A Model for Estimation of Aircraft Attrition from Various Ground Air Defence Weapons

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ABSTRACT

A computer-based mathematical model is developed for the estimation of assessment of damage inflicted on an aircraft due to a ground-based air defence gun. It is assumed that the aircraft is approaching the target from an arbitrary direction and does not change its trajectory during gun firing. Dimension of aircraft and trajectory of warhead are assumed to be known. Damage to aircraft is caused due to blast as well as fragments. Aircraft is assumed to be killed if one of its vital parts has been killed.

1. INTRODUCTION

Air defence (AD) guns and missiles are deployed to provide protection against hostile aircraft coming to attack vulnerable areas and vulnerable points. These weapons may be single- or multi-barrel and may fire DA- or VT-fuzed ammunition or warheads. In order to identify a suitable AD weapon for purposes such as acquisition, or design and development or deployment, so that it is desirable to make an assessment of its effectiveness. The problem of assessing the effectiveness of AD weapons to stationary as well as mobile targets has been studied by various authors\(^1\)-\(^5\). While the aircraft has been modelled as a right cylinder and presenting a circular target of some dimensions by the earlier authors, here we have considered the aircraft comprising of various sections modelled as cones, cylinders, wedges, etc. Further, the aircraft is

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not considered to be necessarily radially approaching the target, which has been the assumption in most of the earlier works. In the present report, we have discussed damage to aircraft body due to explosive charge as well as due to fragments, when warhead/ammunition explodes in the near vicinity of the aircraft. Kill criterion has been taken as the minimum number of fragments required to penetrate and kill a particular part. In the case of blast waves, it is assumed that the probability of kill is one, based upon the impulse transmitted to the structure. A typical aircraft and a typical AD gun with DA/VT-fuzed ammunition has been considered for the validation of the model. However, the model is quite general and can be used for all types of aircraft/weapons.

The aim of the paper is to develop a computer-based mathematical model for the assessment of damage inflicted on an aircraft, using an AD weapon. It is assumed that the dimensions and orientation of the aircraft and the shell/warhead are known.

2. MATHEMATICAL MODEL

An aircraft can be considered to be divided into a number of parts some of which are vital parts such as cockpit, engine, fuel tank, control unit, etc. Aircraft can be considered as killed if at least one of these vital components is killed. Damage to aircraft is caused by the blast when explosion is in its near vicinity and by fragments, if it is at a distance. In the present report we have studied damage due to blast as well as fragmentation effect. A DA-fuzed ammunition defeats the target by first making a physical impact and then exploding. While the penetration is governed by the kinetic energy of the projectile at the point of impact, the structural damage is decided by the pressure transmitted to the aircraft body due to the explosion of the charge. In the case of VT-fused ammunition, it first reaches in the vicinity of the target and explodes at a predetermined distance. Fragments thus formed, hit at various parts of the target and cause damage. Kill at the target aircraft is based on the kill of its vital parts. A vital part is assumed to be killed if required amount of energy is transmitted to the part by the fragments.

In the following section, models for DA- as well as VT-fused ammunition have been developed.

3. MODEL FOR DA-FUZED AMMUNITION

The probability of kill of an aircraft depends on various functions such as kill of its vital parts, probability of hit, probability of fuze-functioning, etc. It is not necessary that even if a part of aircraft is damaged fully, aircraft is killed. It is known from war experiences, that quite a number of aircraft return to friendly areas even after being damaged heavily. Probability of kill $P_k$ of an aircraft component can be defined as

$$P_k = P_h P_f P_{kf}$$  

where $P_h$ is the single shot hit probability of the ammunition at the component; $P_f$ is the probability of fuze-functioning; and $P_{kf}$ is the probability of kill of the component.
given that the ammunition has hit the component and the fuze has functioned.
Evaluation of these three probabilities will be discussed in the following sections.

3.1 Single Shot Hit Probability

For the purpose of finding single shot hit probability (SSHP) of a round of ammunition, we consider an earthfixed rectangular frame of reference \( G-XYZ \) in which the origin \( G \) is at the weapon position and the axes of the frame \( G-XYZ \) (Frame-I) are orthogonal and a moving orthogonal frame of reference \( O-UVW \) (Frame-II) in which the origin \( O \) is at the geometrical centre of the aircraft and the axes \( OU, OV, OW \) respectively are along the rolling, pitching and yawing axis of the aircraft (Fig. 1). Then, direction cosines \((l_0, m_0, n_0)\) of the line \( GO \) are given by

\[
l_0 = \cos A \cos E; \quad m_0 = \sin A \cos E; \quad n_0 = \sin E
\]  

Figure 1  Coordinate-frames of reference
where $A$ and $E$ are respectively the angles in azimuth and elevation of the aircraft
(Fig. 1). If $(x,y,z)$ and $(u,v,w)$ respectively are the coordinates of a point on the
aircraft with respect to fixed and moving frame of reference, then the relations between
the coordinates $(x_p, y_p, z_p)$ and $(u_p, v_p, w_p)$ of a point $P$ at the aircraft are easily seen
to be as follows:

$$x_p = x_0 + l_1 u_p + l_2 v_p + l_3 w_p$$
$$y_p = y_0 + m_1 u_p + m_2 v_p + m_3 w_p$$
$$z_p = z_0 + n_1 u_p + n_2 v_p + n_3 w_p$$

(3)

where $(x_0, y_0, z_0)$ are the coordinates of the aircraft centre with respect to the $G$-$XYZ$
frame and $(l_i, m_i, n_i), i = 1, 2, 3$ are respectively the direction cosines of $OU, OV,$
$OW$ with respect to fixed frame of reference. Now direction cosines of line $GP$ are
given by

$$l_p = x_p/GP; \quad m_p = y_p/GP; \quad n_p = z_p/GP$$

(4)

and the angle $\theta$ between the lines $GP$ and $GO$ is given by (Fig. 1(b))

$$\theta = \cos^{-1} \left( l_p l_0 + m_p m_0 + n_p n_0 \right)$$

(5)

Now consider a plane (referred to as the $D$-plane) at right angles to $GO$ passing
through the point $O$ (Fig. 1). Consider a two-dimensional frame $O$-$ST$ in the $D$-plane
such that $OT$ is in the vertical plane and $OS$ in the horizontal plane through $O$. Then
the direction cosines of the $OS$-axis with respect to the $G$-$XYZ$ frame are (see
Appendix)

$$\left( \frac{m_0}{\sqrt{1 - n_0^2}}, \frac{-l_0}{\sqrt{1 - n_0^2}}, 0 \right)$$

and the direction cosines of $OT$-axis are

$$\left( \frac{-l_0 n_0}{\sqrt{1 - n_0^2}}, \frac{-m_0 n_0}{\sqrt{1 - n_0^2}}, \frac{\sqrt{1 - n_0^2}}{\sqrt{1 - n_0^2}} \right)$$

If $Q$ be the point in which the line $GP$ (produced, if necessary) meets the $D$-plane,
then

$$GQ = GO \cos \theta, \quad OQ = GO \tan \theta$$

(6)

Therefore, coordinates of the point $O$ in the $G$-$XYZ$ frame turns out to be

$$x_q = GQ l_0; \quad y_q = GQ m_0; \quad z_q = GQ n_0$$

(7)

Also, the direction cosines of the line $OQ$ with respect to the $G$-$XYZ$ frame are

$$l_q = (x_q - x_0)/OQ; \quad m_q = (y_q - y_0)/OQ; \quad n_q = (z_q - z_0)/OQ$$

(8)
Finally, the coordinates \((s_q, t_q)\) of the point \(Q\) in the \(D\)-plane are given by

\[
s_q = OQ \cos \phi; \quad \text{and} \quad t_q = OQ \cos \psi
\]

where

\[
\cos \phi = l_q l_s + m_q m_s + n_q n_s; \quad \text{and} \quad \cos \psi = l_q l_t + m_q m_t + n_q n_t
\]

\((l_q, m_q, n_q)\) being the direction cosines of the \(S\)-axis with respect to the \(G\)-\(XYZ\) frame and \((l_t, m_t, n_t)\) are the direction cosines of the \(T\)-axis with respect to the \(G\)-\(XYZ\) frame.

Let \(F_p\) be the shape of a typical part of the aircraft body bounded by line segments with vertices \(P_i\) \((i=1,2,\ldots,n)\), then corresponding points \(Q_i\) \((i=1,2,\ldots,n)\) of the projection of the part on \(D\)-plane can be determined as explained above, and a corresponding figure \(F_q\) can be obtained. The figure \(F_q\) is such that a hit on this will imply a hit on the figure \(F_p\) of the aircraft body. Similar analogy can be extended for other parts of the aircraft even those parts which are bounded by curved segments.

Finally, if \(\sigma_s\) and \(\sigma_t\) be the standard deviations of the normal distribution governing the points of impact of the rounds on the aircraft, then SSHP on a figure \(F_p\) of the aircraft is given by

\[
P_s = \frac{1}{2\pi \sigma_s \sigma_t} \int_{F_p} \exp \left( -1/2 \left( \frac{s^2}{\sigma_s^2} + \frac{t^2}{\sigma_t^2} \right) \right) \, ds \, dt
\]

It is assumed that the round has been aimed at the centre of the aircraft. The parameters \(\sigma_s\) and \(\sigma_t\) can be computed from the system errors of the weapon in the azimuth and elevation respectively.

### 3.2 Probability of Fuze-Functioning

The probability of fuze-functioning \(P_f\) for a DA-fuzed ammunition is constant and a part of the data, and has been taken as 1.0 in the present case.

### 3.3 Probability of Kill

The probability of kill in one round of DA charge may be taken as 1.0 as the explosive energy released by the shell is much higher than the energy required by any of vital components of the aircraft to kill it.

### 4. MODEL FOR VT-FUZED AMMUNITION

The VT fuzed ammunition shell first reaches in the vicinity region (Fig. 2) of the target aircraft then its fuze functions and explodes into fragments having high kinetic energy by vicinity region, it is meant, the region around the aircraft's structure in which VT-fuzed shell can sense the aircraft and explode. Some of these fragments penetrate the structure of the target aircraft causing damage to its components.
Figure 2. The position of aircraft and the VT-shell when the shell is likely to burst at a point CS in the vicinity region of the aircraft (all parameters are referred w.r.t. Frame-I).

The probability of kill of a component of the aircraft in one round can be given as

$$P_k = \int_0^{RL} PLD_c(r)Pdf(r)dr + \int_{RU}^{RL} PLD(r)Pdf(r)P_{km}(r)dr$$  \hspace{1cm} (11)$$

where $RL$ is the distance from the surface of the aircraft within which, if the VT shell explodes the shock wave itself can damage the aircraft's component; $RU$ is the vicinity limit, a distance from the surface of the aircraft beyond which the shell cannot explode. Vicinity shell is a shell formed by an imaginary surface at a distance $r$ from the surface of the aircraft around it. $PLD_c(r)$ is the probability that the VT shell will land around the component at a distance $r$; $Pdf(r)$ is the probability that the fuze will function at a distance $r$ from the aircraft's surface; $PLD(r)$ is the probability that VT shell will land around the aircraft at a distance $r$ from the surface of the aircraft; $P_{km}(r)$ is the probability that at least $k$ number of fragments of mass $\geq m$ will penetrate the component's structure; and $k$ is the number of lethal fragment hits required to kill the component.

4.1 Determination of $RL$

The estimation of $RL$ can be done on the basis of critical impulse failure criterion. This criterion essentially states that structural failure under transient loadings can be correlated to a critical impulse applied for a critical time duration where the latter is assumed to be one-quarter of the natural period of free vibration of the structure. The critical impulse can be expressed as:
\[ I_c = (\rho/E)^{1/2} \cdot t \cdot \sigma_y \]

where \( E \) is Young’s modulus, \( \rho \) is density of material, \( t \) is thickness, and \( \sigma_y \) is dynamic yield strength.

In applying this method to skin panels supported by transverse longitudinal members, for example, one first calculates the critical impulse and natural period of the panel. Incident pressure pulse having a duration of one-quarter of the natural period or more having an impulse at least equal to \( I_c \) will cause rupture of the panel at the attachments.

If the distance of point of explosion from the target is \( z \), then

\[
\frac{p_b}{p_a} = \frac{808[1 + (z/4.5)^2]}{\sqrt{1 + (z/0.048)^2} \sqrt{1 + (z/0.32)^2} \sqrt{1 + (z/1.35)^2}}
\]

where \( p_b \) is the incident blast wave, and \( p_a \) is the atmospheric pressure. Time duration \( t_d \) of positive phase of shock is given by the following relation,

\[
\frac{t_d}{w^{1/3}} = \frac{980[1 + (z/0.02)^4]}{[1 + (z/0.02)^3](1 + (z/0.74)^4) \sqrt{1 + (z/6.9)^2}}
\]

where \( w \) is the charge weight in kg. Reflected pressure \( p_r \) can be given as

\[
p_r = \frac{p_d (p_b/p_a + 7) (p_b/p_a + 1)}{(p_b/p_a + 7)}
\]

Therefore the total impulse \( I \) can be given by

\[
I = \int_0^{t_d} p_r dt
\]

If the reflected pressure pulse has been assumed to be a triangular pressure pulse, then

\[
I = \frac{P_r t_d^2}{2}
\]

Taking the dimensions of the panel as \( a \) and \( b \), the natural frequency \( \omega \) of fundamental mode is

\[
\omega = \pi^2 \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{\frac{E \cdot t^3}{12(1 - \nu^2) \rho}}
\]

where \( E \) is the Young’s modulus, \( t \) is the thickness, \( \rho \) is the density, and \( \nu \) is the Poisson’s ratio. Then the natural period \( T \) of panel is \( T = 2\pi/\omega \).
Keeping in view the above relation, we can simulate the value of \( z \) for which \( I \geq I_0 \). The simulated value will be equal to \( RL \).

### 4.2 Probability of Landing

The probability of landing of VT-shell at distance \( r \) from the surface of the aircraft can be estimated as

\[
PLD(r) = \frac{1}{2\pi\sigma_x\sigma_z} \int_{S_r} \exp \left\{ -1/2 \left( \frac{x^2}{\sigma_x^2} + \frac{z^2}{\sigma_z^2} \right) \right\} \, ds \, dt
\]  

(19)

where \( S_r \) is the projected region of the vicinity shell over the \( D \)-plane (as defined earlier), and \( \sigma_x, \sigma_z \) are the system errors of the firing gun in azimuth and the elevation planes.

### 4.3 Probability of Fuze-Functioning

The probability of fuze-functioning at a miss distance 4.5 m is 0.8 and it decreases rapidly with the increase of distance, such that at a distance 6 m, it is 0.2 and 6.5 m it can be treated as 0. Probability of fuze-functioning can be defined as

\[
Pf(r) = \frac{1}{C} Pf(r)
\]

where \( C = \int_0^{6.5} Pf(r) \, dr \), and

\[
Pf(r) =
\begin{align*}
0.8 & \quad r \leq 4.5 \\
-0.4r + 2.6 & \quad 4.5 < r \leq 6 \\
-0.4r + 2.6 & \quad 6 < r \leq 6.5 \\
0.0 & \quad r \geq 6.5
\end{align*}
\]

(20)

The probability distribution is shown in the Fig. 3.

![Figure 3](image-url)  

Figure 3  Probability of fuze-functioning vs miss distance.
Estimation of Aircraft Attrition Ground AD Weapons

4.4 Probability that at least \( k \) Number of Fragments will Penetrate

Probability that at least \( k \) number of fragments of each of mass \( P_{km} \) will penetrate the component, if the VT-shell bursts in the vicinity shell at a distance \( r \) from the aircraft’s surface is given by

\[
P_{km}(r) = 1 - \sum_{N=0}^{k-1} \frac{(M_r)N}{N!} e^{-M_r}
\]

where \( M_r \) is the average number of fragments penetrating the component. If the VT-shell burst in a vicinity shell at a distance \( r \), then \( M_r \) is given by

\[
M_r = 0.5 \left( \frac{1}{N_p} \sum N^k \right)
\]

where \( N_p \) is the total number of points in the vicinity shell and \( N^k \) is the number of fragment hits to a component with impact velocity greater than \( (V50)_o \) if the shell explodes at \( k \)-th point of the vicinity shell at a distance \( r \) from the surface of the aircraft. \( (V50)_o \) is shown in Fig. 4, and \( N^k \) is evaluated in section 4.5. The factor 0.5 in Eqn (22) is used because of the definition of \( (V50)_o \).

![Figure 4. Typical V50-ballistic limits for aircraft structural materials.](image)

4.5 Expected Number of Fragment Hits

Let the shell burst at a point \( p^k \) in the vicinity shell of the aircraft. The fragments of the shell moves in the conical angular zones with respect to the axis of the VT-shell. Let there be \( n \) such uniform conical zones, uniform in the sense that the ejection of the fragments per unit solid angle is the same within a particular zone. Size of VT-shell is very small as compared to that of the aircraft, therefore it can be assumed that the fragments are ejecting as if they are coming from the centre of the shell.

We define \( z^k_{i,j+1} \) as the zone which is the intersection of two solid cones, with vertex at a point \( p^k \) and the interaction of two solid cones whose slant surfaces make angles \( a_i, q_i, 1 \) respectively with the axis of the shell. And let \( n^k_{i,j+1} \) be the total number of fragments of mass greater than \( m \), in the angular zone \( z^k_{i,j+1} \).
Fragments per unit solid angle in the $z^k_{i,i+1}$ th angular zone can be given as

$$f_{i,i+1}^k = \frac{n_{i,i+1}^k}{2\pi(\cos \alpha_i^k - \cos \alpha_{i+1}^k)}$$ (23)

where $\alpha_i$, $\alpha_{i+1}$, are explained in Eqn (27).

Let $\omega_{i,i+1}^k$ be the solid angle subtended by the component in the $z^k_{i,i+1}$ angular zone and $f_{i,i+1}^k$ is the fragment density therein, then the total number of fragment hits to the component is given by

$$N^k = \sum_{i=1}^{\infty} \omega_{i,i+1}^k f_{i,i+1}^k$$ (24)

4.6 Solid Angle Subtended by a Component in an Angular Zone

The solid angle subtended in the angular zone $z_{i,i+1}$ (the results of this section are independent of $k$ and are true for all values of $k$) by a component at the centre of gravity (CG) of the shell is determined by the intersecting surface of the component and the angular zone $z_{i,i+1}$ mathematically (Fig. 5).

![Figure 5. Scenario of solid angle subtended in an angular zone.](image)

$$\omega_{i,i+1} = \sum_{A_{i,i+1}} \delta \omega$$

$$\delta \omega = \frac{1}{R_A^2} \cos \theta | \delta A$$ (25)

where $A_{i,i+1}$ is the intersecting surface of the component and the zone $z_{i,i+1}$ which will differ in stationary and dynamic cases; $\delta A$ is the small area on the surface $A_{i,i+1}$; $R_A$ is the distance between CG of the shell and the mid-point of $\delta A$; and $\theta$ is the angle between $R_A$ and normal to the surface at the mid-point of $\delta A$.

Value of $\delta \omega$ is evaluated in Eqn (32). Following is the example to evaluate the solid angle subtended by a component of the aircraft in the different angular zones of the VT-shell, when the shell burst at any arbitrary point $C_s$ in the vicinity region of
the aircraft. Similar method can be developed to any component of the aircraft having well defined surface.

Let the VT-fuzed shell burst at a point $C_1$ in the vicinity region of the aircraft, say at time $t = 0$. At the time $t$, let the coordinates of the CG of the VT-shell be $(x_s, y_s, z_s)$ and the velocity of the shell is $V_s$ in the direction $(l_s, m_s, n_s)$ which is also the direction of its axis, with respect to Frame-I which is fixed in space.

Further let at the time of burst, $(x_a, y_a, z_a)$, be the coordinates of the centre of the aircraft which is also the origin of the Frame-II and let $(l_i, m_i, n_i), i = 1$ to $3$ be the direction cosines of the aircraft's axes (i.e., axes of the Frame-II) with respect to Frame-I and this aircraft (Frame-II) is moving with velocity $V_a$ in the direction $(l_i, m_i, n_i)$ in Frame-I.

Let the coordinates of the CG of the shell at the time of burst $(t = 0)$ be $(x_s, y_s, z_s)$, and $(u_s, v_s, w_s)$ with respect to the two frames of reference. Transformation from one system of coordinates to other is given as

$$x_1 = x_s + x_0$$
$$y_1 = y_s + y_0$$
$$z_1 = z_s + z_0$$

Let us assume that VT-shell bursts in stationary position with reference to Frame-I and $a_i, a_{i+1}$, are the angles which the boundaries of the conical angular zone of fragments $z_{i,i+1}$ make with the positive direction of the shell axis and $V_{F_i}, V_{F_{i+1}}$ are the corresponding velocities of the fragments of these boundaries.

When shell bursts in a dynamic mode, the directions and velocities of fragments, as observed in a stationary frame will be

$$a'_i = \tan^{-1} \left( \frac{V_2}{V_1} \right)$$
$$V_{F'_i} = \left( V_1^2 + V_2^2 \right)^{1/2}$$

where

$$V_1 = V_s + V_{F_i} \cos (a_i)$$
$$V_2 = V_{F_i} \sin (a_i)$$

Fragments emerging from $C_1$, in an angular zone $z_{i,i+1}$ will be confined in a cone making angles $a'_i$ and $a_{i+1}$ respectively with the axis of the shell. Intersection of this cone with the surface of the aircraft is say an area $P_1, P_2, P_3$, and $P_4$. Divide the surface enveloping $P_1, P_2, P_3, P_4$ into a finite number of rectangular areas $\delta A = \delta l \delta b$ (say) where $\delta l$ and $\delta b$ are dimensions of the rectangular element (Fig. 5).

If point $P$, whose coordinates with respect to Frame-II are $(u_p, v_p, w_p)$, is the middle point of area $\delta A$, then solid angle of area $\delta A$ subtended at the centre of the shell and angular zone to which it belongs is determined by simulation.
Coordinates at point \( P \), at any time after burst, with respect to a fixed frame are

\[
\begin{align*}
    x_{pl} &= x_p + V_a l_{1} v t; \\
    y_{pl} &= y_p + V_a m_{1} w t; \quad \text{and} \quad z_{pl} &= z_p + V_a n_{1} u t
\end{align*}
\]  

(28)

Where \( V_a \) is the velocity of the aircraft and \((l_{1}, m_{1}, n_{1})\) are direction cosines of velocity vector with reference to Frame-I. If \( \phi \) is the angle between shell axis and line \( C_{i}P \), where \( P \) is the position of point \( P \) at time \( t \), then first step is to determine the angular zone \( a'; a' + 1 \) in which \( \phi \) lies.

Fragment may come to the point \( P \), from angular zone \( z_{i,i+1} \) with velocity \( V_{F,i}' \), \( V_{F,i+1}' \) depending upon \( \phi \) is close to \( a' \) or \( a' + 1 \).

Distance travelled by the fragments along the line \( C_{i} - P \), in time \( t \), is

\[
D_{f} = V_{F}' \cdot t
\]

where \( V_{F}' = \) selected \((V_{F,i}' V_{F,i+1}')\)

(29)

In Eqn (29), value of \( V_{F}' \) is selected from \( V_{F}' \) and \( V_{F,i+1}' \) depending that \( \phi \) is nearer to \( a' \) or \( a' + 1 \).

Actual distance between point \( C_{i} \) and \( P \), is

\[
D_{i} = \left[ \sum (x_{i} - x_{pl})^2 \right]^{1/2}
\]

(30)
From Eqns (29) and (30) we simulate \( t \) such that \( D_s = D_z \) for confirmed impact. Velocity of impact of a fragment \( V_{\text{strike}} \) can be given as

\[
V_{\text{strike}} = (V_F^2 + V_a^2 - 2V_F V_a \cos \beta)^{1/2}
\]

where \( \beta \) is the angle between the positive direction of aircrafts velocity vector and fragment velocity vector. Figure 4 shows the graphs of velocity of fragment \((V50)_h\) versus the penetration in aircraft structural materials is shown. We define \((V50)_\theta\) as the velocity of fragment hitting the component at an angle \( \theta \) with the normal to the surface, so that its probability of penetrating the component is 50 per cent.

If \( \theta \) is the angle of impact, then

\[
(V50)_\theta = (V50)_0 \cos \theta
\]

(31)

where \((V50)_0\) is the required velocity of impact at zero degree angle of obliquity and can be obtained for various thickness of plates and different kinds of projectiles.

If velocity of impact \( V_{\text{strike}} \) is greater than \((V50)_\theta\) then the solid angle \( \delta\omega \), subtended by the small rectangular element, in the angular zone \( z_{i+1} \) is given by

\[
\delta\omega = \frac{\delta A \cdot \cos \theta i}{D_z^2}
\]

which is added to the Eqn (25).

5. CUMULATIVE KILL PROBABILITY

As the aircraft is considered to have been divided into \( y \) parts, let \( P_i(j) \) be the single shot kill probability of a typical vital part due to \( i \)th burst of fire, each burst having \( n \) rounds. The cumulative kill probability of a typical vital part (say, \( j \)th) in \( N \) burst of fire can be given as

\[
CKP(j) = 1 - \prod_{i=1}^{N} [1 - P_i(j)]
\]

(33)

Further the aircraft can be treated as killed if at least one of its vital part is killed. Thus the \( CKP \) for the aircraft as a whole can be given as.

\[
CKP = 1 - \prod_{j=1}^{Y} [1 - CKP(j)]
\]

(34)

6. DATA USED

A typical aircraft was used to validate the model given in the present paper. Data used as input to the model for the aircraft is as follows:

6. Target Aircraft

Radius of the fuselage = 0.86 m
Distance of geometric centre of aircraft from frontal section = 7.82 m
Skin panel size = 15 \( \times \) 25 cm²
Material of the aircraft’s skin: strong aluminium alloy

Density of the strong aluminium alloy ($\rho$) = 2800 kg/m$^3$

Dynamic yield strength (taken) ($\sigma_y$) = $550 \times 10^6$ Pa

Young’s modulus of strong aluminium alloy ($E$) = $75.0 \times 10^9$ Pa

Poisson’s ratio ($\nu$) = 0.33

Table 1. Vital parts and parameters considered in the study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pilot</th>
<th>Fuel tank</th>
<th>Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of vital parts from frontal section (m)</td>
<td>3.30</td>
<td>5.24</td>
<td>10.67</td>
</tr>
<tr>
<td>Width of vital parts (m)</td>
<td>1.94</td>
<td>1.37</td>
<td>2.26</td>
</tr>
<tr>
<td>Equivalent thickness of duraluminium of vital parts assumed (mm)</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Critical energy (in joules) required to kill the vital part$^6$</td>
<td>678.0</td>
<td>339.0</td>
<td>1356.0</td>
</tr>
<tr>
<td>Estimated numbers of fragments to produce the required energy</td>
<td></td>
<td></td>
<td>2</td>
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<tr>
<td>Velocity $V_{50}$ for $0^\circ$ angle of oblique (Fig. 4) (m/s)</td>
<td>699.5</td>
<td>594.5</td>
<td>487.6</td>
</tr>
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</table>

6.2 Weapons

An air defence twin barrel gun with DA/VT-fuzed ammunition is considered for this study, with the following parameters.

System error : 3 mrad
Firing rate : 5 rounds/s/gun barrel
Probability of fuze-functioning DA : 0.99
VT : 0.8 within distance $r \leq 4.5$ m
0.2 at distance $r = 6$ m
0.0 at distance $r \geq 6.5$ m

Time of continuous firing of guns : 3 s
Maximum range of gun : 5000 m
Minimum range of gun : 500 m
Maximum detecting range : 10000 m

7. RESULTS AND DISCUSSION

The model was run for data given above. The aircraft have been considered coming across the gun position at an altitude of 100 m and at a speed 300 m/s. The twin barrel gun starts engaging the target aircraft from the range of 2000 m for a period of 3s.

The number of fragments required to defeat a vital part of an aircraft is calculated on the basis of energy criteria$^6$. Tables 2 and 3 give the kill probability of various vital parts and cumulative kill probability (CKP) of the aircraft as a whole. The results so obtained for the typical aircraft have been presented in Fig. 6 for DA- and VT-fuzed ammunition.
### Table 2
Number of rounds vs CKP of the various vital parts and aircraft as a whole for a typical aircraft due to DA-fuzed ammunition

<table>
<thead>
<tr>
<th>No. of rounds</th>
<th>Aircraft</th>
<th>Pilot</th>
<th>Fuel tank</th>
<th>Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.0045</td>
<td>.0015</td>
<td>.0013</td>
<td>.0017</td>
</tr>
<tr>
<td>4</td>
<td>.0092</td>
<td>.0030</td>
<td>.0027</td>
<td>.0035</td>
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<td>.0047</td>
<td>.0042</td>
<td>.0054</td>
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<tr>
<td>8</td>
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<td>.0064</td>
<td>.0058</td>
<td>.0074</td>
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<td>.0083</td>
<td>.0075</td>
<td>.0095</td>
</tr>
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<td></td>
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<td></td>
<td>.0374</td>
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<td>.0113</td>
<td>.0142</td>
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<tr>
<td>16</td>
<td>.0441</td>
<td>.0146</td>
<td>.0133</td>
<td>.0168</td>
</tr>
<tr>
<td>18</td>
<td>.0514</td>
<td>.0171</td>
<td>.0156</td>
<td>.0196</td>
</tr>
<tr>
<td>20</td>
<td>.0592</td>
<td>.0197</td>
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<td>.0227</td>
</tr>
<tr>
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<td>.0335</td>
<td>.0421</td>
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</tbody>
</table>

### Table 3
Number of rounds vs CKP of the various vital parts and aircraft as a whole for a typical aircraft due to VT-fuzed ammunition

<table>
<thead>
<tr>
<th>No. of rounds</th>
<th>Aircraft</th>
<th>Pilot</th>
<th>Fuel tank</th>
<th>Engine</th>
</tr>
</thead>
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<tr>
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<td>.3369</td>
<td>.3245</td>
<td>.1304</td>
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ACKNOWLEDGEMENTS

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REFERENCES


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APPENDIX

Let \((x_0, y_0)\) be the centre of the D-plan forming a right handed system of axis, \(S\)-axis, \(T\)-axis and \(OG\)-axis as shown in Fig. 1(a). The line \(GO\) is perpendicular to the \(S, T\) plane with direction cosines \((-I_0, -m_0, -n_0)\) where

\[
\begin{align*}
I_0 &= \cos A \cos E = \frac{x_0}{GO} \\
m_0 &= \sin A \cos E = \frac{y_0}{GO} \\
n_0 &= \sin E = \frac{z_0}{GO}
\end{align*}
\]

\(S\)-axis which will lie in the so-called azimuth plane will be normal to the elevation plane i.e., normal to the plane \(GOO'\)
Estimation of Aircraft Attrition Ground AD Weapons

Equation of the plane \( GOO' \) is
\[
I_1x + m_1y + n_1z = 0
\]
Since this plane passes through the three points \((0, 0, 0)\), \((x'_o, y'_o, z'_o)\), and \((x'_o, y'_o, 0)\), we have,
\[
x_0l_z + m_0y_z = 0
\]
\[
x_0l_z + m_0y_z + n_0z_z = 0
\]
Solving Eqns (3) & (4) along with
\[
l_z^2 + m_z^2 + n_z^2 = 1
\]
One gets the direction cosines of \( OS \)-axis as
\[
\left( \frac{m_0}{\sqrt{1 - n_0^2}}, \frac{-l_0}{\sqrt{1 - n_0^2}}, 0 \right)
\]
Similarly we get the directions cosines of \( OT \)-axis as
\[
\left( \frac{-n_0l_0}{\sqrt{1 - n_0^2}}, \frac{-m_0n_0}{\sqrt{1 - n_0^2}}, \frac{l_0}{\sqrt{1 - n_0^2}} \right)