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Lorenz Curves Determine Partial Orders for Comparing Network Structures

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ABSTRACT

Networks can be studied from different points of view. In this paper, it is shown that the traditional Lorenz curve and some of its generalisations can be used for characterising inequality in network properties. Each type of Lorenz curve determines a partial order in a set of networks and a Gini-type index can be associated to each of these curves. The following types of Lorenz curves, each related to a different type of inequality in network properties and resulting in a different partial order, are briefly discussed: (a) Classical Lorenz curves and evenness measures; (b) intrinsic diversity profiles (or k-dominance curves) and associated measures of diversity; (c) generalised Lorenz curves as introduced by Shorrocks, that are not scale invariant but take absolute numbers into account; (d) weighted Lorenz curves for comparisons with an internal or external standard; and (e) Lorenz-type curve introduced to perform source per source comparisons of items. This paper claims that the Lorenz curves and the Gini index are universal tools for studying inequality, including inequality in network properties.

Keywords: Inequality, networks, Lorenz curves, Gini index

1. INTRODUCTION

With the advent of the internet and the world wide web, and renewed interest in social networks, business networks, and citation networks, the scientific activities related to network theory have, over the latest decade, grown at an unprecedented scale[1-2]. Not only the number of network studies has grown, but also the size itself of the networks studied. As the studied networks have become so big visualising them adequately becomes almost impossible. In such cases methods of descriptive statistics may still provide useful information about structural characteristics of the networks. Moreover, different approaches, studying various aspects of network structure, are necessary. Whatever the approach taken, precise definitions are called for. Degree centrality (in-degree, out-degree in a directed network), closeness centrality, and betweenness centrality are all characteristics of nodes[2-3]. When the network is subdivided into groups, the nodes have global and local Q-measure values[4-5].

In this paper, the general term 'array of node properties' has been used to indicate the ordered set of values of one of these characteristic values. More generally, the terminology of sources producing items (nodes having certain network characteristic values) have been used. For the sake of simplicity, the paper will mainly focus on undirected networks and exclude networks without any link. It is recalled that the *h*-index has been defined for node properties too[6]. This aspect, however, will not be studied here. Only 'complete' networks-not samples-studied at a precise moment in time will be discussed.

Dynamical aspects and estimation problems fall outside the scope of this contribution. Modelling using continuous curves is not discussed either. Many of the points made here are already made in the sociological, economical, and ecological literature. Hence, only most appropriate references have been given. The terms 'network' and 'graph' are used interchangeably.

2. EVENNESS

The number of nodes in a network and the inequality or evenness in their properties are two basic notions related to network structures. The number of nodes is a simple notion, which is, at least theoretically, unambiguous. In practice, however, there might be serious difficulties as to when to consider two objects as different (what is a webpage?) or whether or not to include particular actors (is this author a computer scientist, and hence she belongs to the corresponding network of computer scientists or is she active in information retrieval?). In any case, evenness is an even more subtle notion. It is defined as the relative contribution of nodes, actually present or assumed to be present. The author agrees with Taillie[7] and Gosselin[8] that the classical Lorenz curve[9] is the perfect representation of evenness ranking, being independent of the number of nodes concerned[10].

Besides giving rise to a partial order satisfying the permutation invariance and the scale invariance property, this ranking also satisfies the Dalton's transfer principle and the replication property. A short explanation of these notions has been included.Any array of node properties gives rise to precisely one classical Lorenz curve (while the opposite is not true at all). It is assumed that the reader is familiar with classical Lorenz curves (Fig. 1) but just recall that there are two versions: a concave one and a convex one. For the convex form, the node properties are ranked from low to high and ranking from high to low yields the concave variant (Fig. 2). These two approaches are mathematically equivalent. The convex form uses the array $X = (x_1, \ldots, x_N)$ ranked from smallest to largest. The corresponding Lorenz curve connects the origin (0,0) with the points

$$\left(\frac{j}{N}, \frac{\sum_{i=1}^{j} x_i}{\sum_{i=1}^{N} x_i}\right) \text{for } j = 1,.....N.$$

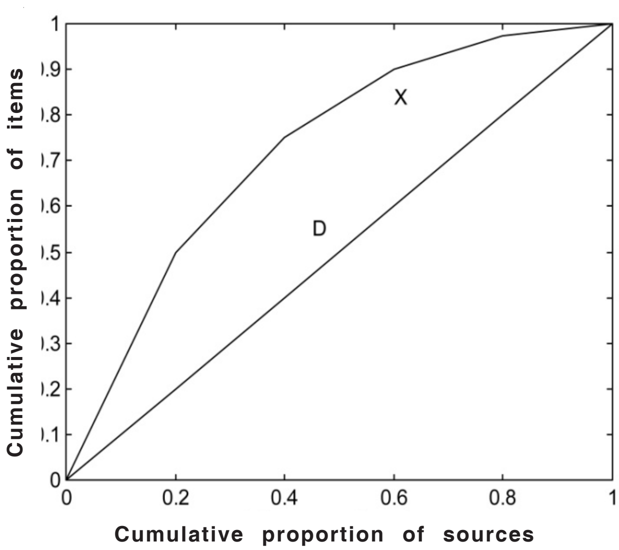

Figure 1. Convex Lorenz curve for evenness. *X*—Lorenz curve of the array (1, 3, 6, 10, 20), while D—diagonal representing perfect evenness.

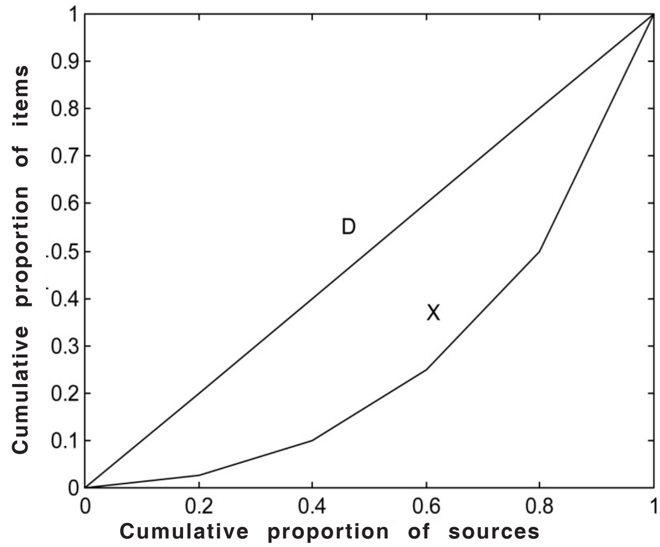

Figure 2. Concave form of the same situation as in Fig. 1.

Considering the convex form, one obtains a partial order where one Lorenz curve corresponds to a higher degree of evenness as another one if it is situated above the other. Highest evenness occurs for the diagonal line, corresponding with perfect evenness. As Lorenz curves may intersect, this construction yields a partial (not a complete or total) order, referred to as the Lorenz dominance order.

Permutation invariance refers to the fact that evenness is not a property of individual nodes but of a network as a whole. Scale invariance means that node properties differing by a proportionality factor are considered equivalent. As Lorenz curves are drawn using proportions they are automatically scale invariant. Dalton's transfer principle[11] states that when a node property, say the degree centrality of a node with a small number of connections decreases in favour of a node that has already more connections, evenness must decrease. It has been shown that the Lorenz order meets this transfer principle. Finally, the replication property, also due to Dalton, says that the evenness of any replication of a network must be equal to the evenness of the original network. In other words, the evenness of the node array (2,3,5,8) is equal to that of (2,3,5,8,2,3,5,8)= (2,2,3,3,5,5,8,8), which is equal to that of (2,2,2,2,2,3,3,3,3,3,5,5,5,5,5,8,8,8,8,8). As replication has no influence on the Lorenz curve also this requirement is clearly satisfied. It is noted that this property is probably of less practical importance in a network context. Rings and fully connected networks are perfectly even for degree centrality.

An evenness function *E* is a function that associates a non-negative number to each network array. It must, moreover, respect the Lorenz partial order. This means that if array X has L_X as its Lorenz curve and array Y has L_Y as its Lorenz curve, and if L_X is strictly above L_Y (in the convex form) then $E(X)>E(Y)$. The examples[10] of functions E satisfying this requirement are[10]:

Gini Evenness Index

The Gini evenness index G, is equal to twice the area under the convex Lorenz curve, or 1 minus twice the area between the Lorenz curve and the diagonal. If $X = (x_j)_{j=1,\ldots,N}$ denotes a network property array, the Gini evenness index is calculated as:

$$G'(X)=\frac{2}{\mu N^2}\left(\sum_{j=1}^{N}(N+1-j)\,x_j\right)-\frac{1}{N} \qquad (1)$$

where, the x_js are ranked from low-to-high and μ denotes the mean of the set $\{x_j\}$.

The modified Gini-Simpson index, Ë is defined as:

$$\Lambda(X)=\frac{1}{N\sum_{j=1}^{N}\left(\frac{x_j}{S}\right)^2} \qquad (2)$$

where, S denotes the sum of the X-array:

$$S=\sum_{j=1}^{N}x_j$$

The reciprocal of the coefficient of variation: *1/ V*. This evenness measure is defined as $\frac{1}{V}=\frac{\mu}{\sigma}$, where, σ denotes the standard deviation of the set $\{x_j\}$ (the components of the array X).

The adapted Shannon-Wiener index, *H',* is:

$$H'(X)=\frac{1}{\ln(N)+\sum_{j=1}^{N}\left(\frac{x_j}{S}\ln\left(\frac{x_j}{S}\right)\right)} \qquad (3)$$

Note that this index is not defined (or equal to ∞) for the perfect evenness situation. It can be concluded that striving for a total order representing evenness is a futile endeavour. A partial order is the best one can achieve.

3. INTRINSIC DIVERSITY PROFILES: COMBINING EVENNESS WITH NUMBER OF NODES PRESENT

In many situations one can include the number of nodes into a measure describing the whole network. For instance, one can make a distinction between different fully-connected networks. It has been shown[12] that if no zeros occur in the property array, then the intrinsic

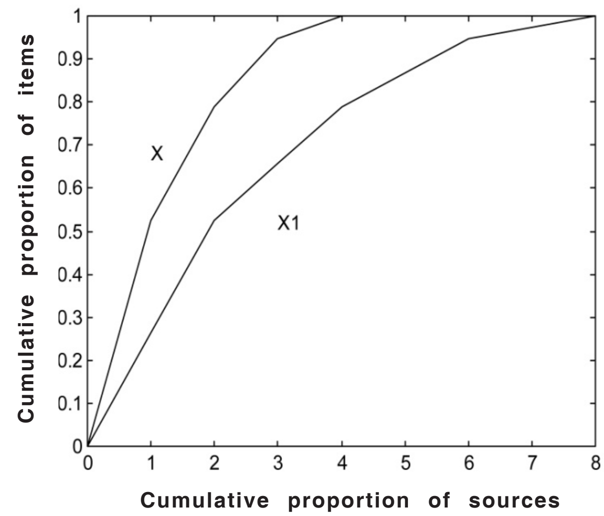

Figure 3. Intrinsic diversity profiles for two arrays with the same evenness: X = (1,3,5,10) = (10,5,3,1) and X1 = (10,10,5,5,3,3,1,1); X -<< X1.

diversity profile as introduced by Patil and Taillie[13] in ecological investigations is the best possible choice. Under the term 'best possible', one means that the resulting partial order is the strongest one among a number of reasonable candidates[12]. Also note that the intrinsic diversity profiles are equivalent to *k*-dominance curves as introduced by Lambshead[14] These curves can be drawn as follows: assume that node properties are ranked from high-to-low. Then the cumulative number of nodes is shown (not normalised) on the abscissa. As ordinates the cumulative proportion of the network property under investigation is used. Figure 3 illustrates the concept. These profiles introduce a partial order denoted as: -<<. If X -<< X1, then the intrinsic diversity profile of X never lies under the profile for X1. In this partial order, perfect evenness for N nodes is less 'diverse' (borrowing a term from ecology) than perfect evenness for N + 1 nodes (while their evenness is the same). A measure respecting the partial order for intrinsic diversity profiles (-<<) is obtained by taking twice the area between the intrinsic diversity profile, the line x = 0 (the ordinate axis) and the line y = 1. This measure is called the adapted Gini coefficient (*AG*). It is calculated as:

$$AG(X)=\frac{2}{S}\left(\sum_{j=1}^{N}j\,x_j\right)-1 \qquad (4)$$

where, $X = (x_1, x_2, \ldots, x_N)$ and the x_j's are ordered from high to low; S is the sum of the x_j's as before. Applying an atan-transformation yields a normalised function (a function taking values between 0 and 1).

Recall that one requires that no array contains a zero value otherwise problems arise with the construction of intrinsic diversity profiles. Yet, zeros may occur in practice. Indeed, when studying a specified set of nodes it may happen that some are not connected at all. Consider now, for instance, $X = (4, 2, 1, 0)$ and $Y = (4, 2, 1)$. The evenness of X is strictly smaller than that of Y; hence one would expect that the intrinsic diversity of Y is larger than that of X. This does not happen, showing that intrinsic diversity profiles cannot cope with null classes. Another variation on the Lorenz curve theme, namely shifted Lorenz curves, does not have this drawback. Here, the Lorenz evenness curve is essentially shifted $1/(2N)$ to the right. Concretely, the shifted Lorenz curve connects the origin with the points

$$\left(\frac{j+0.5}{N}, \frac{\sum_{i=1}^{j} x_i}{\sum_{i=1}^{j} x_i}\right)$$

where $j = 0, \ldots, N\text{-}1$, $x_0 = 0$; the curve ends in the point (1,1). Data are ranked from smallest to largest. For more details about this construction, one can refer to paper by Rousseau[15].

4. SHORROCKS' GENERALISED LORENZ CURVES

In many studies, not only the number of nodes and the relative properties are important, but also the values of these properties themselves. The previous approaches did not take this aspect into account. Economists studying household incomes face the same problem. Besides, the number of households and the inequality between their incomes, also the absolute income plays a role. To cope with this problem Shorrocks[16] proposed the use of generalised Lorenz curves. Rousseau[12] referred to these curves as intrinsic stability profiles. A generalised Lorenz curve is simply a convex Lorenz curve where each ordinate value is multiplied by the average value under consideration, μ (average income in Shorrocks' case). This yields a new partial order where situations with a higher average number of property values are considered to be more unequal than those with a lower average value. This approach clearly has the disadvantage that it is not anymore scale-invariant, but in cases where the counting measure is used, as for degree centrality, this is of no importance. Also, here twice the area under the convex generalised Lorenz curve is an acceptable diversity measure of Gini-type. This Gini measure, however, is not anymore bounded by one, but again, one may apply an atan-transformation if this is considered desirable (Fig. 4).

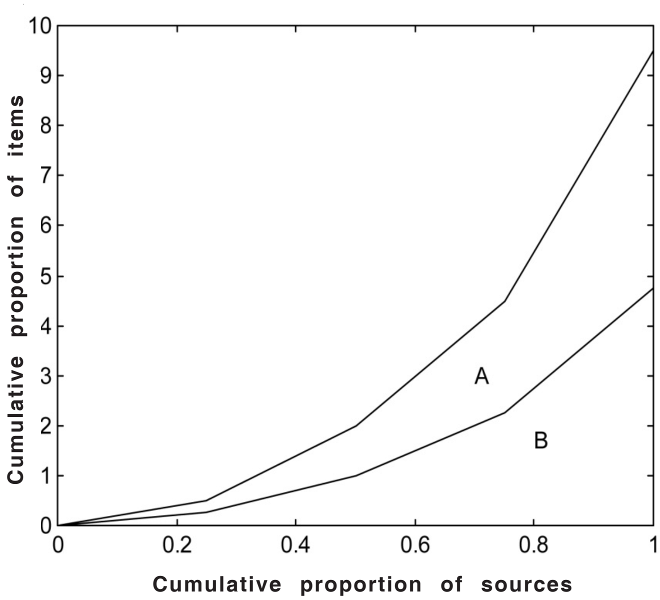

Figure 4. Generalised Lorenz curves: *B* corresponds to the array (1, 3, 5, 10) with an average of 19/4, while *A* corresponds to the array (2, 6, 10, 20) with an average of 38/4=19/2.

5. COMPARISON WITH AN AVERAGE OR HYPOTHESISED SITUATION: WEIGHTED LORENZ CURVES

Sometimes one is not interested in the actual inequality among node properties, but it is more interesting to study how different the distribution of values is wrt to an average or a hypothesised situation. An average can be considered as an internal standard, a hypothesised situation as an external standard. Often this external standard is a factor hypothesised to be the main factor in explaining observed differences in node properties.

Consider the following hypothetical table of cross-classified data (Table 1): a row classification consisting of M rows and a column classification consisting of N columns, with $M, N \geq 2$, e.g. paper by Rousseau[17]. It is assumed that columns are different types of actors, while rows refer to different clusters or regions. Cell values give the sum of all node centralities.

Table 1. Hypothetical cross tabulation (3x4) of actors and regions

Species	*A1*	*A2*	*A3*	*A4*	Average: *R*
Clusters or regions					
R1	20	140	200	140	125
R2	25	10	20	95	37.5
R3	15	90	80	65	62.5
Average: A	20	80	100	100	

Given the data of Table 1, one might assume that for actor type 3 available funding (just as an example) is the main factor explaining the observed difference in links between regions. In that case, one may use the 3-array of available funding (not shown) and compare it to the $A3$-column. Here, funding acts as an external standard. On the other hand, one might also be interested in studying how the evenness of the actor array of a particular region compares with average actor linking. This average array plays then the role of an internal standard. Note that we are not considering the complete analysis of a given contingency table. This is another type of study. In both cases (external or internal standard), one uses weighted Lorenz curves and measures respecting the partial order determined by these weighted Lorenz curves. Weighted Lorenz curves are constructed as follows. Let $S = (s_1, s_2, \ldots, s_N)$ denote the standard array and let $X = (x_1, x_2, \ldots, x_N)$ denote the studied array. This array is the one we want to compare with the standard. Note that indices must correspond and that the studied array and the standard, one have in a natural way the same number of cells (or sources). If, e.g., X denotes the number of links of actors in different regions and S denotes funding, then x_j and s_j must refer to the same region. It is assumed, moreover, that none of the components of the standard array S is zero. In order to construct the weighted Lorenz curve for comparison with a standard the components of both arrays are first ordered in such a way that

$$\frac{x_1}{s_1} \geq \frac{x_2}{s_2} \geq \cdots \geq \frac{x_N}{s_N} \tag{5}$$

This ranking leads to a concave curve. Using the other ranking leads to a convex representation. Next we normalise the arrays S and X, leading to arrays W and AX, where components are determined as:

$$a_j = \frac{x_j}{\sum_{k=1}^{N} x_k} \text{ and } w_j = \frac{s_j}{\sum_{k=1}^{N} s_k} \tag{6}$$

Note that normalising does not change the order. Finally, the weighted Lorenz curve is defined as the broken line connecting the origin (0, 0) to the points with components

$$\left(\sum_{k=1}^{j} w_k, \sum_{k=1}^{j} a_k \right)_{j=1,\ldots,N} \tag{7}$$

For a fixed standard, these weighted Lorenz curves again introduce a partial order in the set of N-arrays. Fig. 5 provides an example of a weighted Lorenz curve.

Functions that respect this partial order are sometimes referred to as measures of asymmetric relative diversity[18]. The term 'relative' refers to the fact that one compares with a standard. The term 'asymmetric' stresses the fact that the roles of the standard and the array under study cannot be interchanged. Examples of such measures are:

(a) Weighted Gini diversity index[19]:

$$G_W'(X) = 1 - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| w_i a_j - w_j a_i \right| \tag{8}$$

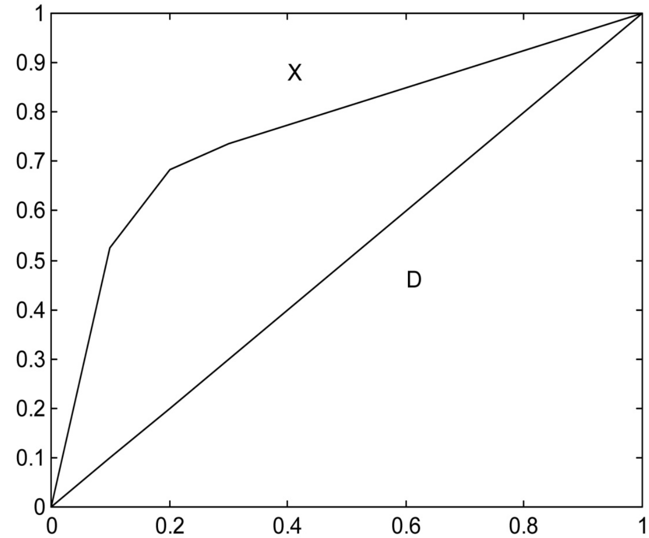

Figure 5. Weighted Lorenz curve (concave form) for the array X = (1, 3, 5, 10) compared with S = (1, 1, 7, 1); after re-ranking and normalising the curve based on A_X = (10/19, 3/19, 1/19, 5/19) and W = (1/10, 1/10, 1/10, 7/10) are drawn.

The interpretation of this index is the same as that for the (unweighted) Gini index, namely, twice the area above the weighted concave Lorenz curve. When the weighted Lorenz curve coincides with the diagonal, the weighted Gini diversity index is 1.

(b) Asymmetric (or weighted) inverse squared coefficient of variation:

$$V_w^2(X) = \frac{1}{\sum_{j=1}^{N} \frac{\left(a_j - w_j\right)^2}{w_j}} - 1 \tag{9}$$

This measure takes values between 0 and ∞.

6. SOURCE PER SOURCE COMPARISON OF NODE PROPERTIES: DIFFERENCES OF RELATIVE VALUES

In this approach, introduced by Egghe and Rousseau[20], one directly compares relative arrays. Relative contributions of the same sources (nodes), but at different times are compared by taking differences. These differences may be positive or negative, and one compares with the zero-array.

A type of Lorenz curve adapted to this situation has been constructed as follows. Let $X = (x_j)_{j=1,\ldots,N}$ and $Y = (y_j)_{j=1,\ldots,N}$ be two N-arrays and let $A = (a_j)_{j=1,\ldots,N}$ and $B = (b_j)_{j=1,\ldots,N}$ denote their relative arrays (sum of all components equal to one). Then the components of the difference array $D = (d_j)_{j=1,.., N}$ with $d_j = a_j - b_j$ are ranked from largest-to-smallest. Finally, again using partial sums, one puts:

$$t_j = \sum_{k=1}^{j} d_k = \sum_{k=1}^{j} \left(a_k - b_k\right) \qquad (10)$$

The corresponding Lorenz curve is obtained by joining the origin (0, 0) with the points with coordinates

$$\left(\frac{j}{N}, t_j\right)_{j=1,\cdots,N} \qquad (11)$$

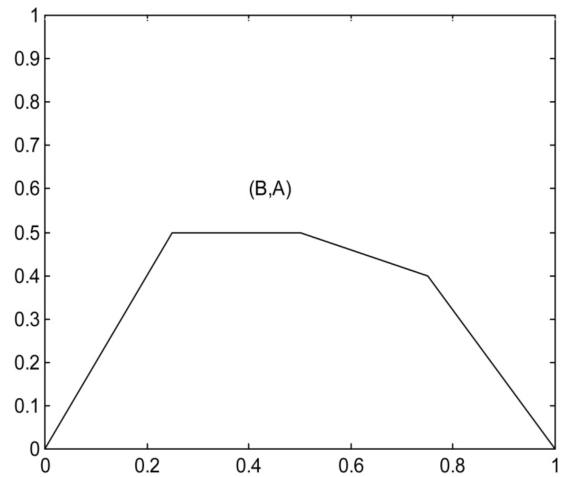

Figure 6. Lorenz bridge for the arrays B = (0.6, 0.2, 0.1, 0.1); A = (0.1, 0.2, 0.2, 0.5).

Note that this curve always ends in the point (1, 0). For this reason one may call it the Lorenz bridge. (See Fig. 6 as an example).

Similar to the other Lorenz curves also the Lorenz bridge leads to a partial order and functions respecting this partial order are the ones one is interested in. Such functions are sometimes referred to as measures of symmetric relative concentration. Here, the term 'relative' again refers to the fact that one compares with a standard (the zero line). Examples of such measures are:

Gini diversity (bridge) function:

$$G_r^{'}(X,Y) = 1 + \frac{1}{N}\sum_{j=1}^{N} j d_j \qquad (12)$$

where, the d_j are ranked in decreasing order. This is the area of the unit square (namely 1) minus the area under the Lorenz bridge. The Gini bridge function takes the value one in the case of perfect correspondence.

Another acceptable measure of symmetric relative diversity is the adapted Simpson index of relative difference:

$$V_r^2 = \frac{1}{N\sum_{j=1}^{N} d_j^2} \qquad (13)$$

This measure is related to the squared coefficient of variation, hence the notation V_r^2. It can be concluded that the exact relation between different forms of Lorenz curves and the appropriate measures follows from a general mathematical theory put forward by Egghe[21].

7. OTHER DEVELOPMENTS AND RECENT USE OF THESE NOTIONS

7.1 TIP-and TOP-Curves

In socio-economic studies, scientists have often focused their attention on people with the lowest income. They introduced the notion of poverty line, a threshold line or value such that if someone's income falls below this threshold income, this person is considered to live in poverty. Inequality among the poor with respect to the whole situation, e.g., the whole country, is then studied by an adaptation of Shorrocks' generalised Lorenz curves, the so-called TIP-curves[22]. In network studies, as in research evaluation studies, however, one is usually more interested in the most productive sources rather than in the low producers. For this reason, the opposite of TIP-curves, called TOP-curves has been introduced, as these are designed to study the most productive sources[23,24]. Important notions when studying the most productive sources are incidence, intensity and inequality among the top. For more information one can refer to papers by Rousseau[23], *et al.* and Rafols & Meyer[25].

7.2 There is More to Inequality Measurement Than Variety and Evenness

It has recently been argued[25] that besides the total number of sources (variety) and their evenness (balance) there is another dimension to inequality measurement[25]. This third factor is the disparity. This is a measure for the degree to which sources themselves are different. The Rao-Stirling measure (or quadratic entropy) is proposed when all three aspects are to be taken into account[26,27]. Rafols & Meyer[25] used this approach in their study on interdisciplinarity. The Rao-Stirlling measure is further studied by Leydesdorff & Rafols[28] in order to find out if it can really play a role in benchmarking interdisciplinarity on the journal level. They found that this measure was rather sensitive to the distance measure involved in its calculation.

7.3 Applications of Lorenz-type Curves and Gini Index in Informetrics

The Rafols-Meyer article mentioned above brings us to a short review of recent applications of Lorenz curves or inequality measures in informetrics, but no attempt is made at being complete. These studies are not necessarily applicable to networks. Egghe[29] showed that the *g*-index[30] is an indicator that respects the Lorenz dominance order in the case of a fixed number of sources. Liu & Rousseau[31] use the Gini index to investigate if different *h*-type indices are able to separate classes in a library classification system, and this based on book loans, while Chiang[32], *et al*. use a page view Gini index to determine the Web 2.0-ness of Web users. It turns out that the standard Lorenz curve is a well-known tool in the field as shown e.g. in Larivière[33], *et al*. Yet, the more special Lorenz-type curves and associated measures are rarely used. It is noted though that Egghe & Rousseau[34] characterise own-group preference by a weighted Lorenz curve, while Albarrán[35], *et al*. use the idea of incidence, intensity, and inequality (as in TOP-curves) to study high and low citation impacts. Burrell[36] emphasised that concentration studies should incorporate a time dimension and illustrated how this can be done.

8. CONCLUSIONS

It is concluded that the Lorenz curve and Gini index are universal tools for studying many aspects of network theory, or more general, any unequal situation. It is important to be precise in stating the aim of diversity or evenness measurements, and carefully choose the corresponding Lorenz curve and Gini index[17].

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