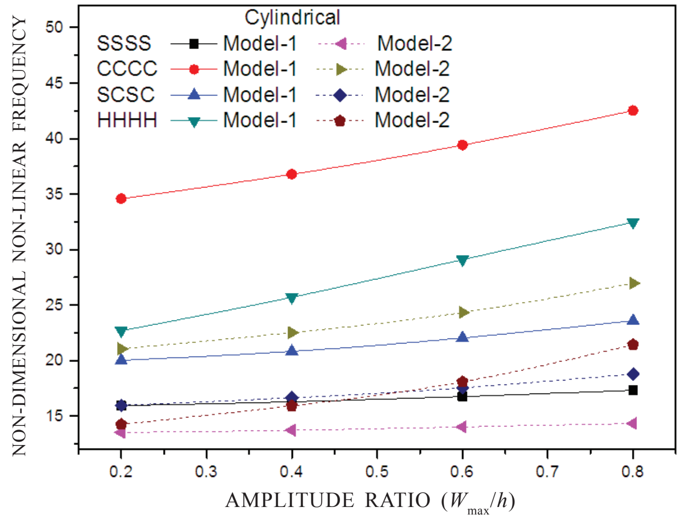
Owing to their spatial curvature, the stretching energy for the shell panels are high as compared to bending energy. The type of shell (deep/shallow) is recognized by its curvature ratio (R/a) and it has the considerable effect on their vibration behaviour. In order to investigate the effect, the linear and nonlinear free vibration responses are computed for each type of shell geometries (spherical /cylindrical/ ellipsoid/hyperboloid/ flat) for two curvature ratios (R/a = 10 and 100, except flat) and presented in Table 7. It is observed that the ellipsoid shell panel has the highest fundamental frequency among all and the frequency values are decreasing as the panel becomes flat. It can also be seen that all the panel geometries are showing the hard spring type of behaviour.

It is well known that the stiffness plays a major role in determining the structural responses and it greatly depends on the type of constraint conditions. It is true that the stiffness of the structure/structural component increases as the number of constraints increases. The effect of four different support conditions (SSSS, CCCC, SCSC, and HHHH) on the nonlinear free vibration behaviour of laminated composite single/doubly (cylindrical/hyperboloid/ellipsoid) curved panels have been analysed in this example and the responses are presented in Figs. 7 (a) - 7(c). It is seen that the frequency parameters are higher and lower for the clamped and simply supported cases irrespective of the geometry. This is because as the number of constraints decreases, the stiffness of the structure decreases and the frequency values decreases monotonically.

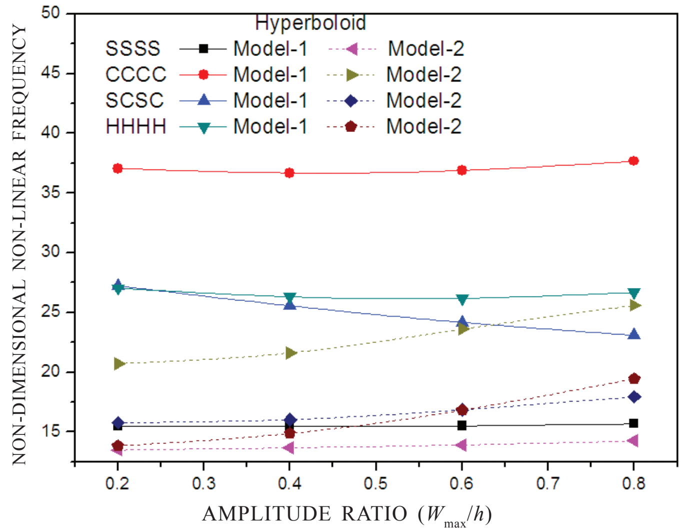
Generally membrane stresses are dominant in any shell structure due to which these structures are capable of carrying the extra amount of load in comparison to the flat panel. In addition to that the thickness ratio (a/h) is the most sensitive geometrical parameter for the analysis of the natural frequencies of the composite shell structures and it is more pronounced in the case of the thin structures. In this example, the effect of thickness ratio (a/h) on the nonlinear vibration behaviour of the laminated composite curved panel has been investigated. The responses are computed for four thickness ratios (a/h = 20, 30,

Table 7. Effect of geometry and curvature ratio on the non-dimensional linear and nonlinear frequency parameter of laminated Carbon/epoxy flat and curved panels

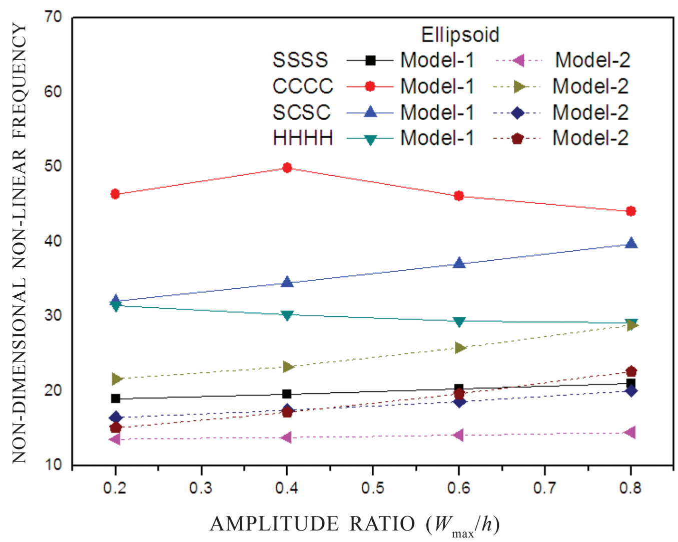
Shell type	R/a	W_{max}/h				
		0.0	0.2	0.4	0.6	0.8
Spherical	10	20.43	21.1147	21.8546	22.6334	23.4628
	100	15.347	15.4912	15.7455	16.1051	16.5605
Cylindrical	10	16.715	17.1619	17.6938	18.3074	18.9892
	100	15.302	15.4031	15.6157	15.9355	16.3608
Ellipsoid	10	25.423	26.223	27.0585	27.9232	28.814
	100	15.422	15.6083	15.9022	16.298	16.7912
Hyperboloid	10	16.676	16.3300	16.0810	15.9381	15.9142
	100	15.301	15.3163	15.444	15.6815	16.033
Flat	∞	15.287	15.3445	15.5146	15.7942	16.1779
	∞	15.287	15.3445	15.5146	15.7942	16.1779



(a)



(b)



(c)

Figure 7. Effect of support conditions on the non-dimensional non-linear frequency parameter of laminated carbon/ epoxy curved panels (a) Cylindrical (b) Hyperboloid (c) Ellipsoid.

50 and 100) and presented in Figs. 8 (a) - 8(c). It was observed that the nonlinear frequency responses are higher and lower for the cylindrical and the hyperbolic panel, respectively. It is also observed that the responses are showing hard spring type of behaviour for Model-1 and soft spring type of behaviour for Model-2, irrespective of the panel geometries.

Figure 9 shows the first six mode shapes of square (a/b = 1) simply-supported thin (a/h = 100) symmetric cross-ply $[(0°/90°)_s]$ laminated composite plate. It is well known that the mode shapes show the direction of vibration only but not give any idea related to the numerical value. It can be clearly seen that the present responses are within the expected line.

5. CONCLUSIONS

The linear and nonlinear frequency responses of the laminated Carbon/Epoxy composite single/doubly curved shell panels have been analysed numerically using new geometrical nonlinear model. The shell panel model has been developed using two HSDT mid-plane kinematics in conjunction with Green-Lagrange geometrical nonlinearity. The governing motion equation of the vibrated panel structure is obtained by solving the total Lagrangian functional using the classical Hamilton's principle and discretised with the help of FE steps. The desired nonlinear responses are computed by solving the present higher-order FE nonlinear equations via the direct iterative method. Also, the frequency responses are computed experimentally and the results obtained using both the numerical and the simulation models (ANSYS using APDL code) are compared with the available published results including the present experimental values. For the present experimental and numerical analysis purpose, the composite properties are obtained experimentally. The comparison study reveals that the HSDT type of mid-plane kinematics in conjunction with Green-Lagrange geometrical nonlinearity is inevitable for the accurate analysis of the laminated structure when the structure experiences the large amplitude vibration. Lastly, few examples related to different design parameters are solved using the presently developed higher-order FE models (Model-1 and Model-2) to investigate their effect. It is revealed that the Model-1 follow a hard spring type of behaviour irrespective of the geometrical and material parameters in contrast to the Model-2. Further, the nondimensional frequency responses decrease with the curvature ratios whereas increase with increasing values of the thickness ratios. Similarly, the frequency parameters are following an increasing trend when the number of constraints at the edges increase, regardless the geometrical configuration of the panel.

REFERENCES

1. Amabili, M. Nonlinear vibrations of laminated circular cylindrical shells: Comparison of different shell theories. *Compos. Struct.,* 2011, **94**, 207–220. doi: 10.1016/j.compstruct.2011.07.001
2. Chakravorty, D.; Bandyopadhyay, J.N. & Sinha, P.K. Finite element free vibration analysis of doubly curved laminated composite shells. *J. Sound Vibration,* 1996, **191**(4), 491-504. doi: 10.1006/jsvi.1996.0136

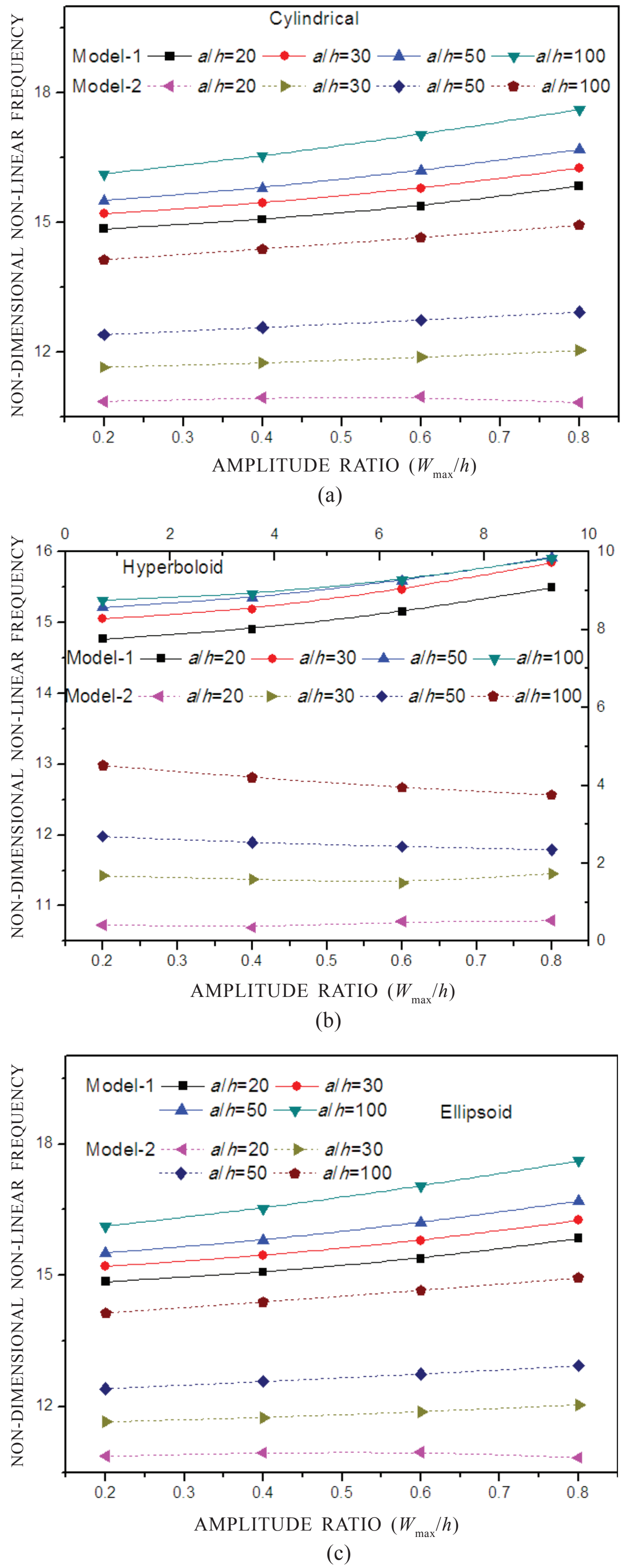


Figure 8. Effect of thickness ratio on the non-dimensional non-linear frequency parameter of laminated carbon/epoxy curved panels (a) Cylindrical (b) Hyperboloid (c) Ellipsoid.

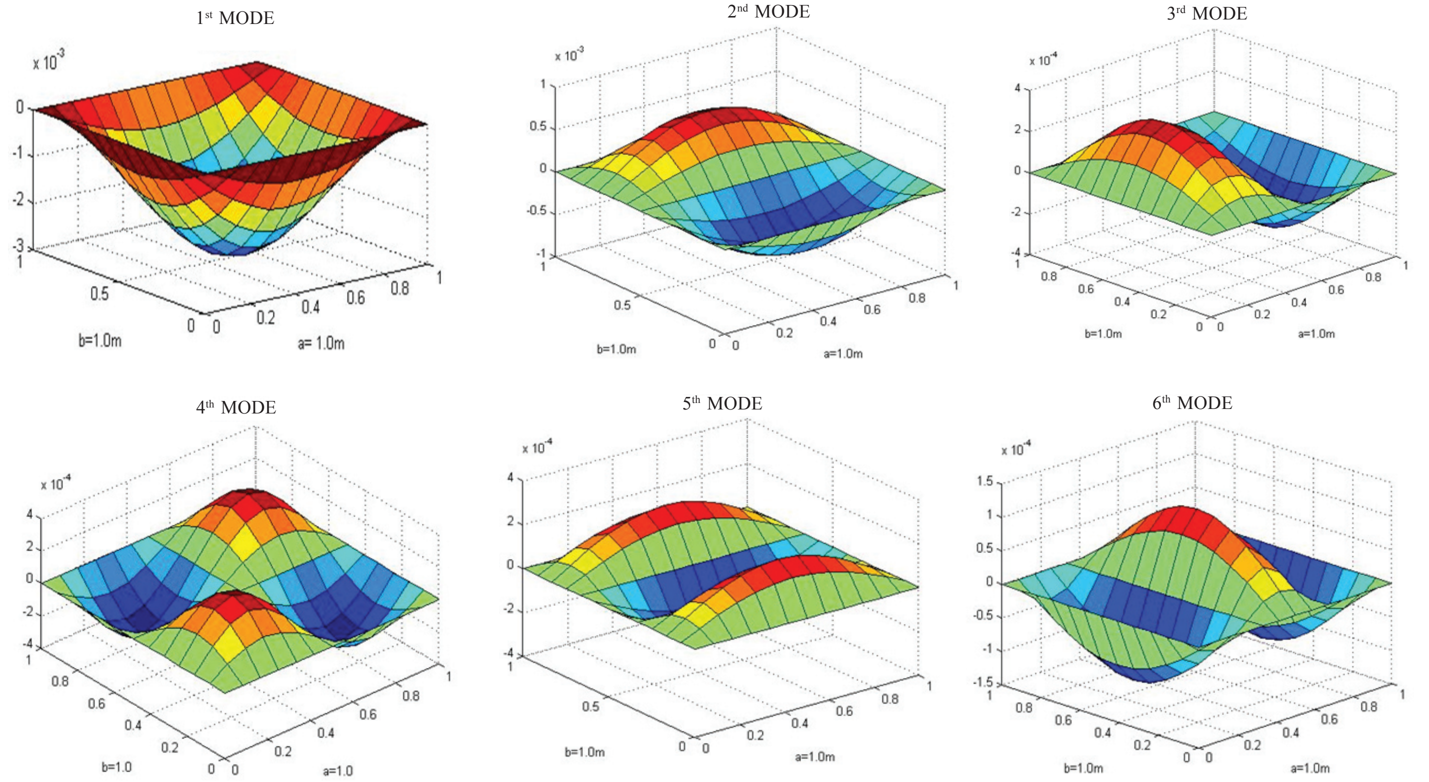


Figure 9. First six mode shapes of a simply supported cross ply laminated carbon/epoxy flat panel.

3. Qatu, M.S.; Sullivan, R.W. & Wang, W. Recent research advances on the dynamic analysis of composite shells: 2000–2009. *Compos. Struct.,* 2010, **93**, 14–31. doi: 10.1016/j.compstruct.2010.05.014
4. Alijani, F. & Amabili, M. Non-linear vibrations of shells: A literature review from 2003 to 2013. *Int. J. Non-Linear Mech.,* 2014, **58**, 233–257. doi: 10.1016/j.ijnonlinmec.2013.09.012
5. Ganapathi, M. & Patel, B.P. A new C^1 eight-noded plate element for static and dynamic analyses of composite laminates. *Def. Sci. J.*, 2000, **50**(3), 317-323. doi: 10.14429/dsj.50.3738
6. Lei, X.; Rui, W.; Yong, L. & Jin, L. The effect of woven structures on the vibration characteristics of glass fabric/epoxy composite plates. *Def. Sci. J.*, 2011, **61**(5), 499-504. doi: 10.14429/dsj.61.296
7. Mahmoodi, S.N.; Jalili, N. & Khadem, S.E. An experimental investigation of nonlinear vibration and frequency response analysis of cantilever viscoelastic beams. *J. Sound Vibration*, 2008, **311**, 1409–1419. doi: 10.1016/j.jsv.2007.09.027
8. Jeyaraj, P. Buckling and Free vibration behaviour of an isotropic plate under arbitrarily varying temperature distributions. *Int. J. Struct. Stability Dynam.,* 2012, **13**(6), 1250071. doi: 10.1142/S021945541250071X.
9. Khdeir, A.A.; Reddy, J.N. & Frederick, D. A study of bending, vibration and buckling of cross-ply circular cylindrical shells with various shell theories. *Int. J. Eng. Sci*., 1989, **27**(11), 1337–1351. doi:10.1016/0020-7225(89)90058-X
10. Reddy, J.N. & Liu, C.F. A higher-order shear deformation theory of laminated elastic shells. *Int. J. Engg. Sci*., 1985, **23**, 319-330. doi: 10.1016/0020-7225(85)90051-5
11. Reddy, J.N. A refined nonlinear theory of plates with transverse shear deformation. *Int. J. Solids Structures,* 1984, **20**, 881-896. doi:10.1016/0020-7683(84)90056-8
12. Shin, D.K. Large amplitude free vibration behaviour of doubly curved shallow open shells with simply-supported edges. *Composite Structure*, 1997, **62**, 35-49.
13. Liu, C.F. & Huang, C.H. Free vibration of composite laminated plates subjected to temperature changes, *Composite Structure*, 1996, **60**(1), 95-101. doi:10.1016/S0045-7949(96)00215-5
14. Kant, T. & Swaminathan, K. Analytical solutions for free vibration of laminated composite and sandwitch plates based on a higher-order-refined theory. *Composite Structure*, 2001, **53**, 73-85. doi: 10.1016/S0263-8223(00)00180-X
15. Kerur, S. & Ghosh, A. Active control of geometrically non-linear transient response of smart laminated composite plate integrated with AFC actuator and PVDF sensor. *J. Intell. Mater. Syst. Struct.*, 2011, **22**(11), 1149-1160. doi: 10.1177/1045389X11414222
16. Kishore, M.D.V.H.; Singh, B.N. & Pandit, M.N. Nonlinear static analysis of smart laminated composite plate. *Aerospace Sci. Technol.*, 2011, **15**, 224–235. doi: 10.1016/j.ast.2011.01.003
17. Naidu, N.V.S. & Sinha, P.K. Nonlinear free vibration analysis of laminated composite shells in hygrothermal environments. *Composite Structure*, 2007, **77**, 475–483. doi: 10.1016/j.compstruct.2005.08.002

18. Nanda, N. & Bandyopadhyay, J.N. Nonlinear free vibration analysis of laminate composite cylindrical shells with cutouts. *J. Reinf. Plast. Compos.,* 2007, **26**, 1413-1427. doi: 10.1177/0731684407079776
19. Ngo-Cong, D.; Mai-Duy, N.; Karunasena, W. & Tran-Cong, T. Free vibration analysis of laminated composite plates based on FSDT using one-dimensional IRBFN method. *Composite Structure*, 2011, **89**,1–13. doi: 10.1016/j.compstruc.2010.07.012
20. Pradyumna, S. & Bandyopadhyay, J.N. Static and free vibration analyses of laminated shells using a higher order theory. *J. Reinf. Plast. Compos.,* 2008, **27**, 167-186. doi: 10.1177/0731684407081385
21. Tornabene, F.; Viola, E. & Fantasize, N. General higher-order equivalent single layer theory for free vibrations of doubly-curved laminated composite shells and panels. *Composite Structure*, 2013, **104**, 94–117. doi: 10.1016/j.compstruct.2013.04.009
22. Singh, V.K. & Panda, S.K. Nonlinear free vibration analysis of single/doubly curved composite shallow shell panels. *Thin-Walled Struct.,* 2014, **85**, 341-349. doi: 10.1016/j.tws.2014.09.003
23. Mahapatra, T.R.; Kar, V.R. & Panda, S.K. Nonlinear free vibration analysis of laminated composite doubly curved shell panel in hygrothermal environment, *J. Sandwich Struct. Mat.,* **17**(5), 2015, 511-545. doi: 10.1177/1099636215577363
24. Tarn, J.Q. An asymptotic variational formulation for dynamic analysis of multilayered anisotropic plates. *Comput. Methods Appl. Mech. Engg*., 1996, **130**, 337-353. doi: 10.1016/0045-7825(95)00906-X
25. Lukovskii, I.A. & Timokha, A.N. Asymptotic and variational methods in non-linear problems of the interaction of surface waves with acoustic fields. *J. Appl. Maths Mech.*, 2001, **65**, 463-470. doi:10.1016/S0021-8928(01)00052-1
26. Lee, C.Y. & Hodges, D.H. Asymptotic construction of a dynamic shell theory: Finite-element-based approach. *Thin-Walled Structures*, 2009, **47**, 256-270. doi: 10.1016/j.tws.2008.08.004
27. Lee, C.Y. & Hodges, D.H. Dynamic variational-asymptotic procedure for laminated composite shells – Part I: Low-frequency vibration analysis. *J. Appl. Mech*., 2009, **76**, 011002-1-7. doi: 10.1115/1.3002761
28. Lee, C.Y. & Hodges, D.H. Dynamic variational asymptotic procedure for laminated composite shells – Part II: High-frequency vibration analysis. *J. Appl. Mech*., 2009, **76**, 011003-1-7. doi: 10.1115/1.3002762
29. Yifeng, Z. & Yu, W. A variational asymptotic approach for hygrothermal analysis of composite laminates. *Composite Structure*, 2011, **93**, 3229–3238. doi: 10.1016/j.compstruct.2011.06.003
30. Yifeng, Z.; Lei, C. & Yu, W. Variational asymptotic modeling of the thermo mechanical behavior of composite cylindrical shells. *Composite Structure*, 2012, **94**, 1023-1031. doi: 10.1016/j.compstruct.2011.10.002
31. Benchouaf, L. & Boutyour, E. H. Nonlinear vibrations of buckled plates by an asymptotic numerical method. *C. R. Mecanique*, 2016, **344**, 151-166. doi: 10.1016/j.crme.2016.01.002
32. Dash, P. & Singh B.N. Nonlinear free vibration of piezoelectric laminated composite plate. *Finite Elem. Anal. Des.,* 2009, **45,** 686-694. doi: 10.1016/j.finel.2009.05.004
33. Reddy, J.N. Mechanics of laminated composite: Plates and shells-Theory and analysis. 2nd ed., CRC press, 2004.
34. Cook, R.D.; Malkus, D.S; Plesha, M.E. & Witt, R.J. Concepts and applications of finite element analysis. 4th Ed., John Wiley and Sons, Singapore, 2009, 202-246.
35. Reddy, J.N. An introduction to nonlinear finite element analysis. 3rd Ed., Oxford University press, 2009.
36. Dym, C.L. Principles of mathematical modeling. 2nd Ed., Elsevier Academic Press, 2004.
37. Rossinag T. D. & Fletch N.H. Nonlinear vibrations in plates and gongs, *J. Acoust. Soc. Am.*, 1983, **73**, 345-351. doi: 10.1121/1.388816
38. Parhi, P.K.; Bhattacharyya, S.K. & Sinha, P. K. Hygrothermal effects on the dynamic behavior of multiple delaminated composite plates and shells. *J. Sound Vibration*, 2001, **248**, 195-214. doi: 10.1006/jsvi.2000.3506
39. Panda, H.S.; Sahu S.K. & Parhi, P.K. Effects of moisture on the frequencies of vibration of woven fibre composite doubly curved panels with strip delaminations. *Thin-Walled Struct.* 2014, **78**, 79–86. doi: 10.1016/j.tws.2014.01.010

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CONTRIBUTORS

Mr Chetan Kumar Hirwani is pursuing his PhD in the Department of Mechanical Engineering, NIT, Rourkela. He has been working on the nonlinear behaviour of delaminated composite plates/shells since last one and half year via experimental and numerical study of different kind of advanced composite components including the simulation. His area of interest is finite element analysis of damaged modelling of laminated composite plate and shell structures.

In the current study, he has contributed towards the development of the linear and nonlinear FEM code in MATLAB environment for computing the numerical responses and extended his expertise to carry out the experimental work.

Dr Subrata Kumar Panda received his PhD from IIT Kharagpur, India, in 2009. He is working as an Assistant Professor in Department of Mechanical Engineering, NIT, Rourkela. His current research interest includes nonlinear solid mechanics, smart composite structures, nonlinear FEM, experimental vibrations, functionally graded materials, SMA, PZT and magnetostrictive material, bio-mechanical analysis of functional materials.

His contribution in the current study, is to develop the nonlinear mathematical formulation and knowledge sharing during experimentation as well as paper writing.

Dr Trupti Ranjan Mahapatra received his PhD from KIIT University, India in 2015. He is working as an Associate Professor in School of Mechanical Engineering, KIIT University. He worked in the field of laminated curved composite structures under hydrothermal environment. His of current research interest includes smart (SMA, PZT and magnetostrictive material) composite structures, nonlinear FEM, Functionally Graded Material and so on.
In the current study, he contributed towards numerical result generation, interpretation of the same and report writing.

Ms Sushree S. Sahoo completed MTech (Research) from Department of Mechanical Engineering NIT, Rourkela. Her skills and expertise includes laminated composites, finite element analysis pertaining to mechanical behaviour of composite materials, experimental vibration analysis.
In the current study, she has contributed in the development of the experimental set up, obtain experimental data and validate with the numerically computed values.

Mr Vijay Kumar Singh is a PhD scholar in Department of Mechanical Engineering, NIT, Rourkela. He has been working on the nonlinear behaviour of laminated composite plates/shells since last four years using experimental and numerical simulation. His recent area of interest is finite element modelling of smart composite plate and shell structures.
In the current study, he is involved in development of the numerical model and validation of the numerical result with corresponding experimental values.

Dr B.K. Patle has completed his PhD in robotics and structural optimisation from NIT, Rourkela, Odisha. Presently working as an Assistant Professor and HOD at RSR RCET Bhiali, Chhatishgarh, India. His current research interest with laminated structural analysis.
In the current study, his expertise towards proper work plan for the experimental analysis and also contributed during final paper writing.