A NOTE ON PIEZOMETRIC EFFICIENCY IN AN ORTHODOX GUN

R. N. BHATTACHARYYA

Maharaja Manindra Chandra Collage, Calcutta

(Received 31 July 1970; revised 11 November 1971)

It has been shown that the muzzle velocity and the piezometric efficiency both may be increased for a suitable composite charge in comparison with those of the single charge. The condition stipulated is that the length of the gun should be sufficiently large in comparison to the initial free space behind the shot. The composite charge considered here consists of two components such that the pressure driving the shot remains absolutely constant throughout the period when the second component burns.

In a paper Ray\(^1\) discussed the possibility of getting constant driving pressure in an orthodox gun all through the period when the second component of the composite charge burns. He showed that in order to satisfy this condition the central ballistic parameter corresponding to the second component will be \(2/\gamma\) and the second component has the form factor equal to \(-1\). The author with the above conditions made a comparative study of the composite charge and the single charge regarding their piezometric efficiency (PE) and muzzle velocity (MV). The case of single charge has already been considered in Part I. Under the constant pressure phase during the second stage of burning the piezometric efficiency has been calculated both analytically and numerically. It has been shown that the piezometric efficiency and the muzzle velocity of the composite charge cannot always be made greater than that of the single charge. But if the length of the gun be sufficiently large in comparison to the initial free space behind the shot, then the PE and MV may be increased in comparison with that of the single charge. Also both the quantities increase if \(M_1\) be increased i.e. if the ratio \(C_1/C\) be decreased except perhaps for the progressive propellants.

**FIRST STAGE OF BURNING**

Assuming the values of \(\gamma\)'s of the two charges to be equal and neglecting the covolume effect, the basic equations in the first stage of burning as given by Kapur\(^2\) are

\[
F_1C_1Z_1 + F_2C_2Z_2 = p \left[ K_0 + A x - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} \right] + \frac{1}{2} \omega (\gamma - 1) v^2
\]

\[
\omega v \frac{dv}{dx} = \omega \frac{dv}{dt} = A p
\]

\[
D_i \frac{df_i}{dt} = - \beta_i p
\]

\[
Z_i = (1 - f_i) (1 + \theta_i f_i), \quad (i = 1, 2)
\]

Then equations are to be integrated with the initial conditions

\[x = v = p = Z_1 = Z_2 = 0, \quad f_1 = f_2 = 1\]

From (2) and (3) integrating with the initial conditions one gets

\[v = \frac{AD_1}{\beta_1 \omega} (1 - f_1) = \frac{AD_2}{\beta_2 \omega} (1 - f_2)\]

At all burnt of the first component

\[v_{B1} = \frac{AD_1}{\beta_1 \omega}\] by putting \(f_1 = 0\)

Also the value of \(f_2\) at burnt of the first component is

\[f_{2B1} = 1 - \frac{D_1/\beta_1}{D_2/\beta_2} = 1 - \frac{1}{\alpha_0}\]
where
\[ \alpha_0 = \frac{D_2/\beta_2}{D_1/\beta_1} \]  \hfill (7)

when
\[ \frac{D_2}{\beta_2} > \frac{D_1}{\beta_1} \]
i.e. the charge \( C_1 \) burns out first.

**SECOND STAGE OF BURNING**

Following Kapur\(^2\), the equations at this stage are
\[
F_1 C_1 + F_2 C_2 Z_2 = p \left[ K_0 + A \alpha - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} \right] + \frac{1}{2} \omega (\gamma - 1) v^2
\]  \hfill (8)
\[
\omega \frac{dv}{dx} = A p
\]  \hfill (9)
\[
D_2 \frac{df_2}{dt} = - \beta_2 p
\]  \hfill (10)
\[
Z_2 = (1 - f_2) (1 + \theta_2 f_2)
\]  \hfill (11)

These equations are to be integrated under constant pressure phase i.e. \( p = p_{B1} \). The initial conditions are
\[
x = x_{B1}, \quad v = v_{B1}, \quad Z_2 = Z_{2B1}, \quad f_2 = f_{2B1} = 1 - \frac{1}{\alpha_0}
\]  \hfill (12)

Also in the second stage of burning
\[
v = v_{B1} + \frac{A D_2}{\beta_2 \omega} \left( 1 - \frac{1}{\alpha_0} - f_2 \right)
\]  \hfill (13)

Now Ray\(^1\) obtained the condition that Second component burns under constant pressure which is the pressure at burn of the first component. The condition, he obtained, is
\[
\theta_2 = -1 \quad \text{and} \quad M_2 = \frac{2}{\gamma}
\]  \hfill (14)

**DETERMINATION OF MAXIMUM PRESSURE**

From equation (1), (4), (5) and (14) one gets
\[
F_1 C_1 \frac{\beta_1 \omega}{A D_1} v \left( 1 + \theta_1 - \frac{v \beta_1 \omega}{A D_1} \theta_1 \right) + F_2 C_2 \frac{\beta_2 \omega^2}{A^2 D_2^2} v^2 = A p (x + t) + \frac{1}{2} \omega (\gamma - 1) v^2
\]

where
\[
A I = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}
\]

By (2) we get the differential equation
\[
\frac{dx}{x + t} = \frac{F_1 C_1 \beta_1 (1 + \theta_1)}{A D_1} + v \left( \frac{1}{2} - \frac{\theta_1}{M_1} \right)
\]  \hfill (15)

where
\[
\theta_1 
eq -1
\]

when \( \theta_1 \neq \frac{M_1}{2} \), integrating the equation (15) with the condition that \( x = 0, v = 0 \) one gets
\[
\log \left( 1 + \frac{x}{t} \right) = \frac{2 M_1}{M_1 - 2 \theta_1} \log \left[ 1 + v \frac{A D_1}{F_1 C_1 \beta_1 (1 + \theta_1)} \frac{M_1 - 2 \theta_1}{2 M_1} \right]
\]  \hfill (15a)
Now \(x = x_{B1}\) when \(v = v_{B1}\)
we have from (6)
\[
-x_{B1} = l \left[ \left\{ \frac{M_1 + 2}{2 (1 + \theta_1)} \right\} \left( \frac{2 M_1}{M_1 - 2 \theta_1} \right) - 1 \right]
\]
Again when \(\theta_1 = \frac{M_1}{2}\), equation (15) on integration leads to
\[
x_{B1} = l \left[ e \left( \frac{M_1}{1 + \theta_1} \right) - 1 \right]
\]
Again for \(\theta_1 \neq \frac{M_1}{2}\), differentiating both sides of (15a) with respect to \(x\), we have
\[
\frac{dv}{dx} = \frac{F_1 C_1 \beta_1}{A D_1} \frac{1 + \theta_1}{l} \left( 1 + \frac{x}{l} \right) - \frac{M_1 + 2 \theta_1}{2 M_1}
\]
and from (2) one gets,
\[
p = \frac{2 F_1 C_1}{A l} \frac{(1 + \theta_1)^2}{M_1 - 2 \theta_1} \left[ \left\{ \frac{M_1 + 2}{2 (1 + \theta_1)} \right\} - \left( \frac{2 \theta_1}{M_1} \right) - \left( 1 + \frac{x}{l} \right) - \frac{M_1 + 2 \theta_1}{2 M_1} \right]
\]
Hence from (16)
\[
P_{B1} = \frac{2 F_1 C_1}{A l} \frac{(1 + \theta_1)^2}{M_1 - 2 \theta_1} \left[ \left\{ \frac{M_1 + 2}{2 (1 + \theta_1)} \right\} - \left( \frac{4 \theta_1}{M_1 - 2 \theta_1} \right) - \left( \frac{M_1 + 2}{2 (1 + \theta_1)} \right) - \frac{M_1 + 2 \theta_1}{M_1 - 2 \theta_1} \right]
\]
for \(\theta_1 = \frac{M_1}{2}\) similarly we have the expression for \(P_{B1}\) as
\[
P_{B1} = \frac{F_1 C_1 (1 + \theta_1)}{A l} e - \frac{M_1}{1 + \theta_1}
\]
where
\[
p = \frac{F_1 C_1 (1 + \theta_1)^2}{A l M_1} \left( 1 + \frac{x}{l} \right) - \log \left( 1 + \frac{x}{l} \right)
\]
Now to determine the maximum pressure for \(\theta_1 \neq \frac{M_1}{2}\), we have noticed that \(\frac{d^2 p}{dx^2} < 0\) when \(M_1 \neq 2 \theta_1\).
Hence the pressure will be maximum at \(x\) given by
\[
1 + \frac{x}{l} = \left( \frac{M_1 + 2 \theta_1}{4 \theta_1} \right) \left( \frac{2 M_1}{M_1 - 2 \theta_1} \right)
\]
But for \(\theta_1 = 0\) (22) gives no finite values of \(x\) and hence the maximum pressure will occur at burnt of the first component.
Hence
\[
P_{\text{max}} \mid \theta_1 = 0 = \frac{2 F_1 C_1}{A l} \frac{1}{M_1 + 2}
\]
From (18) and (22), the expression for the maximum pressure for \(\theta_1 \neq 0\) is given by
\[
P_{\text{max}} = \frac{2 F_1 C_1}{A l} \frac{(1 + \theta_1)^2}{M_1 - 2 \theta_1} \left[ \left\{ \frac{M_1 + 2 \theta_1}{4 \theta_1} \right\} - \left( \frac{4 \theta_1}{M_1 - 2 \theta_1} \right) - \left( \frac{M_1 + 2 \theta_1}{4 \theta_1} \right) - \left( \frac{M_1 + 2 \theta_1}{M_1 - 2 \theta_1} \right) \right]
\]
Similarly for \(\theta_1 = \frac{M_1}{2}\) the expression for maximum pressure is found to be
\[
P_{\text{max}} = \frac{F_1 C_1}{2 A l} \frac{(1 + \theta_1)^2}{\theta_1} \frac{1}{\epsilon}
\]
Now we consider the second stage of burning to deduce $v_{B2}$ and $x_{B2}$, the pressure remaining constant during this stage. From (13) by putting $f_1 = 0$ and the value of $v_{B1}$, we have

$$v_{B2} = \frac{A D_1}{\beta_1 \omega} \alpha_0$$

(26)

From (9) and (10) we have

$$\frac{d\nu}{df_2} = -\frac{A D_2}{\beta_2 \omega},$$

which by (10) and $p = p_{B1}$ gives

$$\frac{d^2 \nu}{df_2^2} = -\frac{A D_2^2}{\beta_2^2 \omega p_{B1}}$$

and on integrating we have

$$\frac{dx}{df_2} = -\frac{A D_2^2}{\beta_2^2 \omega p_{B1}} (1 - f_2)$$

Again integrating with the condition that

$$x = x_{B1}, f_2 = f_{2B1} = 1 - \frac{1}{\alpha_0},$$

we have

$$x = x_{B1} + \frac{A D_2^2}{2 \beta_2^2 p_{B1} \omega} \left[ (1 - f_2)^2 - \frac{1}{\alpha_0^2} \right]$$

Putting the value of $x_{B1}$ and $p_{B1}$ when $\theta_1 = \frac{M_1}{2}$

$$\frac{x_{B2}}{l} = \left\{ \frac{M_1 + 2}{2(1 + \theta_1)} \right\} \frac{2M_1}{M_1 - 2\theta_1} \left[ 1 + \frac{A^2 D_2^2}{4 \beta_2^2 \beta_1 C_1 \omega (1 + \theta_1)^2} \left( \frac{M_1 - 2\theta_1}{M_1 - 2\theta_1} \right) \right] - 1$$

(27)

and when

$$\theta_1 = \frac{M_1}{2},$$

$$\frac{x_{B2}}{l} = e^{\frac{M_1}{1 + \theta_1}} + \frac{M_1}{2(1 + \theta_1)} \left( \alpha_0^2 - 1 \right) e^{\frac{M_1}{1 + \theta_1} - 1}$$

(28)

**DETERMINATION OF MUZZLE VELOCITY**

After all burnt the gases expand adiabatically and the corresponding equations are

$$\omega \nu \frac{dv}{dx} = Ap$$

$$p = p_{B1} \left( \frac{x_{B2} + l}{x + l} \right)^\gamma$$

(29)

Integrating equations (29) we have

$$\nu^2 = v_{B2}^2 + \frac{2A}{\omega (1 - \gamma)} p_{B1} (x_{B2} + l)^\gamma \left[ (x + l)^{1-\gamma} - (x_{B2} + l)^{1-\gamma} \right]$$

(30)

Now for $\theta_1 = \frac{M_1}{2}$ if $V_1$ be the muzzle velocity and $\eta_1$ is the corresponding quantity in non-dimension.

form then by putting $x = d$ (length of the gun) in (30)
BHATTACHARYYA: Piezometric Efficiency in an Orthodox Gun

\[ V_1^2 = \frac{A^2 D^2}{\beta_1^2 \omega^2} \left[ \alpha_0^2 - \frac{2 A l}{\omega (1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D^2} \, P_{Bi} \, M_0' \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]  

and

\[ \eta_1^2 = M_1^2 \left[ \alpha_0^2 - \frac{2 A l}{\omega (1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D^2} \, P_{Bi} \, M_0' \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]

where

\[ M_0' = \left\{ \frac{M_1 + 2}{2 (1 + \theta_1)} \right\} \frac{2 M_1}{M_1 - 2 \theta_1} + \frac{A D_2^2}{2 \beta_2^2 \omega P_{Bi} l} \left( 1 - \frac{1}{\alpha_0^2} \right) \]

For \( \theta_1 = \frac{M_1}{2} \), we have,

\[ V_2^2 = \frac{A^2 D^2}{\beta_1^2 \omega^2} \alpha_0^2 - \frac{2 A l}{\omega (1-\gamma)} \, P_{Bi} \left\{ e^{\frac{M_1}{1 + \theta_1}} + \frac{M_1 (\alpha_0^2 - 1)}{2 (1 + \theta_1)} e^{\frac{M_1}{1 + \theta_1}} \right\} \]

\[ \cdot \left[ 1 - \left( \frac{M_1}{e^{1 + \theta_1} + \frac{M_1 (\alpha_0^2 - 1)}{2 (1 + \theta_1)} e^{\frac{M_1}{1 + \theta_1}}} \right)^{1-\gamma} \right] \]

and

\[ \eta_2^2 = M_1^2 \left[ \alpha_0^2 - \frac{2 A l}{\omega (1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D^2} \, P_{Bi} \, M_0' \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]

where

\[ M_0 = e^{\frac{M_1}{1 + \theta_1}} + \frac{M_1 (\alpha_0^2 - 1)}{2 (1 + \theta_1)} e^{\frac{M_1}{1 + \theta_1}} \]

The mean pressure is given by

\[ \bar{p} = \frac{\omega}{2 A d} \frac{V^2}{A} \]

Hence to determine Piezometric efficiency we are to consider three cases:

**Case 1.** \( \theta_1 = \frac{M_1}{2} \)

Piezometric efficiency

\[ \frac{l}{d} \theta_1 = e M_1 \left[ \alpha_0^2 - \frac{2 M_0 (1 + \theta_1)}{M_1 (1 - \gamma)} e^{\frac{M_1}{1 + \theta_1}} \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]

**Case 2.** \( \theta_1 = 0 \)

Piezometric efficiency

\[ \frac{l}{d} \frac{M_1 (M_1 - 2 \theta_1)}{4 (1 + \theta_1)^2} \left[ \alpha_0^2 - \frac{2 l \beta_1^2 \omega}{A D_1^{3(1 - \gamma)}} P_{Bi} M_0' \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]

\[ \cdot \left\{ \frac{M_1 + 2}{2 (1 + \theta_1)} \right\} - \left( \frac{\frac{4 \theta_1}{M_1 - 2 \theta_1}}{\frac{4 \theta_1}{M_1 - 2 \theta_1}} \right) - \left( \frac{\frac{M_1 + 2}{2 (1 + \theta_1)}}{\frac{M_1 + 2}{2 (1 + \theta_1)}} \right) \frac{M_1 + 2 \beta_1}{M_1 - 2 \theta_1} \]

**Case 3.** \( \theta_1 \neq \frac{M_1}{2} \)

Piezometric efficiency

\[ \frac{l}{d} \frac{M_1 (M_1 - 2 \theta_1)}{4 (1 + \theta_1)^2} \left[ \alpha_0^2 - \frac{2 l \beta_1^2 \omega}{A D_1^{3(1 - \gamma)}} P_{Bi} M_0' \left\{ 1 - \left( \frac{1 + d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \]

\[ \cdot \left( \frac{\frac{M_1 + 2 \theta_1}{4 \theta_1}}{\frac{M_1 + 2 \theta_1}{4 \theta_1}} \right) - \left( \frac{\frac{M_1 + 2 \theta_1}{4 \theta_1}}{\frac{M_1 + 2 \theta_1}{4 \theta_1}} \right) \frac{M_1 + 2 \beta_1}{M_1 - 2 \theta_1} \]
Now for composite charge there are three possibilities:

(i) $\frac{x_{B1}}{d} > 1$ i.e. shot leaves the gun before all burnt of the first component such that constant pressure phase has not yet been reached. We are not concerned with this stage as we are to consider the cases under constant pressure phase with the second component.

(ii) $\frac{x_{B1}}{d} < 1$ and $\frac{x_{B2}}{d} > 1$ i.e. the shot leaves the gun after all burnt of the first component.

In this stage no analytical expression for $P_E$ is possible and as such during this stage $P_E$ is to be calculated numerically.

(iii) $\frac{x_{B1}}{d} < 1$ and $\frac{x_{B2}}{d} < 1$ i.e. the shot leaves the gun after all burnt of the second component.

The expressions (35), (36) and (37) give the $P_E$ during this stage of burning. However in a recent paper Bhattacharyya discussed the calculation of $MV$ and $P_E$ in an orthodox gun with single charge.

### Numerical Calculations

The numerical calculation is based on the principle as discussed in author's previous paper.

(i) For $l/d = 0.5$, $\theta = 0$, $M = 1$

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>PE</td>
</tr>
<tr>
<td>1.093</td>
<td>0.8114</td>
</tr>
<tr>
<td>70</td>
<td>1.635</td>
</tr>
<tr>
<td>75</td>
<td>1.592</td>
</tr>
<tr>
<td>80</td>
<td>1.320</td>
</tr>
</tbody>
</table>

(ii) For $l/d = 0.3$, $\theta = 0$, $M = 1$

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>PE</td>
</tr>
<tr>
<td>1.370</td>
<td>0.7656</td>
</tr>
<tr>
<td>60</td>
<td>2.712</td>
</tr>
<tr>
<td>70</td>
<td>2.321</td>
</tr>
<tr>
<td>75</td>
<td>2.487</td>
</tr>
</tbody>
</table>

(iii) For $l/d = 0.2$, $\theta = 0$, $M = 1$

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>PE</td>
</tr>
<tr>
<td>1.562</td>
<td>0.6332</td>
</tr>
<tr>
<td>50</td>
<td>3.130</td>
</tr>
<tr>
<td>60</td>
<td>3.050</td>
</tr>
<tr>
<td>70</td>
<td>2.912</td>
</tr>
</tbody>
</table>
BHATTACHARYYA: Piezometric Efficiency in an Orthodox Gun

(iv) For \( l/d = 0.1, \ \theta = 0, \ M = 1 \)

<table>
<thead>
<tr>
<th>Single Charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV PE</td>
<td>MV PE</td>
</tr>
<tr>
<td>1.833 0.3207</td>
<td>30 5.010 0.8039</td>
</tr>
<tr>
<td></td>
<td>40 4.025 0.8775</td>
</tr>
<tr>
<td></td>
<td>50 4.533 0.8425</td>
</tr>
<tr>
<td></td>
<td>60 4.012 0.8135</td>
</tr>
</tbody>
</table>

(v) For \( l/d = 0.5, \ \theta = 1, \ M = 1 \)

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV PE</td>
<td>MV PE</td>
</tr>
<tr>
<td>1.330 0.7522</td>
<td>65 3.012 0.7625</td>
</tr>
<tr>
<td></td>
<td>80 2.013 0.5230</td>
</tr>
<tr>
<td></td>
<td>88 1.721 0.4293</td>
</tr>
</tbody>
</table>

(vi) For \( l/d = 0.2, \ \theta = 1, \ M = 1 \)

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV PE</td>
<td>MV PE</td>
</tr>
<tr>
<td>1.709 0.4920</td>
<td>70 3.023 0.5129</td>
</tr>
<tr>
<td></td>
<td>80 3.125 0.4559</td>
</tr>
<tr>
<td></td>
<td>85 2.662 0.4229</td>
</tr>
</tbody>
</table>

(vii) For \( l/d = 0.1, \ \theta = 1, \ M = 1 \)

<table>
<thead>
<tr>
<th>Single charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV PE</td>
<td>MV PE</td>
</tr>
<tr>
<td>1.944 0.4525</td>
<td>75 3.859 0.6016</td>
</tr>
<tr>
<td></td>
<td>80 3.529 0.5662</td>
</tr>
<tr>
<td></td>
<td>85 3.422 0.5449</td>
</tr>
</tbody>
</table>

(viii) For \( l/d = 0.1, \ \theta = -0.5, \ M = 1 \)

<table>
<thead>
<tr>
<th>Single Charge</th>
<th>Composite charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV PE</td>
<td>MV PE</td>
</tr>
<tr>
<td>1.787 0.8400</td>
<td>80 3.526 0.3902</td>
</tr>
<tr>
<td></td>
<td>85 3.223 0.4205</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The author has much pleasure in expressing his gratefulness to Prof. A. Ray for the continuous interest shown by him during the preparation of this work and for the benefit of discussion with him.

REFERENCES