1. INTRODUCTION

Long-range rocket projectile is a kind of widely used weapon equipment in many countries as its range is distant and its fire power is enormous. Because the striking precision of classical rocket projectile can not fulfill the requirement of accurate striking, guidance rocket was designed to solve this problem. Guidance rocket projectile need implement flight control and correcting ballistic in the flight course to hit the target precisely. In this case, the information of position and flight velocity must be measured in the flight period. In the current navigation systems, global positioning system (GPS) and inertial measuring unit (IMU) are used widely1-3, besides, celestial navigation system (CNS) and geomagnetism navigation system (GNS) are adopted too4,5.

As the projectile body is manufactured by steel, it is easy to be magnetised in magnetism environment, and complex magnetic field will procedure in the flight course when the projectile slicing the magnetism field and when the electromechanical system works, so magnetometer is not a good choose for projectile navigation. Because projectile’s roll angular velocity is high in flight course, the celestial sensor can not capture the star constellation accurately, so the CNS can not be used on projectile. GPS and IMU are not yielding to those constrain, they can be used in the projectile navigation. The measure error of GPS does not accumulate with time, but the current integration model used on rocket projectile and other guidance projectile is loose integration, in this case, GPS signal may lose lock as its angular velocity is fast, and there are some passive factors such as orbit error of satellite, signal propagation error, and so on7,8. Ultra-tight integration model was proposed to solve this problem7-9. In this paper, authors introduced this method into projectile navigation.

2. ULTRA-TIGHT INTEGRATION

MEMS IMU is used widely in guidance projectiles, because its cost is low and cubage is small, but its measuring bias accumulate quickly. GPS is often used to restrain the error accumulation. Its integration model can be expressed as:

![Diagram of GPS/IMU Integration](image_url)

Figure 1: Loose integration of GPS and IMU.

As shown in Fig. 1, IMU and GPS measuring the position information separately, and fusing the position information at last. In this case, the GPS positioning bias can’t be reduced further, so the ultimate fusion result can’t achieve the optimal result. To solve this problem, we constrain the GPS receiver’s output noise via IMU real-time output before GPS export its
position information\textsuperscript{3,4}, and then integrating the IMU position information with GPS position information. The integration principle is as shown in Fig. 2.

**Figure 2. Ultra-tight integration of GPS and IMU.**

Figure 2 shows that the signal capturing contains the carrier phase tracking and phase locking, and carrier phase tracking is the basis of all the subsequent signal processing\textsuperscript{2,6}, so the carrier phase tracking course is important enough. Traditionally, the carrier phase is get from Phase Locked Loop (PLL) in the GPS receiver, and the PLL principle can be expressed like Fig. 3.

**Figure 3. PLL signal process procedure.**

To improve the measuring accuracy of carrier phase, the common method is to improve the PLL’s tracking precision of satellite signal\textsuperscript{7}. Tracking error of PLL is related to tracking loop’s tape width\textsuperscript{8}. With the assistant of IMU, we can solve the contradiction between loop noise and dynamic performance. Via measuring the projectile velocity through IMU, we can estimate the Doppler shift between projectile and satellite. Using IMU estimate the Doppler shift between projectile and satellite, and then integrating the IMU position and introduce it into the oscillator control part\textsuperscript{9,10}. Using IMU estimate the Doppler shift between projectile and satellite, and then integrating the IMU position and introduce it into the oscillator control part\textsuperscript{9,10}. Using IMU estimate the Doppler shift between projectile and satellite, and then integrating the IMU position and introduce it into the oscillator control part\textsuperscript{9,10}.

**Figure 4. IMU-based phase locking loop principle.**

3. **NOVEL FEDERATED FILTER**

3.1 **Federated Kalman Filter**

Federated Kalman filter theory is a special form of distributed Kalman filtering fusion method proposed by American scholars Carlson in 1998. It’s composed of several sub-filters and a main filter, is a block-estimation and two-step cascade distributed filter fusion method\textsuperscript{9}, by using the information distribution principle, The optimal fusion output is obtained. It’s a distributed filtering fusion method with high-precision, high fault tolerance and low compute burden properties\textsuperscript{11,12}.

From a structural point, it’s different from the parallel filtering algorithm, in which the sub-filter is completely independent on each other, each sub-filter in the federated filter sharing state information from master filter and get measuring information form corresponding sensors. Its structure can be indicated\textsuperscript{11,12} as shown in Fig. 5.

For general discrete system state model, we can express the state transition as:

\[
X(k) = \Phi(k / k - 1)X(k - 1) + \Gamma(k)W(k)
\]

where \(X(k)\) is the state vector of time of \(k\), \(\Phi(k / k - 1)\) is state transition matrix from time \(k - 1\) to time \(k\), \(\Gamma(k)\) is system noise matrix, \(W(k)\) is Gaussian noise, and its variance is \(\mathbf{Q}(k)\).

The process noise variance inverse matrix \(\mathbf{Q}^{-1}\) to represent state information of the equation is used. In addition, the state estimate can be present by variance inverse matrix \(P^{-1}\), \(R^{-1}\) is variance inverse matrix of measure noise of measure. If the state estimate vector, the system estimation variance matrix, the state vector variance matrix are denoted with \(\hat{X}, \mathbf{Q}, P\) for local filter \(i\), and the main filter’s state estimate vector, the system estimation variance matrix, the state vector variance matrix are denoted by \(\hat{X}, \mathbf{Q}, P\), the main filter’s information will be distributed to every local filter by the follow regulation\textsuperscript{9,11}:

\[
P_{i}^{-1} \hat{X}_{g} = P_{i}^{-1} \hat{X}_{i} + P_{i}^{-1} \hat{X}_{g} + \cdots + P_{i}^{-1} \hat{X}_{n}
\]

\[
P_{i}^{-1} = P_{i}^{-1} + P_{i}^{-1} + \cdots + P_{i}^{-1}
\]

\[
\hat{X}_{g} = P_{g}^{-1} \sum_{i=1}^{n} P_{i}^{-1} \hat{X}_{i}
\]

\[
\hat{X}_{i} = \hat{X}_{g}
\]

\[
P_{i}^{-1} = \beta_{i} P_{g}^{-1}
\]

\[
\mathbf{Q}_{g}^{-1} = P_{g}^{-1} + P_{g}^{-1} + \cdots + P_{g}^{-1}
\]

\[
\mathbf{Q}_{i}^{-1} = \beta_{i} \mathbf{Q}_{g}^{-1}
\]

where \(\beta_{i}\) is information distribute coefficient, and comply with the following regulation:

\[
\sum_{i=1}^{n} \beta_{i} + \beta_{m} = 1
\]

where \(0 \leq \beta_{i} \leq 1, \quad i = 1, 2, \cdots, n\).

If the system combined by \(n\) individual local sensor systems, and each systems carry out measuring independently. So there will be \(n\)
individual measuring date, for local sensor $i$, its state equation
and measuring equation can be noted as:

$$X_i(k) = \Phi_i(k/k-1)X_i(k-1) + \Gamma_i(k)W_i(k)$$

$$Z_i(k) = H_i(k)X_i(k) + V_i(k)$$

where $X_i(k)$ is local system’s state vector, and $Z_i(k)$ local
system’s measuring vector, where $V_i(k)$ is Gaussian noise
array of the corresponding local system.

Compared with other distributed fusion algorithms, federated kalman filter has the especial advantage in the
information feedback and information distribution course. In
the distribution process, the information $Q_i(k)$ and $P_i(k)$
of each local system will be distributed by this regulation:

$$
\begin{align*}
Q_i(k) &= \beta_i(Q_i(k-1)) \\
X_i(k-1) &= X_i(k-1)
\end{align*}
$$

As can be seen from Eqn. (12), after each integration
circulation, each sub-filter is re-obtained from the optimum
initial value of the integration output of the main filter, the sub-
filter also get the initial value which has been optimised to avoid
the error cumulative. After the distribution of information,
each sub-filter complete time update alone according their own
recursion equation, the process can be expressed as:

$$
\begin{align*}
\dot{X}_i(k-1) &= \Phi_i(k/k-1)\dot{X}_i(k-1) \\
P_i(k/k-1) &= \Phi_i(k/k-1)P_i(k-1)\Phi_i^T(k/k-1) \\
&+ \Gamma_i(k/k-1)Q_i(k-1)\Gamma_i^T(k/k-1)
\end{align*}
$$

and $i=1,2,...,n$.

As the main filter does not receive the measuring value, measurement update is complete only in sub-systems, so there
is no main filter measurement update. Measurement update
only conducted in each sub-filter, the update process is:

$$
\begin{align*}
P_i^{-1}(k) &= P_i^{-1}(k/k-1) + H_i^T(k)R_i^{-1}(k)H_i(k) \\
P_i^{-1}(k)\dot{X}_i(k) &= P_i^{-1}(k/k-1)\dot{X}_i(k/k-1) \\
&+ H_i^T(k)R_i^{-1}(k)Z_i(k)
\end{align*}
$$

where $i=1,2,...,n$.

From Eqns (10)-(16), we can get $\dot{X}_i(k)$. Then the estimation of the sub-filter will be integrated in the main filter, and we can get the optimal estimation. Fusion process can be expressed:

$$
\begin{align*}
P_g &= [P_1^{-1} + P_2^{-1} + ... + P_n^{-1}]^{-1} \\
\dot{X}_g &= P_g [P_1^{-1}\dot{X}_1 + P_2^{-1}\dot{X}_2 + ... + P_n^{-1}\dot{X}_n] = P_g \sum_{i=1}^{n} P_i \dot{X}_i
\end{align*}
$$

After fusion finished, the optimal output of main filter will
be assigned to each sub-filter again, for the aim of computing
easily, distribution coefficient taken:

$$
\begin{align*}
\beta_i = \frac{\text{trace}(P_i)}{\sum_{i=1}^{n} \text{trace}(P_i)} \\
\beta_n &= 1 - \sum_{i=1}^{n-1} \beta_i
\end{align*}
$$

Federal filter’s notable feature is that, the filtering
processing is done in the sub-filter filtering, and the integration
processing is completed in the main filter, and the main filter
will give a feedback to sub-filter. Therefore, the performance
of the sub-filter will largely affected the overall performance of
filtering fusion.

As shown in Fig. 5, the optimal fusion part get information
from local filter, so the output accuracy of the local filter is
rather important for the fusion result.

### 3.2 Strong Tracking Cubature Filter

For the rocket projectile, the filter state equation can be
expressed as:

$$X_{k+1} = X_k + X_{k+1}\Delta t$$

where $X$ is the state vector, and $X = (x, y, z, v_x, v_y, v_z, C)$, which means the position component, the velocity component of
three axis in inertial coordinate, and the ballistic coefficient.

As there exist the effect of attack angle and Magnus force
moment, the flight course is rather complex, and the real motion
state is not suit the state Eqn. (20) well, that is the motion model
is not exact enough. Meanwhile, the measuring noise of GPS
and IMU is not strict Gaussian noise, the traditional kalman filter can’t fit this condition well. Unscented kalman filter (UKF) was introduced to solve this problem\textsuperscript{10-12}, but the date stability performance of UKF has not been proved up to now, as its basic principle is UT transform, and used $2n+1$ sigma point, the amount of computing is huge. As the advantage of high dimension accuracy is high, and date stability is outstanding\textsuperscript{13,14}, cubature filter was adopt in this paper.

Cubature kalman filter (CKF) is the same as unscented kalman filter, approximating the random variable density function by weight sampling, the difference between UKF and CKF is that, CKF produce sigma point through cubature regular instead of UT regular. For common nonlinear state equation and observation equation, given it satisfy such format:

\[
X(k) = f_{t}(X(k-1)) + \Gamma(k)W(k) \\
Z_{t}(k) = h_{t}(X(k)) + V_{k}(k)
\]

Cubature filter based time update course is, given the posterior probability density function $p(x(k-1))$ of time $k-1$ is known, we can decompose error variance $P(k-1)$ via Cholesky decomposition:\textsuperscript{14}:

\[
P(k-1) = S(k-1)S^{T}(k-1)
\]

Through calculating Cubature point we can get:

\[
\chi_{t}(k / k-1) = S(k-1)\xi_{t} + \hat{X}(k-1)
\]

where $\xi_{t} = \sqrt{\frac{m}{2}[I]}$, $i = 1,2,...,m,m = 2n$. \textsuperscript{[1]} means the element in the cubature point \textsuperscript{14}.

Propagating Cubature point through state equation:\textsuperscript{14}:

\[
\chi_{t}(k / k-1) = f_{t}(\chi_{t}(k / k-1))
\]

Estimating the state prediction value of $k$ time:

\[
\hat{X}(k / k-1) = \frac{1}{m} \sum_{i=1}^{m} \chi_{t}(k / k-1)
\]

So the prediction value of error variance of $k$ time can be expressed as:

\[
P(k / k-1) = \frac{1}{m} \sum_{i=1}^{m} \chi_{t}(k / k-1)\chi_{t}^{T}(k / k-1) - \hat{X}(k / k-1)\hat{X}^{T}(k / k-1) + Q(k-1)
\]

After calculating the value of $P(k / k-1)$, decomposing $P(k / k-1)$ by Cholesky decomposition:\textsuperscript{14,15}:

\[
P(k / k-1) = S(k / k-1)S^{T}(k / k-1)
\]

On the basis, calculating Cubature point:

\[
\chi_{t}(k / k-1) = S(k / k-1)\xi_{t} + \hat{X}(k / k-1)
\]

Propagating Cubature point via measuring equation:

\[
z_{t}(k / k-1) = h_{t}(\chi_{t}(k / k-1))
\]

\[
\hat{Z}(k / k-1) = \frac{1}{m} \sum_{i=1}^{m} z_{t}(k / k-1)
\]

\[
P_{zz}(k / k-1) = \frac{1}{m} z_{t}(k / k-1)z_{t}^{T}(k / k-1) - \hat{Z}(k / k-1)\hat{Z}^{T}(k / k-1) + R(k)
\]

So we can get the state estimation:

\[
X(k) = X(k / k-1) + K(k)(Z(k) - \hat{Z}(k / k-1))
\]

\[
K(k) = P_{zz}(k / k-1)P_{zz}(k / k-1) + R(k)
\]

We can get the current state variance matrix at the same time:

\[
P(k) = P(k / k-1) - W(k)P_{zz}(k / k-1)W^{T}(k)
\]

CKF algorithm is based on spherical-radial cubature criteria, for all nonlinear state equation, there isn’t linearisation course of the model, but spread cubature point through the equation, so it is applicable to all forms of non-linear models. This article brought CKF as sub-filters in federated filter, which can reduce the adverse impact of the filter caused by nonlinear factor, meanwhile, reduce the computing amount.

Literature proved the validity of cubature filter in the complex environment\textsuperscript{14,15}, and provide a reference for application, but the filtering result is still not satisfy our requirement well when using traditional cubature filter, as said in above, the motion model is not exact enough. To solve this problem, literature\textsuperscript{15,16} proposed a strong tracking approach. On this basis, the strong tracking principle which was used in extend kalman filter was introduced into cubature filter in this paper to weaken the effect of modeling error. The operational process is, for Eqns (32)-(33), the calculate course is improved as:

\[
P_{zz}(k / k-1) = \lambda(k)\sum_{i=1}^{m} z_{t}(k / k-1)z_{t}^{T}(k / k-1) - \hat{Z}(k / k-1)\hat{Z}^{T}(k / k-1) + R(k)
\]

\[
P_{zz}(k / k-1) = \lambda(k)\sum_{i=1}^{m} \chi_{t}(k / k-1)\chi_{t}^{T}(k / k-1) - \hat{X}(k / k-1)\hat{X}^{T}(k / k-1)
\]

\[
P(k) = \lambda(k)P(k / k-1) - W(k)P_{zz}(k / k-1)W^{T}(k)
\]

\[
\lambda(k) = \begin{cases} 
\lambda(0) & \lambda > 0 \\
1 & \lambda \leq 0
\end{cases}
\]

\[
\lambda(0) = \frac{\text{tr}N(k)}{\text{tr}M(k)}
\]

\[
V_{r}(k) = \begin{cases} 
(r(1)r^{T}(1)) & r \text{ is the observe vector prediction residual error, and:}
\end{cases}
\]

\[
\lambda(k) = \begin{cases} 
\lambda(0) & \lambda > 0 \\
1 & \lambda \leq 0
\end{cases}
\]

\[
\lambda(0) = \frac{\text{tr}N(k)}{\text{tr}M(k)}
\]

\[
V_{r}(k) = \begin{cases} 
(r(1)r^{T}(1)) & r \text{ is the observe vector prediction residual error, and:}
\end{cases}
\]
\[ M(k) = \frac{1}{m} \sum_{i=0}^{m} (z_i(k/k-1) - \hat{Z}(k/k-1))(z_i(k/k-1) - \hat{Z}(k/k-1))^T \]

where \( 0 < \rho \leq 1 \) is the forgetting factor, \( \beta \) is the reduction factor.\(^{17,18}\)

### 3.3 Vector Sharing Principle

In traditional federated filter, as expressed in Fig. 5, the allocation factor \( \beta \) determine the fusion result, and the parameter \( \beta \) is a scalar, which can’t reflect the effect of observability degree. So we improved it with vector sharing principle.\(^{19,20}\)

After the improvement, the fusion structure of GPS/IMU based federated filter is as shown in Fig. 6.

In the figure,

\[ b_i = \sqrt{\frac{1}{2}} (A_i + r)^{-1} \]

and

\[ A_i = \text{diag}(a_{i1}, a_{i2}, \cdots, a_{in}) \]

\[ a_{ij} = \frac{1}{\lambda_{ij}} + \frac{1}{\lambda_{ij} + 1} + \cdots + \frac{1}{\lambda_{ij}} \]

\[ \lambda_{ij} = \text{diag}(\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{in}) \]

\[ r_i = \text{diag}(r_{i1}, r_{i2}, \cdots, r_{in}) \]

\[ \sigma_{ij} = \text{diag}(\sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{in}) \]

\[ \sigma_{ij} \] is the observability matrix singular value.\(^{19,20}\)

### 4. SIMULATION

To test that if this method proposed in this paper is useful and available, we carried out simulation based on the ballistic data. The simulation principle is, we used 6DOF ballistic equation to estimate the data of position, velocity, acceleration, attitude, attitude velocity. Then, we added initial bias onto the ballistic data as the initial measuring data, the initial bias is form MEMS gyro and accelerometer, the performance of gyro is, constant drift: \( 0.5^\circ/h \), white noise variance \( 0.05^\circ \), the first order coefficient \( 5 \times 10^{-6} \); the performance of accelerometer is: constant drift 100 \( \mu \)g, white noise variance 100 \( \mu \)g; at last, we input the ballistic data into Satellite analog signal generator to get GPS signal. To get the best result, we set the GPS output signal intermediate frequency is 7 MHz, sampling frequency is 30 MHz, carrier phase loop noise bandwidth is 10 Hz, damping factor is 0.7, the code loop bandwidth is 2.5 Hz, carrier loop gain is 0.3 and code loop gain is 0.5, under the assistant of IMU, carrier bandwidth is 0.3 Hz, and code loop is changed into the first order and its bandwidth is 0.5 Hz.

In the simulation, the flight time is set 600 s. In the course, we count the position and velocity error of two different
methods as shown in Figs. 7 and 8. The loose integration method is fusing the output information of GPS and INS with UKF federated filter, and the ultra-tight integration method is proposed in this paper. The positioning accuracy of the proposed method is higher than traditional GPS/INS integration method as viewed from Figs. 7 and 8.

From the figures we can see that, we can get stable navigation in less than 100 s, and the constringency time of ultra-tight integration is less than the loose integration method. As used ultra-tight integration structure and the vector share federated filter, the navigation accuracy is improved greatly, and is not more than 2 m, which can meet the requirement of guidance rocket.

5. CONCLUSION

A new navigation method for long-range rocket projectile based on information fusion algorithm was proposed. The simulation result shows that the positioning accuracy of the method proposed is higher than traditional method. As the computing account of cubature filter is less than UKF, this approach considered the computing time and output precision at the same time, provided a new method for engineering application.

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CONTRIBUTORS

Dr Handong Zhao, received his MSc from Beijing Institute of Technology in 1993, and received his PhD from Nanjing University of Science and Technology in 2008. Presently working as a Professor at North University of China. His research interest include: inertial navigation, integrated navigation and rocket projectile guidance and control. In the current study, he has contributed in the ultra-tight integration structure design and integration algorithm.

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