Optimal Decision Procedure to Declare Human Fatigue and Prediction of 3-Mean Repair Times with One Repairman

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ABSTRACT

This paper gives an optimum procedure to decide whether fatigue is present in a repairman and if it is true prediction of the 3-mean repair times of the three failed systems.

Keywords: Mean repair times, optimal decision procedure, human fatigue

NOMENCLATURE

MRT Mean repair time
A, B, C Three support systems for which MRTs are needed.
η1, η2 Multiplying constant for t3
α Type I error
β Power function
t1, t2, t3 3-MRTs of support systems A, B, C, respectively
zb A random variable which takes values between 1 or 0, depending on whether fatigue of repairman exists or not.
S \[ S = \sum_{i=1}^{M} Z_i \]
M Total number of MRTs for which repair completions are over, i.e., \( M = 3 \)

1. INTRODUCTION

Consider a multisystem model \((N + 3)\), where \(N\) is the number of systems, in the major system which has three additional systems called support systems. These support systems are denoted by A, B and C. The repair of A, B and C is taken by a single repairman in the order A, B, C. The mean repair completion times for the three support systems A, B and C (3-MRTs) are denoted by symbols \( t_1, t_2\) and \( t_3\). The human fatigue problems enter the 3-MRT process since there is one repairman only. In such cases, it is easy to think and formulate the whole problem as problems in the decision-making and decision procedures.

2. DECISION-MAKING PROBLEM

The decision-making problem is formulated as

\( H_0 \) No fatigue exists in the repairman
\( H_1 \) Fatigue exists in the repairman, or in other words \( t_1 < t_2 < t_3 \)

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2.1 Decision Procedure

**Step 0** Choose \( \alpha \) (type I error)

**Step 1** If \( t_1 < t_2 < t_3 \), then \( Z_b = 1 \), otherwise \( Z_b = 0 \)

**Step 2** Compute \( S = \sum_{b=1}^{M} Z_b \)

**Step 3** Put \( M = 1, 2, 3 \) in summation \( S \) successively

**Step 4** Determine \( k \), such that \( binf(k;1/6,3) = \alpha \)

**Step 5** Reject \( H_0 \) iff \( S = 2 \) and if \( S \neq 1 \).

2.2 Analysis

Define \( h(t_1) = \int_{t_1}^{t_2} \int_{t_2}^{t_3} \exp f(t) \exp f(t) \) (1)

It is to be remembered that Eqn (1) is true regardless of \( H_0 \) and \( H_1 \).

It can be easily shown that

\[
P_\alpha = Pr \{ Z_b = 1 \; \text{regardless of} \; H_0, H_1 \} = E \{ h(t_1) \} \tag{2}
\]

where \( E \) is the expected value.

Now

\[
Pr \{ S = r; H_0 \; \text{is true} \} = binm(r;1/6,3) \tag{3}
\]

\[
Pr \{ S = r; H_1 \; \text{is true} \} = binm(r;P_1,3) \tag{4}
\]

It can easily be proved that \( P_1 < 1/6 \) under \( H_1 \).

Hence, from Neyman-Pearson lemma, decision procedure [Eqns (1) to (4)] is optimum. The performance of decision rule [Eqn (3)] is evaluated for two illustrations. The power function is:

\[
\beta = binf(3; P_1; 3) \tag{5}
\]

The Eqn (5) can be illustrated for two cases by putting the expression for

\[
F(t_1) = F(t_3 - 2\Delta) \\
F(t_2) = F(t_3 + \Delta) \\
F(t_3) = \exp f(t)
\]

The second illustration can be given for

\[
F(t_1) = F(\eta_1 t_3), \; \eta_1 < 1 \\
F(t_2) = F(\eta_2 t_3), \; \eta_2 < 1 \\
F(t_3) = \exp f(t)
\]

3. CONCLUSION

Two problems have been analysed. The first problem is a decision procedure to decide whether human fatigue is present in the repairman, when \( M = 3 \). By setting probabilities \( P_0, P_1 \) and \( P_2 \) equal to \( t_1, t_2 \) and \( t_3 \), respectively, one can easily predict \( t_1, t_2 \) and \( t_3 \) (3-MTRs) after deciding that there is human fatigue in the repairman.

Type I error and \( M \) as \( \alpha = 0.05 \) and \( M = 3 \), respectively can be chosen. The value of \( k \) in the relation \( S \leq k \) should be chosen as one among any values for \( k = 2 \) or \( k = 3 \).

REFERENCES