Range-only Target Localisation using Geometrically Constrained Optimisation

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ABSTRACT

The problem of optimal range-only localisation of a single target is of considerable interest in two-dimensional
search radar networking. For coping with this problem, a range-only target localisation method using synchronous
measurements from radars is presented in the real ellipsoidal earth model. In the relevant radar localisation scenario,
geometric relationships between the target and three radars were formed. A set of localisation equations was derived
on range error in such a scenario. Using these equations, the localisation task has been formulated as a nonlinear
weighted least squares problem that can be performed using the Levenberg- Marquardt (LM) algorithm to provide
the optimal estimate of the target’s position. To avoid the double value solutions and to accelerate the convergence
speed for the LM algorithm, the initial value was approximately given according to observations from two radars. In
addition, the relative validity has been defined to evaluate the performance of the proposed method. The performance
of the proposed approach is evaluated using two simulation experiments and a real-test experiment, and it has been
found to possess higher localisation accuracy than the other conventional method.

Keywords: Range-only target localisation, target localisation, nonlinear optimisation, Cramer-Rao lower bound,
Levenberg-Marquardt algorithm, radar networking, two-dimensional search radar

1. INTRODUCTION

Two-dimensional (2-D) search radar networking, which is
generally utilised in the first line of air defence, has been widely
applied in modern airspace control1-3. This fact generally has
two reasons: on one hand, 2-D radars are cheaper than three-
dimensional (3-D) radars; on the other, single 2-D radar can’t
directly determine a target’s position since it is merely able to
obtain the range and azimuth without the height of the target.
Consequently, it is a valuable task-how to utilise the resources
and measurements from multiple 2-D radars to estimate the
position of a target. To simplify the description, all radars
below denote two-dimensional search radars.

According to the type and working mode of sensors, the
target localisation methods mainly form three categories: active
localisation using range4-7, passive localisation using azimuth8-
11, and combined localisation using both range and azimuth12-15.
Moreover, three factors are often taken into consideration in
radar networking4,12:

(a) a radar possesses higher precision in range than in azimuth
since its emission lobes approximating to spindles have
certain widths in the horizontal plane;
(b) range error has less influence on localisation results than
azimuth error in propagation; and
(c) a localisation dead area exists near the baseline between
two radars while only two radars are employed to locate

a target. For the above reasons, this research utilises three
radars and their range for target localisation.

In the traditional localisation methods, only one
assumption is given for simplifying the design procedures
as one such assumption is, all radars are placed in the same
horizontal plane and their north direction is consistent or parallel
to each other by ignoring the earth’s curvature. However, these
assumptions are often unsatisfied in big scenes due to the wide
coverage of radar networks15. The other important information
such as the types, performances, working modes, number
and geometric distribution of radars are omitted, which is the
important consideration in real localisation process. In addition,
the least squares estimator and its modified approaches are
often applied in various applications for target localisation
problems5,7,11,13. Based on the above facts, a range-only target
localisation method is presented in this paper. In the proposed
method, the localisation model is established using range from
three radars in the real ellipsoidal earth model, and the target
localisation problem is formulated as a nonlinear weighted
least squares problem through a set of localisation equations
driven on range error. Then Levenberg-Marquardt (LM)
algorithm is applied to solve the localisation equations. At last,
two simulation experiments and a real-test experiment were
used to show both the validity and feasibility of the proposed
method.
2. TARGET LOCALISATION MODEL

As shown in Fig. 1, n radars $R_i$ ($i = 1, 2, \cdots, n$) are placed at the corresponding geographical coordinates $(B_i, L_i, H_i)$ in the localisation model, where $B_i$, $L_i$ and $H_i$ denote the longitude, latitude, and height coordinate respectively; and their observations $z_i = (r_i, \theta_i)$ are received simultaneously, where $r_i$ and $\theta_i$ denote the range and azimuth of the target $T$. Theoretically, $n$ range spheres with each corresponding radius $r_i$ and its centre at the position $R_i$ intersect at the point, the target’s position. Hence, at least three radars are required for range-only target localisation in this case. Therefore, three radars and their range information are applied to locate a target in this research.

Taking the earth curvature into consideration, the geographical coordinate $(B_i, L_i, H_i)$ of each radar should be converted into the geocentric coordinate $(X_i, Y_i, Z_i)$:

$$
\begin{align*}
X_i &= (N_i + H_i) \cos L_i \cos B_i \\
Y_i &= (N_i + H_i) \cos L_i \sin B_i \\
Z_i &= \left[ N_i (1 - \rho^2) + H_i \right] \sin L_i
\end{align*}
$$

where $R_e$ and $r_e$ represent the long and short radii of the earth respectively, $\rho = \sqrt{1 - \frac{r_e^2}{R_e^2}}$ is the earth eccentricity ratio, and

$$
N_i = R_e \sqrt{1 - \rho^2 \sin^2 L_i}
$$

Under the assumption of radars with range error, a set of geometrical equations are derived according to the geometrical relationships between the target and three radars in the localisation model as follow:

$$
\rho_i^2 = (X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2, \quad i = 1, 2, 3
$$

where $X = (X, Y, Z)$ is the true geocentric coordinate of the target. In consideration of range error, Eqn (2) can be rewritten in the following formula:

$$
\rho_i^2 = (X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2 + 2r_i \nu_i - v_i^2, \quad i = 1, 2, 3
$$

where $v_i$ is range error of the $i$th radar. Based on Eqn (3), the localisation equations are derived as follow:

$$
d_i(X) = r_i - \sqrt{(X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2}, \quad i = 1, 2, 3
$$

Furthermore, one can define the objective function in consideration of different influence of each radar’s range error on localisation results as follow:

$$
F(X) = \sum_{i=1}^{3} \left( \frac{d_i(X)}{\sigma_i} \right)^2
$$

where $\sigma_i$ denotes the standard deviation of range error on the $i$th radar. As a result, the localisation task can be formulated as a nonlinear weighted least squares problem to provide the optimal estimate such as the minimum value for $F(X)$.

3. PROPOSED METHOD AND ITS VALIDITY ANALYSIS

To overcome a nonlinear least square problem, many methods have been developed such as the Newton, Gauss-Newton, and LM algorithm. The LM algorithm has the advantages on both the local convergence property of the Gauss-Newton algorithm and the global convergence property of the gradient descent algorithm, and it is one of the most effective algorithms to solve a nonlinear least square problem with small margins$^{13,16}$. As a result, the LM algorithm is applied to solve target localisation problems in the following.

3.1.1 The LM algorithm for Solving the Localisation Equations

The optimal estimate for the target’s position is the value $X$ while $F(X)$ obtains its minimum value. To minimise $F(X)$, the vector is defined as:

$$
d(X) = (d_1(X), d_2(X), d_3(X))^T
$$

Moreover, introducing the diagonal matrix $\Lambda$, whose main diagonal elements are $\sigma_1^2$, $\sigma_2^2$ and $\sigma_3^2$ respectively, the objective function (Eqn(5)) can be rewritten as:

$$
F(X) = d(X)^T \Lambda^{-1} d(X)
$$

Then define the Jacobi matrix of $d(X)$ at the point $X = (X', Y', Z')$:

$$
J(X) = \begin{bmatrix}
\frac{\partial d_1(X)}{\partial X} & \frac{\partial d_1(X)}{\partial Y} & \frac{\partial d_1(X)}{\partial Z} \\
\frac{\partial d_2(X)}{\partial X} & \frac{\partial d_2(X)}{\partial Y} & \frac{\partial d_2(X)}{\partial Z} \\
\frac{\partial d_3(X)}{\partial X} & \frac{\partial d_3(X)}{\partial Y} & \frac{\partial d_3(X)}{\partial Z}
\end{bmatrix}
$$

Given $f_i(X) = d_i(X)/\sigma_i$, $i = 1, 2, 3$, and $f(X) = (f_1(X), f_2(X), f_3(X))^T$, the gradient vector for $F(X)$ can be obtained as:

$$
g(X) = 2J(X)^T f(X)
$$

The second derivative matrix (Hesse matrix) for $F(X)$ is:

$$
G(X) = 2J(X)^T J(X) + 2 \sum_{i=1}^{3} f_i(X) \nabla^2 f_i(X)
$$

Setting

$$
S(X) = \sum_{i=1}^{3} f_i(X) \nabla^2 f_i(X)
$$

And substituting $S(X)$ in Eqn (10), yields the following expression:

$$G(X) = 2J(X)^T J(X) + 2S(X)$$

(12)

To solve Eqn (12) for $X$, the recursive formulaires of the Newton algorithm\(^\text{13}\) is introduced:

$$\left( J_k^T J_k + S_k \right) \delta_k = -J_k^T f_k$$

(13)

$$X_{k+1} = X_k + \delta_k$$

(14)

where $\delta_k$ is the $k$\textsuperscript{th} iterative step length. Due to the large computational complexity on $S_k$ and the possible singularity on $J_k^T J_k$ in the Newton algorithm, Levenberg and Marquardt utilised the following equ (15) to replace Eqn (13) for calculating $\delta_k$\(^\text{14}\):

$$\left( J_k^T J_k + \mu_k I \right) \delta_k = -J_k^T f_k$$

(15)

where $I$ stands for the unit matrix, and $\mu_k \geq 0$ refers to the control parameter in the iterative process. Consequently, Eqn (14) together with Eqn (15) is called the LM algorithm. The LM algorithm is applied in the proposed method to solve the localisation equations. As a result, the main design procedure of the proposed method is illustrated as Table 1.

Table 1. Design procedure by the proposed method

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Input: Initialise the target’s position $X_0$, the allowable error $\epsilon &gt; 0$, the suitable control parameter $\mu$.</td>
</tr>
<tr>
<td>2.</td>
<td>Output: the target’s position $X_k$ at time $k = 1, 2, \cdots$.</td>
</tr>
<tr>
<td>3.</td>
<td>Initialise the localisation model: the geocentric coordinate $(X_i, Y_i, Z_i)$, range $r_i$ and range error $\sigma_i$ of each radar $R_i$, the diagonal matrix $\Lambda$, and the unit matrix $I$.</td>
</tr>
<tr>
<td>4.</td>
<td>while $|\delta_k| &lt; \epsilon$ do</td>
</tr>
<tr>
<td>5.</td>
<td>Calculate the value of the functions $d_i^0$ and $f_k$, and the Jacobi matrix $J_k$.</td>
</tr>
<tr>
<td>6.</td>
<td>Solve $\delta_k$ according to Eqn (15).</td>
</tr>
<tr>
<td>7.</td>
<td>Update $X_k = X_k + \delta_k$ and $k = k + 1$;</td>
</tr>
<tr>
<td>8.</td>
<td>end</td>
</tr>
</tbody>
</table>

3.1.2 The Initial Value of the Target’s Position

To avoid double value solutions\(^\text{16}\) and accelerate the convergence speed for the LM algorithm, the initial value $X_0$ for the proposed method, which is the approximate position of the target $T$, can be calculated according to the observations $z_i = (r_i, \theta_i)$ of two radars selected from three radars. Referring to Fig. 2, a local Cartesian coordinate system was constructed, where the position of the radar $R_i$ is as the origin $O$ and $x$ axis is coincident or parallel to the north direction. The position of the radar $R_0$, is denoted as $A$. In the coordinate system, $T'$ is the projection of the target $T$ onto the plane $xOy$. Finally, the coordinate $x_0 = (x_0, y_0, z_0)$ of the target $T$ in the local Cartesian coordinate system can be calculated as:

$$
\begin{align*}
    x_0 &= r_0 \sin(\theta_0 - \theta_i) \cos \theta_i / \sin(\theta_2 - \theta_i) \\
    y_0 &= r_0 \sin(\theta_2 - \theta_0) \sin \theta_i / \sin(\theta_2 - \theta_i) \\
    z_0 &= \left[ r_i^2 - r_0^2 \sin^2(\theta_2 - \theta_0) / \sin^2(\theta_2 - \theta_i) \right]^{1/2}
\end{align*}
$$

(16)

where $r_0$ and $\theta_0$ are the magnitude and angle of the vector $OA$ respectively. Consequently, the approximate geocentric coordinate $X_0$ can be given by transforming $x_0$ from the local coordinate system to the geocentric coordinate system.

3.2 Validity Analysis of the Proposed Method

The Cramer-Rao lower bound (CRLB)\(^\text{6,7,12}\) is the minimum mean square error (MMSE), which the unbiased estimators can reach, and it is normally used as a performance metric for localisation methods. In this paper, the localisation CRLB is driven, and the relative validity is defined as the ratio of MMSE of the proposed method to the CRLB for estimating its localisation results as below.

3.2.1 The CRLB on Localisation Results

Express $Z = (X, Y, Z)^T$ as the estimate of the target’s position and $\Delta Z$ as the likelihood function of the observation set $Z = \{ r_i, i = 1, 2, 3 \}$, which is given as follows:

$$
\Delta Z = p(Z|X) = \prod_{i=1}^{3} p(r_i | X) 
$$

(17)

The CRLB on localisation results is determined in the following formula:

$$
P_{CRLB}^{-1} = E \left\{ \nabla_X \ln(\Delta Z) \nabla_X \ln(\Delta Z) \right\}_{X = X_{true}}
$$

(18)

where $X_{true}$ is the true position of the target. The Gaussian density of the range $r_i$ is obtained by the following equation:

$$
p(r_i | X) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left\{ -\frac{1}{2} \left[ \frac{(r_i - f_i(X))^2}{\sigma_i^2} \right] \right\}
$$

(19)

Then the logarithm likelihood function of the observation set $Z$ can be expressed as:

$$
\ln(\Delta Z) = -\frac{1}{2} \sum_{i=1}^{3} \left[ \ln(2\pi\sigma_i^2) + \frac{(r_i - f_i(X))^2}{\sigma_i^2} \right]
$$

(20)

By substitution of Eqn. (20) into Eqn. (18), $P_{CRLB}(X)$ is simplified as:
\[ P_{\text{CRLB}}(X) = \left( J^T \Lambda^{-1} J \right)^{-1} \]  

Consequently, \( P_{\text{CRLB}}(X) \) is used to calculate the relative validity of the proposed method.

3.2.2 Relative Validity Analysis of the Proposed Method

The relative validity of the proposed method can be defined as:

\[ \text{RE}^{CE} = E(\hat{X} - X)^2 / P_{\text{CRLB}}(X) \]  

Here, \( E(\hat{X} - X)^2 \) acts as the mean square error of the proposed method, and it can be approximately expressed as:

\[ E(\hat{X} - X_{\text{true}})^2 \approx \frac{1}{M} \sum_{i=1}^{N} E(\hat{X}_i - X_{\text{true}})^2 \]  

where \( M \) is the number of the Monte Carlo simulation runs, and \( \hat{X}_i \) is the estimate of the target’s position for the \( i^{th} \) simulation.

4. EXPERIMENT RESULTS AND ANALYSIS

Two simulation experiments and a real-test experiment were carried to estimate the performance of the proposed method in comparison with the least squared target localisation (LSTL) method in the paper. All experiments were conducted using a computer with a dual-core CPU of Intel Pentium 4 2.93 GHz, 1 GB RAM. The program for each experiment was performed using the MathWorks MATLAB 2009a version software.

4.1 Two Simulation Experiments

The detection area, its longitude ranging from 119.0° to 120.0° and its latitude from 39.5° to 40.5°, is equally divided into 21*21 grids. Three radars locate at (119.0°, 39.5°, 0.0 km), (120.0°, 39.5°, 0.0 km) and (119.0°, 40.2°, 0.0 km) respectively. The standard deviation \( \sigma_r \) of each radar’s range error is 100 m.

To intuitively reflect the localisation performance, geometrical dilution of precision (GDOP) was applied to describe the 3D geometric distribution of the localisation error:

\[ \text{GDOP} = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2} \]  

where \( \sigma_X \), \( \sigma_Y \) and \( \sigma_Z \) are the standard deviations of location errors in direction of \( X \), \( Y \) and \( Z \) axes, respectively in the geocentric coordinate system.

Figures 3(a) and 3(b) provide the GDOP of both the proposed method and the CRLB, respectively. Fig. 4 shows the relative validity \( \text{RE}^{CE} \) of the proposed method according to Eqn. (22). As can be seen from Fig. 3, the GDOP of the proposed method is much closed to that of the CRLB. It can be seen from Fig. 4, the values of \( \text{RE}^{CE} \) range from 0.9 to 1.1. Based on the above facts, it verified that the proposed method can approximate the localisation accuracy of the CRLB.

To compare the proposed method with the LSTL method, six target positions were located at the same longitude 119.5° and latitude 40.0° but for different heights ranging from 6
km to 16 km with equal intervals. Figure 5 shows the root mean-squared errors (RMSE) for the target’s position using two methods for 100 Monte Carlo simulation runs. As is seen from Fig. 5, the RMSE of the proposed method are smaller than the LSTL method. Therefore, due to range with higher precision than azimuth, the LM algorithm can provide good estimates for the target’s position through the establishment of the localisation equations using range in the real ellipsoidal earth model.

4.2 Real-test Experiment

The real-test experiment was evaluated using real-tracking data, which is generated from three radars (ID=RI, RII, and RIII) listed in Table 2. The radars are stationed at (119.5150°, 31.75428°, 0.0 km), (119.9560°, 31.96631°, 0.0 km) and (119.5010°, 27.99498°, 0.0 km), respectively. Due to the restricted test condition, the true positions of the targets are

Table 2. Localisation results by the proposed method

<table>
<thead>
<tr>
<th>No.</th>
<th>RI</th>
<th>RII</th>
<th>RIII</th>
<th>Localisation results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(127.4 km, 321.1°)</td>
<td>(171.2 km, 295.8°)</td>
<td>(142.2 km, 307.7°)</td>
<td>(119.5975°, 30.10883°, 3555 m)</td>
</tr>
<tr>
<td></td>
<td>(122.0 km, 321.8°)</td>
<td>(165.6 km, 295.5°)</td>
<td>(136.5 km, 307.8°)</td>
<td>(119.5999°, 30.12459°, 3553 m)</td>
</tr>
<tr>
<td></td>
<td>(119.3 km, 322.1°)</td>
<td>(162.9 km, 295.2°)</td>
<td>(133.8 km, 307.7°)</td>
<td>(119.6010°, 30.13263°, 3551 m)</td>
</tr>
<tr>
<td></td>
<td>(116.6 km, 322.6°)</td>
<td>(160.1 km, 295.0°)</td>
<td>(131.0 km, 307.8°)</td>
<td>(119.6021°, 30.14066°, 3549 m)</td>
</tr>
<tr>
<td></td>
<td>(113.9 km, 323.0°)</td>
<td>(157.4 km, 294.8°)</td>
<td>(128.3 km, 307.9°)</td>
<td>(119.6031°, 30.14870°, 3547 m)</td>
</tr>
<tr>
<td></td>
<td>(111.2 km, 323.4°)</td>
<td>(154.8 km, 294.6°)</td>
<td>(125.3 km, 307.8°)</td>
<td>(119.6038°, 30.15554°, 3545 m)</td>
</tr>
<tr>
<td></td>
<td>(108.6 km, 323.8°)</td>
<td>(151.9 km, 294.4°)</td>
<td>(122.6 km, 307.9°)</td>
<td>(119.6050°, 30.16387°, 3543 m)</td>
</tr>
<tr>
<td></td>
<td>(105.9 km, 324.2°)</td>
<td>(149.1 km, 294.1°)</td>
<td>(129.9 km, 308.0°)</td>
<td>(119.6064°, 30.20248°, 3522 m)</td>
</tr>
<tr>
<td></td>
<td>(103.2 km, 324.8°)</td>
<td>(146.4 km, 293.9°)</td>
<td>(116.9 km, 307.9°)</td>
<td>(119.6068°, 30.17934°, 3538 m)</td>
</tr>
<tr>
<td></td>
<td>(100.6 km, 325.3°)</td>
<td>(143.6 km, 293.7°)</td>
<td>(114.3 km, 308.0°)</td>
<td>(119.6078°, 30.18768°, 3544 m)</td>
</tr>
<tr>
<td>Row1</td>
<td>(207.0 km, 145.9°)</td>
<td>(131.1 km, 178.2°)</td>
<td>(169.7 km, 123.7°)</td>
<td>(120.2639°, 30.33694°, 4466 m)</td>
</tr>
<tr>
<td></td>
<td>(204.4 km, 146.1°)</td>
<td>(128.4 km, 177.9°)</td>
<td>(166.7 km, 123.8°)</td>
<td>(120.2172°, 30.34018°, 4409 m)</td>
</tr>
<tr>
<td></td>
<td>(199.1 km, 146.6°)</td>
<td>(123.0 km, 177.2°)</td>
<td>(161.1 km, 123.7°)</td>
<td>(120.1298°, 30.34829°, 4289 m)</td>
</tr>
<tr>
<td></td>
<td>(196.5 km, 146.9°)</td>
<td>(120.4 km, 176.9°)</td>
<td>(158.3 km, 123.6°)</td>
<td>(120.0927°, 30.35384°, 4233 m)</td>
</tr>
<tr>
<td></td>
<td>(193.8 km, 147.2°)</td>
<td>(117.7 km, 176.6°)</td>
<td>(155.5 km, 123.6°)</td>
<td>(120.0565°, 30.36031°, 4178 m)</td>
</tr>
<tr>
<td></td>
<td>(191.2 km, 147.5°)</td>
<td>(114.9 km, 176.2°)</td>
<td>(152.7 km, 123.6°)</td>
<td>(120.0256°, 30.36825°, 4128 m)</td>
</tr>
<tr>
<td></td>
<td>(188.6 km, 147.9°)</td>
<td>(112.2 km, 175.8°)</td>
<td>(149.9 km, 123.5°)</td>
<td>(119.9952°, 30.37557°, 4079 m)</td>
</tr>
<tr>
<td></td>
<td>(186.1 km, 148.1°)</td>
<td>(109.5 km, 175.3°)</td>
<td>(147.1 km, 123.5°)</td>
<td>(119.9723°, 30.37970°, 4034 m)</td>
</tr>
<tr>
<td></td>
<td>(183.5 km, 148.4°)</td>
<td>(106.8 km, 174.9°)</td>
<td>(144.4 km, 123.4°)</td>
<td>(119.9498°, 30.38749°, 3993 m)</td>
</tr>
<tr>
<td></td>
<td>(180.8 km, 148.8°)</td>
<td>(104.2 km, 174.4°)</td>
<td>(141.5 km, 123.3°)</td>
<td>(119.9308°, 30.39474°, 3954 m)</td>
</tr>
<tr>
<td></td>
<td>(97.9 km, 295.3°)</td>
<td>(139.8 km, 307.0°)</td>
<td>(86.5 km, 320.9°)</td>
<td>(119.6078°, 30.12378°, 3570 m)</td>
</tr>
<tr>
<td></td>
<td>(95.2 km, 294.7°)</td>
<td>(137.0 km, 306.9°)</td>
<td>(83.7 km, 321.1°)</td>
<td>(119.6084°, 30.13151°, 3567 m)</td>
</tr>
<tr>
<td></td>
<td>(92.5 km, 294.2°)</td>
<td>(134.1 km, 306.6°)</td>
<td>(80.9 km, 321.4°)</td>
<td>(119.6093°, 30.13901°, 3565 m)</td>
</tr>
<tr>
<td></td>
<td>(89.8 km, 293.5°)</td>
<td>(131.4 km, 306.3°)</td>
<td>(78.1 km, 321.7°)</td>
<td>(119.6100°, 30.14754°, 3562 m)</td>
</tr>
<tr>
<td></td>
<td>(84.4 km, 292.3°)</td>
<td>(125.8 km, 306.2°)</td>
<td>(72.4 km, 322.3°)</td>
<td>(119.6113°, 30.16232°, 3557 m)</td>
</tr>
<tr>
<td></td>
<td>(81.8 km, 291.8°)</td>
<td>(122.9 km, 306.0°)</td>
<td>(69.9 km, 322.9°)</td>
<td>(119.6122°, 30.17198°, 3553 m)</td>
</tr>
<tr>
<td></td>
<td>(79.2 km, 291.2°)</td>
<td>(120.1 km, 306.0°)</td>
<td>(67.0 km, 323.4°)</td>
<td>(119.6127°, 30.17870°, 3550 m)</td>
</tr>
<tr>
<td></td>
<td>(76.6 km, 290.5°)</td>
<td>(117.4 km, 306.0°)</td>
<td>(64.5 km, 324.0°)</td>
<td>(119.6136°, 30.18804°, 3546 m)</td>
</tr>
<tr>
<td></td>
<td>(74.0 km, 289.9°)</td>
<td>(114.7 km, 306.0°)</td>
<td>(61.7 km, 324.6°)</td>
<td>(119.6141°, 30.19589°, 3542 m)</td>
</tr>
<tr>
<td></td>
<td>(97.9 km, 295.3°)</td>
<td>(139.8 km, 307.0°)</td>
<td>(86.5 km, 320.9°)</td>
<td>(119.6078°, 30.12378°, 3570 m)</td>
</tr>
<tr>
<td></td>
<td>(95.2 km, 294.7°)</td>
<td>(137.0 km, 306.9°)</td>
<td>(83.7 km, 321.1°)</td>
<td>(119.6084°, 30.13151°, 3567 m)</td>
</tr>
</tbody>
</table>
unknown. The radar performance parameters are represented as follow: the maximum detection distance 230.0 km, range error 200.0 m and azimuth error 1°. Table 2 lists the localisation results by the proposed method. As can be seen from Table 2, the propose method has good performance in estimate accuracy on the targets’ positions. Therefore, it is verified that the proposed method is feasible to locate a target in real case.

5. CONCLUSIONS

In this paper, the characteristics and shortcomings of the traditional target localisation methods in 2-D search radar networking have been analysed. Due to range with higher precision than azimuth for two-dimensional radars, a range-only target localisation method is presented using synchronous measurements from three radars. Considering the real ellipsoid earth model, the equivalent geometric model is introduced in the proposed method. A set of localisation equations is derived on range error in such a scenario. Consequently, the target localisation problem is converted into a nonlinear weighted least squares problem. The LM algorithm is applied to solve the localisation equations and to estimate the target’s position. By giving the initial value approximately, the proposed method is able to avoid the value solutions and accelerate the convergence speed. Furthermore, it can approximate the localisation accuracy of the CRLB through the analysis of the relative validity defined. The simulation results show that the proposed method is effective and has high accuracy, while the real-test result illustrates that the proposed method is feasible.

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REFERENCES


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