Motion State of Fuel within Shell in Projection Acceleration Process

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**ABSTRACT**

The fuel-air explosive (FAE) warheads are charged with the liquid-solid mixture fuel. The fuel is different from conventional solid explosives in physical and mechanical properties. The mass centre of the charged fuel changes during projecting the projectile. In this study, a method to calculate the mass centre change of the charged fuel is suggested and the influence of this change on the projectile motion state in the projection process is discussed. The results show that in projection, the fuel mass centre varies with the projection acceleration and the deformation characteristics of the mixture fuel. The higher is the acceleration, the larger is the displacement of the mass centre. This displacement also increases with the compressibility of the fuel. It constitutes an influence on the state of motion for the whole projectile in the projection process, whose calculation approach is also proposed. The result provides a theoretical basis for the design of the FAE weapons.

**Keywords:** Projection acceleration process, fuel-air explosive, warheads, liquid-solid mixture fuel, FAE, FAE warheads, projectile motion, projection process, FAE weapons

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Fuel mass</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Fuel density</td>
</tr>
<tr>
<td>$G$</td>
<td>Inertia force</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius</td>
</tr>
<tr>
<td>$l$</td>
<td>Height or distance</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Cone angle</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$u_x$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Bulk deformation modulus</td>
</tr>
<tr>
<td>$M$</td>
<td>Moment</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
</tbody>
</table>

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1. INTRODUCTION

The fuel-air explosive (FAE) weapons have already been developed by many countries, or are being developed and improved. High-energy fuel is the central issue of developing the FAE weapons. The liquid-solid fuel mixture, with high density and energy, is widely used in new high-energy FAE warheads\(^1\,\!^2\). However, conventional warheads are generally charged with solid explosives. Compared with the solid explosives, the liquid-solid fuel mixture obviously has different physical state that necessarily has some influence of projection overloading, both on the mechanical responses and on the state of motion for the whole projectile. Whether these influences damage the other elements of projectile system, is the prerequisite to keep a normal action for the projectile.

In developing a particular FAE weapon system, it was found that the assembled fuse did not attain a good result. Some experts put forth the question that the bad result for the fuse may be caused by the different mechanical responses between the liquid-solid fuel mixture and the solid explosives in projection acceleration process. To answer this question, some analyses have been presented which are concerned with the mechanical responses of liquid-solid fuel mixture to projection acceleration. A theoretical method is given on the basis of an application example, which may provide a theoretical basis for correctly appraising the action of this kind FAE weapon system.

2. PROJECTION ACCELERATION PROCESS

In projection acceleration process, the fuel is compressed due to overloading, and its states before and after overloading are shown in Fig. 1. A relation between the acceleration and the inertia force exerted on the fuel is expressed by

\[
a_f m_f = G_f
\]

Figure 1. Displacement of fuel within shell in projection acceleration process: (a) before overloading and (b) after overloading

where \(m_f\), \(a_f\), and \(G_f\) denote the mass of
the charged fuel, the overloading acceleration of projection process, and the inertia force at fuel mass centre, respectively.

Assuming that the shell acceleration is \( a_c \), when the shell comes in contact with the fuel (there is no relative movement between the fuel and the shell), one may obtain \( a_f = a_c \). Only when \( a_f > a_c \), can the fuel be away from the shell (into vibration state). If \( a_f > a_c \), it must exhibit an action induced by the other pushes but not by the shell’s, which makes the acceleration of the fuel larger than that given by the shell. Actually, except the action from the shell, there is nothing from the others. Therefore, in the acceleration process, it is impossible to produce the fuel vibration within the shell.

3. ESTIMATION OF MASS CENTRE CHANGE

The fuel makes contact with the inner wall of the shell without friction. In the projection process, the total force exerted on the cross-section at \( x \) shown in Fig. 1, is given by

\[
G_x = \frac{1}{3} \pi x \left( r_0^2 + r_r^2 + r_x r_0 \right) \rho_f a \quad (x \leq l_{\text{curve}})
\]

\[
G_x = \left[ \frac{1}{3} \pi l_{\text{curve}} \left( r_0^2 + r_r^2 + r_r r_0 + \pi r^2 (x - l_{\text{curve}}) \right) \rho_f a \right] \quad (x > l_{\text{curve}})
\]

where \( a \) is the overloading acceleration.

The mean pressure on the cross-section at a distance \( x \) from the top fuel is:

\[
\sigma_x = \frac{x \rho_f a}{3} \left( \frac{r_0^2}{r_r^2} + \frac{r_r}{r_x} + 1 \right) \quad (x \leq l_{\text{curve}})
\]

\[
\sigma_x = \rho_f a \left[ \frac{l_{\text{curve}}}{3} \left( \frac{r_0^2}{r_r^2} + \frac{r_0}{r} + 1 \right) + x - l_{\text{curve}} \right] \quad (x > l_{\text{curve}})
\]

where \( r_x \approx r_0 + x \tan(\alpha/2) \), \( \alpha \) is the cone angle of the curved part of warhead (approx. value).

Since the warhead structure is symmetrical, the fuel deformation may be approximately treated as a one-dimensional problem in projection overloading process. On a certain cross-section of the fuel, the mean strain is:

\[
\varepsilon_x = \frac{du_x}{dx}
\]

On the basis of elasto-plastic theory analysis\(^1\), the relationship between stress and strain can be expressed as (Appendix 1)

\[
\sigma_x (1 + 2k) = 3k_v \varepsilon_x
\]

Further, the displacement is derived as

\[
u_x = \left( \frac{1 + 2k}{3k_v} \right) \frac{\rho_f a}{3} \left[ \frac{1}{3} \left( k_1 + k_2 + k_3 \right) \right] + k_4 (x \leq l_{\text{curve}})
\]

\[
u_x = \left( \frac{1 + 2k}{3k_v} \right) \frac{\rho_f a}{3} \left[ \frac{l_{\text{curve}}}{3} \left( \frac{r_0^2}{r^2} + \frac{r_0}{r} + 1 \right) \right] \left( l - x \right) + \frac{1}{2} (l^2 - x^2) - l_{\text{curve}} (l - x) \quad (x > l_{\text{curve}})
\]

where \( k \approx 0.35 \), and parameters \( k_1, k_2, k_3 \) and \( k_4 \) are determined by the relations given in the Appendix 1.

When \( x = 0 \), the displacement reaches its maximum.

\[
u_x = \left( \frac{1 + 2k}{3k_v} \right) \frac{\rho_f a}{3} \left[ \frac{l_{\text{curve}}}{3} \left( \frac{r_0^2}{r^2} + \frac{r_0}{r} + 1 \right) \right] \left( l + \frac{1}{2} l^2 - l_{\text{curve}}^2 \right)
\]

For an FAE rocket projectile, the related parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td>539 mm</td>
</tr>
<tr>
<td>( l_{\text{curve}} )</td>
<td>246 mm</td>
</tr>
<tr>
<td>( r )</td>
<td>60 mm</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>30 mm</td>
</tr>
<tr>
<td>( k_v )</td>
<td>1.09 \times 10^6 MPa</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>1460 kg/m³</td>
</tr>
</tbody>
</table>
The fuel mass centre is located at 227 mm away from the bottom of the fuel. When the overloading acceleration \( a \) is equal to 600 mm/s\(^2\), the relative displacements of the fuel at different positions in projection loading process are calculated and plotted in Fig. 2. It can be seen from the theoretical results that the maximum relative displacement at the top fuel is equal to 0.55 mm, while the minimum one at the bottom fuel is 0. At the mass centre, \( x = 312 \) mm, its relative displacement is \( u_x (x = 312 \text{ mm}) = 0.28 \text{ mm} \).

The moment of the charged fuel to the mass centre of whole projectile becomes

\[
M = m_f g l_{f0} \cos \beta
\]

(9)

where \( l_{f0} \) is the distance from the mass centre of the charged fuel to that of the whole projectile after it changes due to overloading and \( \beta \) is the transient projection angle of the body projectile. The mass centre changes in such a short duration that the projection angle does not change largely. Therefore, \( \beta_0 \approx \beta \), and then the moment increment before and after the change of fuel mass centre is:

\[
\Delta M = m_f g \Delta l_{f0} \cos \beta
\]

(10)

where

\[
\Delta l_{f0} = u_x (x = 312 \text{ mm}) = 0.28 \text{ mm}
\]

The change of moment causes the body projectile to tilt and forms a slight rotation around its mass centre. The angular acceleration of this rotation has the following relationship with the moment increment:

\[
\Delta M = J \frac{d\omega}{dt}
\]

(11)

where \( J \) is the angular acceleration of the body projectile and \( \omega \) is the angular velocity of rotation induced by the change of fuel mass centre. Integrating above equation yields:

\[
\omega = \Delta M t / J
\]

(12)

where \( t \) denotes the time corresponding to the change of fuel mass centre, and may be determined by the following method:

At the beginning of projection, the whole projectile exerts an action on the fuel through the bottom cover of the warhead, which produces stress waves in the fuel media. Since the fuel is in a state of liquid-solid mixture, the part behind the wavefront can be regarded impressible. That is to say, in the process as the stress waves propagate from the bottom cover of the warhead to the top of the
charged fuel, the fuel becomes impressible. However, the change of fuel mass centre just finishes on time \( t \) given by

\[
t = \frac{l}{c}
\]  

(14)

where \( c \) is the speed of stress wave propagation in fuel media and is expressed as

\[
c = \frac{d\sigma_s}{\sqrt{\rho_j d e_x}}
\]  

(15)

Substitution of Eqn (5) into Eqn (15) yields:

\[
c = \frac{3k_j}{\sqrt{\rho_j (1 + 2k)}},
\]  

(16)

For the projectile mentioned above, it is calculated that \( c = 659 \) m/s and then from Eqn (14), \( t = 0.00082 \) s. During the change of fuel mass centre in projection overloading process, the body projectile rotates slightly, and the linear velocity of tilt at the fuse is given as

\[
v_{\text{fuse}} = \omega l_{\text{fuse}}
\]  

(17)

where \( l_{\text{fuse}} \) is the distance between the fuse and the mass centre of whole projectile. For the given FAE rocket projectile, \( l_{\text{fuse}} = 1.487 \) m. Using Eqns (11), (13) and (17), the linear velocity of tilt at the fuse is obtained as

\[
v_{\text{fuse}} = l_{\text{fuse}} m_j g \Delta l f \cos \beta / J
\]  

(18)

Now it is known that \( J = 45 \) kg \( \times \) m\(^2\). Substituting all of the parameters into the above equation, one obtains \( v_{\text{fuse}} = 0.306 \) \( \mu \)m/s. Such a velocity cannot damage the clock structure of the fuse. It shows that the assembled fuse can attain good result, which provides a theoretical basis for predicting a normal action of this projectile. Using the suggested approach, more calculations can be done. The results, as one aspect of theoretical guidelines, have been proved by the experiments and are very useful for the design of certain FAE weapons.

5. CONCLUSION

The FAE warheads charged with the liquid-solid mixture fuel have different mechanical responses from the warheads with traditional solid explosives in projection acceleration process. The mass centre of the charged fuel changes in projection acceleration process. It moves backwards to make the distance from the mass centre of the charged fuel to that of the whole projectile small, and the displacement depends on the projection overloading acceleration. The higher the acceleration, the larger the displacement. Meanwhile, the displacement is also related to the deformation properties of the fuel. When the liquid-solid mixture fuel has large compressibility, the backward displacement increases. An FAE weapon is taken as an application example and the backward displacement of the fuel mass centre is calculated and discussed. It shows that the result thus obtained provides a theoretical basis for the design of this FAE weapon.

REFERENCES


