Fault Detection in Systems–A Fuzzy Approach

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**ABSTRACT**

The task of fault detection is important when dealing with failures of crucial nature. After detection of faults in a system, it is advisable to suggest maintenance action before occurrence of a failure. Fault detection may be done by observing various symptoms of the system during its operational stage. Sometimes, symptoms cannot be quantified easily but can be expressed in linguistic terms. Since linguistic terms are fuzzy quantifiers, these can be represented by fuzzy numbers. In this paper, two cases have been discussed, where a fault likely to affect a particular system/systems, is detected. In the first case, this is done by means of a compositional rule of inference. The second case is based on modified similarity measure. For both these cases, linguistic terms have been expressed as trapezoidal fuzzy numbers.

**Keywords:** Fuzzy number, fuzzy relation, occurrence indication relation, conformability indication relation, FMEA, fault detection, failure mode and effect analysis, modified similarity measure, MSM

1. **INTRODUCTION**

The system-failure engineering is primarily concerned with failures and related problems which may include reliability, safety, security, etc. Failure is an unavoidable phenomenon which can be observed in various circumstances, such as space shuttle explosion, nuclear reaction accident, airplane crash, chemical plant leak, etc. The causes of failure are diverse and can be physical human, logical, and even financial. When an engineer designs a component or a system or develops a process, his main objective in such a design is to prevent unacceptable failures to reach the customer. One of the valuable tools in both safety and quality control of systems is the failure mode and effect analysis (FMEA) which has been extensively used in automotive industries, and most manufacturers stipulate the FMEA as the requisite method for ensuring that the quality is built into the design and manufacturing processes of new products.

When dealing with failures of crucial nature, the task of fault detection becomes very important. By fault, it means a system state which deviates from the desired system state. The task of fault
detection may include detecting a fault by observing various symptoms of the system during its operation, detecting where the fault has occurred, and assessing the damage. However, the observed symptoms are frequently vague. For example, in identifying the leaking location in the cooling system of a boiling water reactor (BWR), some of the symptoms observed (according to Pouliiezos and Stavrakakis') are the pressure decrease in the main streamline; high temperature in the building and an increase in the flow rate of the sump in the building. Fuzzy methodology is a natural tool for incorporating symptoms of this kind. Also, it is difficult to quantify such symptoms. Nevertheless, it may be easy to express these in terms of linguistic phrases, e.g., often occurred, seldom occurred, never occurred, etc., which are vague in nature. Because of this vagueness and uncertainty in the system-fault relationship, the usage of fuzzy methodology needs to be explored.

After detection of faults in a system, it is advisable to suggest maintenance action before occurrence of the failure. In the maintenance of systems, repair maintenance and preventive maintenance are the two important constituents. Preventive maintenance is important for systems, wherein failure is of critical nature, e.g., defence systems, nuclear power plants, human systems, etc. Repair maintenance is needed to run the system in the most efficient manner and to maximise the expected profit to the possible extent. Without proper and timely maintenance, even highly reliable systems may not remain in dependable state for long periods, as expected.

According to Sorsa and Koivo, the problem of fault detection could be solved by any of the three methods: (i) the estimation method, (ii) the rule-based reasoning, and (iii) the pattern-recognition technique. Frank used the estimation method for fuzzy residual generation.

Tsukamoto, et al., Asse, and Bastani, et al. have used rule-based reasoning for fault detection problems. Under this, a set of fuzzy relational inequalities is used to describe the intensity of the deterministic relationships existing between the faults (viewed as causes) and the determined symptoms (viewed as effects). If $S$ is the vector of fuzzy symptoms, $F$ is the vector of fuzzy faults, and $R$ is a fuzzy relational matrix describing the intensity of the causal interdependencies existing between the faults and the determined symptoms, then

$$S = F \circ R$$

An alternative idea using a direct symptom-driven fuzzy reasoning strategy was given by Sanchez. He used a heuristic symptoms-faults interdependency $R'$ instead of the casual faults-symptoms relationships, that is

$$F = S \circ R'$$

Though rule-based reasoning has been used by many workers, not much has been done in this field when symptoms are expressed in linguistic phrases, which, in turn, can be expressed as fuzzy numbers.

Peltier and Dubuiisson showed that pattern-recognition techniques could be used to deal with fault detection problems in cars.

In the present study, the following two methods have been presented using trapezoidal fuzzy numbers:

- When a fuzzy set of symptoms is observed in different systems and documentation relating symptoms with faults is available, the fuzzy set of possible faults for different systems can be inferred by means of compositional rule of inference.

- When a fuzzy set of symptoms is observed in a particular system and the normal range of severity of symptoms that can be expected with different faults, are given. Help of a modified similarity measure is taken to determine the distance between the observed symptoms and the different symptoms associated with the faults.

Before the formal solution/methodologies are presented, some important concepts and definitions from a fuzzy set theory are given.

2. DEFINITIONS

- **Fuzzy Relations:** A fuzzy set defined on the cartesian product of crisp sets $X_1, X_2, ..., X_n$ is
known as a fuzzy relation. Here, the tuples \((x_1, x_2, \ldots, x_n)\) may have varying degrees of membership within the relation. Any relation between two sets \(X\) and \(Y\) is known as a binary relation and is usually denoted by \(R(X, Y)\).

- **Composition of Two Binary Relations:** Consider two binary relations \(P(X, Y)\) and \(Q(Y, Z)\). The composition of these two relations is denoted by

\[
R(X, Z) = P(X, Y) \circ Q(Y, Z)
\]  

and is defined as a subset \(R(X, Z)\) of \(X \times Z\) such that \((x, z) \in R\) if and only if there exists at least one \(y \in Y\), such that \((x, y) \in P\) and \((y, z) \in Q\).

The composition operation for fuzzy relations can take several forms. One of the forms of this operation on fuzzy relations is the maximum product composition. It is denoted by \(P(X, Y) \circ Q(Y, Z)\) and defined by

\[
\mu_{P\circ Q}(x, z) = \max \{ \mu_P(x, y) \cdot \mu_Q(y, z) \mid y \in Y \text{ for all } x \in X \text{ and } z \in Z \}
\]  

(2)

An \(\alpha\)-cut of a fuzzy set \(A\) is a crisp set \(A_\alpha\) that contains all the elements of the universal set \(X\) having a membership grade in \(A\) greater than or equal to the specified value of \(\alpha\). Thus

\[
A = \{ x \in X \mid \mu_A(x) \geq \alpha \}, \quad 0 \leq \alpha \leq 1
\]  

(3)

- **Fuzzy Number:** A convex and normalised fuzzy set defined on \(R\) whose membership function is piecewise continuous is called a fuzzy number. A fuzzy set is called normal when at least one of its elements attains the maximum possible membership grade, i.e.,

For all \(x \in R\), \(V_x \mu_A(x) = 1\)

where \(V\) stands for maximum.

A fuzzy set is convex if and only if each of its \(\alpha\)-cut is a convex set. Equivalently, one may say that a fuzzy set \(A\) is convex if and only if

\[
\mu_A(\lambda r + (1 - \lambda)s) \geq \min \{ \mu_A(r), \mu_A(s) \}
\]  

for all \(r, s \in R^n\) and all \(\lambda \in [0,1]\).

- **Trapezoidal Fuzzy Numbers:** A fuzzy number \(A\) is a trapezoidal fuzzy number denoted by \((a_1, a_2, a_3, a_4)\), \(a_1 \leq a_2 \leq a_3 \leq a_4\) if its membership function \(A\) is given by

\[
\mu_A(x) = \begin{cases} 
0 & , x < a_1 \\
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
1 & , a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\
0 & , x > a_4 
\end{cases}
\]  

(5)

When \(a_2 = a_3\), then trapezoidal fuzzy number becomes a triangular fuzzy number.

A trapezoidal fuzzy number can also be characterised by the interval of confidence at level \(\alpha\).

Thus, for all \(\alpha \in [0,1]\)

\[
\bar{A}_\alpha = [(a_2 - a_1) \alpha + a_1, -(a_4 - a_3) \alpha + a_4]
\]  

(6)

where an interval of confidence in \(R\) is an ordinary subset of \(R\) which represents a type of uncertainty.

- **Associated Ordinary Number:** If \(\bar{A} = (a_1, a_2, a_3, a_4)\) is a trapezoidal fuzzy number, then its associated ordinary number is given by

\[
\bar{a} = \frac{a_1 + a_2 + a_3 + a_4}{4}
\]  

(7)

Multiplication, maximum and minimum operations on trapezoidal fuzzy numbers do not necessarily give a trapezoidal fuzzy number. However, one can approximate the results of these operations by a trapezoidal fuzzy number\(^9\).

- **Maximum of Trapezoidal Fuzzy Number:** If \(\bar{A} = (a_1, a_2, a_3, a_4)\) and \(\bar{B} = (b_1, b_2, b_3, b_4)\), then
an approximate trapezoidal fuzzy number will be
\[
\text{AVB} - (a_1 V b_1, a_2 V b_2, a_3 V b_3, a_4 V b_4)
\] (8)

- **Multiplication of Trapezoidal Fuzzy Number with an Ordinary Number:**

If \( \tilde{A} = (a_1, a_2, a_3, a_4) \), then the interval of confidence is:
\[
\tilde{A}_a = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]
\]

Multiplying by an ordinary number, one gets:
\[
b\tilde{A}_a = b [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha] \\
[ba_1 + b(a_2 - a_1)\alpha, ba_4 - b(a_4 - a_3)\alpha]
\]

when \( \alpha = 0 \) and \( \alpha = 1 \), then an approximate value is obtained as
\[
[ab_1, ab_2, ab_3, ab_4]
\] (9)

Normalised distance \( \delta (\tilde{A}_i, \tilde{A}_j) \) between \( \tilde{A}_i \) and \( \tilde{A}_j \) is given by
\[
\delta(\tilde{A}_i, \tilde{A}_j) = \frac{[LD(\tilde{A}_i, \tilde{A}_j) + RD(\tilde{A}_i, \tilde{A}_j)]}{2(\beta_2 - \beta_1)}, 0 \leq \delta \leq 1
\] (10)

If the interval of confidence of two trapezoidal fuzzy numbers \( \tilde{A}_i \) and \( \tilde{A}_j \) are respectively
\[
\tilde{A}_i = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4]
\]
\[
\tilde{A}_j = [(b_2 - b_1)\alpha + b_1, -(b_4 - b_3)\alpha + b_4]
\]
then
\[
LD(\tilde{A}_i, \tilde{A}_j) = [(a_2 - a_1)\alpha + a_1, -(b_2 - b_1)\alpha - b_2]
\]
\[
RD(\tilde{A}_i, \tilde{A}_j) = [-(a_4 - a_3)\alpha + a_4, (b_4 - b_3)\alpha - b_4], \alpha \in [0, 1]
\] (11)

where \( \beta_2 \) and \( \beta_1 \) are the arbitrary values at the right and left chosen in such a way that
\[
\beta_2 - \beta_1 \geq \frac{LD(\tilde{A}_i, \tilde{A}_j) + RD(\tilde{A}_i, \tilde{A}_j)}{2}
\]

3. **METHODOLOGY**

3.1 Case Study I

**Determination of the Fuzzy Set of Possible Faults for Different Systems using Compositional Rule of Inference**

In this case, faults have been detected by rule-based reasoning, i.e., by means of compositional rule of inference, when documentation relating different symptoms to different faults is available. It is assumed that the symptoms and the faults are specified in linguistic terms. Linguistic terms being fuzzy quantifiers can be represented in fuzzy logic by fuzzy numbers. These are then manipulated in terms of operations of fuzzy arithmetic.

Let \( S \) be a crisp universal set of all symptoms, \( F \) be a crisp universal set of all faults, and \( P \) be the universal set of all components/systems.

A fuzzy relation \( M_s \) specifying the degree of presence of symptoms for different systems is given. Based on this information, the fuzzy set of possible faults has to be determined for different systems by the compositional rule of inference. This will be done by constructing two types of relations, an occurrence relation and a conformability relation. An occurrence relation gives the frequency of appearance of a symptom with a particular fault, whereas a conformability relation describes the discriminating power of the symptom to confirm a particular fault. Conclusions can be drawn from these two indication relations. The basic steps for this method are:

**Step 1.** Since documentation concerning relations of the symptoms and the faults involves statements with linguistic terms, express these terms as trapezoidal fuzzy numbers.

**Step 2.** Based on this documentation, construct matrices of relations

- \( M_o \) on the set \( S \times F \), where \( \mu_{M_o}(s, f) \) (\( s \in S, f \in F \)) indicates the frequency of occurrence of symptoms with fault \( f \).
- \( M_c \) on the set \( S \times F \) where \( \mu_{M_c}(s, f) \) corresponds to the degree to which symptom \( s \), confirms the presence of fault \( f \).
The matrix of relation, $M_s$ on the set $P \times S$, where membership grade $\mu_{M_s}(p,s)$ ($p \in P$, $s \in S$) indicate the degree to which the symptom $s$ is present in a system $p$, is given. (It is supposed that this is obtained on observation of the systems).

**Step 3.** Using relations $M_o$, $M_c$, and $M_s$, calculate the indication relations defined on the set $P \times F$ of systems and faults, the relations being occurrence indication relation ($MP_{s_1}$) and conformability indication relation ($MP_{s_2}$), where

$$MP_{s_1} = M_s \odot M_o$$

and

$$MP_{s_2} = M_s \odot M_c.$$

Here $\odot$ stands for the maximum product composition of two binary relations.

**Step 4.** Draw different types of conclusions regarding the presence of faults in the systems, from the derived relations. For instance, one may make a confirmed diagnosis of a fault for a symptom or strongly confirmed diagnosis or excluded diagnosis, and so on.

### 3.2 Case II

**Determination of the Most Likely Fault Affecting the System by a Similarity Measure Called Modified Similarity Measure.**

In this case, faults have been detected by fuzzy clustering. The method uses some form of distance measure to determine the similarity between observed attributes (symptoms) and those present in the existing diagnostic clusters. The method described is a modified version of the method employed by Esogbue and Elder. A measure called modified similarity measure (MSM) has been developed for determining the most likely fault.

Let there be a single system which displays certain symptoms of irregularity while functioning. The observer makes a note of the symptoms in terms of linguistic phrases like a particular symptom is very strongly present, not present, etc. Also, each of the fault-symptom relation is described by trapezoidal fuzzy numbers. The importance of the symptoms for detecting faults are given by a matrix depicting weights of relevance. Then, for finding out which fault is most likely to affect the system, a measure called the MSM has been devised. The steps in this method are briefly outlined as follows:

**Step 1.** Convert the observed symptoms in the system in terms of trapezoidal fuzzy number.

**Step 2.** Write down the interval of confidence for all the trapezoidal fuzzy numbers.

**Step 3.** Find the normalised distance between the system's symptoms with the respective symptoms of the faults.

**Step 4.** Find MSM. If $\mu_w(s_i, f_j)$ denotes the weight of the symptom $s_i$ for fault $f_j$, then the MSM is given by

$$D_{x,f_j} = \left[ \sum_{i=1}^{n} (\mu_w(s_i, f_j) \cdot \delta_{x,s_i} f_{ij})^2 \right]^{1/2}$$

where $\delta_{x,s_i} f_{ij}$ is the normalised distance between the system $x$'s symptoms ($s_1, s_2, ..., s_n$) and the symptoms of fault, $f_j$ ($j = 1, ..., p$).

**Step 5.** The most likely fault for the system is the one for which the similarity measure has the minimum value.

### 4. ILLUSTRATIONS

#### 4.1 Case I

Let there be four symptoms $s_1, s_2, s_3,$ and $s_4$ and two types of faults, $f_1$ and $f_2$. Assume that the analyst has knowledge about the relation between $s_1, s_2, s_3,$ and $s_4$ with $f_1$ and $f_2$ and are given as follows:

- $s_1$ never occurs with fault $f_1$ and never confirms $f_1$. It often occurs with fault $f_2$.  

"
s₂ occurs seldom with f₁ and very seldom confirms f₂. It always occurs with f₂ and always confirms f₂.

s₃ occurs very often with f₁ but seldom confirms f₁. It never occurs with f₂ and never confirms f₂.

s₄ occurs very seldom with f₂ but often confirms f₂.

All missing relational pairs of symptoms and faults are assumed to be unspecified. Since the terms always, often, seldom, etc. are fuzzy quantifiers, one can represent these by trapezoidal fuzzy numbers.

The linguistic terms and the corresponding quadruple representations of fuzzy numbers are shown in Table 1. However, these values are only suggestive.

Table 1. Linguistic terms and their corresponding fuzzy numbers

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always</td>
<td>(0.9,1,1,1)</td>
</tr>
<tr>
<td>Very often</td>
<td>(0.6, 0.8, 0.8, 1)</td>
</tr>
<tr>
<td>Often</td>
<td>(0.5, 0.7, 0.7, 1)</td>
</tr>
<tr>
<td>Unspecific</td>
<td>(0,0,1,1)</td>
</tr>
<tr>
<td>Seldom</td>
<td>(0,0.3,0.3,0.5)</td>
</tr>
<tr>
<td>Very seldom</td>
<td>(0, 0.1, 0.1, 0.3)</td>
</tr>
<tr>
<td>Never</td>
<td>(0, 0, 0, 0.3)</td>
</tr>
</tbody>
</table>

Matrices of relations Mᵦ and Mₑ are given by

\[
\begin{bmatrix}
    f_1 & f_2 \\
    s_1 (0, 0, 0, 0.3) & (0.5, 0.7, 0.7, 1) \\
    M_\epsilon = s_2 (0.0, 1.0, 1.0, 0.3) & (0.9, 1.1, 1) \\
    s_3 (0.0, 0.3, 0.3, 0.5) & (0.0, 0.0, 0.3) \\
    s_4 (0, 0, 1, 1) & (0.0, 1.0, 1.0, 0.3)
\end{bmatrix}
\]

Assume that a fuzzy relation Mₓ specifying the degree of presence of symptoms s₁, s₂, s₃, and s₄ for four systems p₁, p₂, p₃, and p₄ are given:

\[
\begin{bmatrix}
    s_1 & s_2 & s_3 & s_4 \\
    p_1 (0.3, 0.7, 0.6, 1) \\
    M_\epsilon = p_2 (0.2, 0.9, 0.3, 0.7) \\
    p_3 (0.5, 0.4, 0.8, 0.1) \\
    p_4 (1.0, 0.6, 0.6, 0.6)
\end{bmatrix}
\]

Since one has the binary relations Mᵦ defined on S x F, Mₑ defined on S x F and Mₓ defined on P x S, with a common set S, one has to find the composition of Mᵦ and Mₑ for the occurrence indication relation Mᵦₑ and the composition of Mₑ and Mₓ for the conformability indication relation Mₑₓᵦ.

Now

\[
\begin{bmatrix}
    0.3 & 0.7 & 0.6 & 1 \\
    0.2 & 0.9 & 0.3 & 0.7 \\
    0.5 & 0.4 & 0.8 & 0.1 \\
    1 & 0 & 0.6 & 0.1
\end{bmatrix} \odot
\begin{bmatrix}
    (0,0,0,0.3) & (0.5,0.7,0.7,1) \\
    (0.0,3.0,3.0,5) & (0.9,1.1,1) \\
    (0.6,0.8,0.8,1) & (0,0,0,0.3) \\
    (0,0,1,1) & (0.0,1.0,1.0,0.3)
\end{bmatrix}
\]

The first value in the matrix Mᵦₑₓᵦ₁ is calculated as follows:

\[
\text{Max} \left[ \begin{bmatrix} 0.3(0, 0, 0, 0.3), 0.7(0, 0.3, 0.3, 0.5), 0.6(0.6, 0.8,0.8,1), 1(0, 0, 1, 1) \end{bmatrix} \right]
\]

Using Eqns (8) and (7), one gets the approximate values for multiplication and maximum.
Then:

\[
\begin{bmatrix}
(0, 0, 0, 0.09) & (0.15, 0.21, 0.21, 0.3)
\end{bmatrix}
\begin{bmatrix}
V(0.021, 0.21, 0.35) & V(0.63, 0.7, 0.7, 0.7)
\end{bmatrix}
\begin{bmatrix}
V(0.36, 0.48, 0.48, 0.6) & V(0, 0, 0, 0.18)
\end{bmatrix}
\begin{bmatrix}
V(0, 0, 1, 1) & V(0.01, 0.1, 0.3)
\end{bmatrix}
\begin{bmatrix}
(0, 0, 0, 0.06) & (0.10, 0.14, 0.14, 0.2)
\end{bmatrix}
\begin{bmatrix}
V(0.027, 0.27, 0.45) & V(0.81, 0.9, 0.9, 0.9)
\end{bmatrix}
\begin{bmatrix}
V(0.18, 0.24, 0.24, 0.3) & V(0, 0, 0, 0.09)
\end{bmatrix}
\begin{bmatrix}
V(0, 0, 0.7, 0.7) & V(0.007, 0.07, 0.21)
\end{bmatrix}
\begin{bmatrix}
(0, 0, 0, 0.15) & (0.25, 0.35, 0.35, 0.5)
\end{bmatrix}
\begin{bmatrix}
V(0.012, 0.12, 0.20) & V(0.36, 0.4, 0.4, 0.4)
\end{bmatrix}
\begin{bmatrix}
V(0.48, 0.64, 0.64, 0.8) & V(0, 0, 0, 0.24)
\end{bmatrix}
\begin{bmatrix}
V(0, 0, 0.1, 0.1) & V(0.001, 0.01, 0.03)
\end{bmatrix}
\begin{bmatrix}
(0, 0, 0, 0.3) & (0.5, 0.7, 0.7, 1)
\end{bmatrix}
\begin{bmatrix}
V(0, 0, 0, 0) & V(0, 0, 0, 0)
\end{bmatrix}
\begin{bmatrix}
V(0.36, 0.48, 0.48, 0.6) & V(0, 0, 0, 0.18)
\end{bmatrix}
\begin{bmatrix}
V(0, 0, 0.6, 0.6) & V(0.006, 0.06, 0.18)
\end{bmatrix}
\]

Similarly, calculating for \(MP_{r_2}\), one has:

\[
\begin{bmatrix}
\begin{bmatrix}
f_1 & f_2
\end{bmatrix}
\begin{bmatrix}
p_1 & p_2
\end{bmatrix}
\begin{bmatrix}
p_3 & p_4
\end{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
(0.021, 0.11) & (0.63, 0.7, 0.7, 1)
\end{bmatrix}
\begin{bmatrix}
(0.09, 0.7, 0.7, 0.9)
\end{bmatrix}
\begin{bmatrix}
(0.24, 0.24, 0.4)
\end{bmatrix}
\begin{bmatrix}
(0.18, 0.6, 0.6, 0.6)
\end{bmatrix}
\begin{bmatrix}
(0.36, 0.4, 0.4, 0.5)
\end{bmatrix}
\begin{bmatrix}
(0.3, 0.42, 0.42, 0.6)
\end{bmatrix}
\begin{bmatrix}
(0.5450, 0.7575)
\end{bmatrix}
\begin{bmatrix}
(0.3725, 0.8775)
\end{bmatrix}
\begin{bmatrix}
(0.2200, 0.4400)
\end{bmatrix}
\begin{bmatrix}
(0.3450, 0.4350)
\end{bmatrix}
\]

For drawing conclusions, one finds the associated ordinary numbers of the trapezoidal fuzzy numbers (also called defuzzified values):

\[
\begin{bmatrix}
f_1 & f_2
\end{bmatrix}
\begin{bmatrix}
p_1 & p_2
\end{bmatrix}
\begin{bmatrix}
p_3 & p_4
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.36, 0.48, 0.48, 0.6
\end{bmatrix}
\begin{bmatrix}
0.5, 0.6, 0.6, 0.6
\end{bmatrix}
\]

From the defuzzified values of \(MP_{r_1}\), it is seen that fault \(f_1\) is occurring quite strongly in the system \(p_1\), and to some extent, in the system \(p_3\). Also fault \(f_2\) is occurring strongly in systems \(p_1\) and \(p_3\) and very strongly in the system \(p_2\). A fault can be said to be totally confirmed for the system \(p\) if \(MP_{r_1}(p,f) = 1\). While looking at \(MP_{r_1}\), one observes that this cannot be said for any of the systems, one can only say that fault \(f_2\) is strongly confirmed for the system \(p_2\) and more or less confirmed for the system \(p_1\).

4.2 Case II

Let there be a system \(x\) which indicates symptoms \(s_1, s_2, s_3,\) and \(s_4\). The levels of severity of these symptoms are shown in Table 2.

<table>
<thead>
<tr>
<th>Symptoms</th>
<th>Level of severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1)</td>
<td>Almost absent severity</td>
</tr>
<tr>
<td>(s_2)</td>
<td>Very high severity</td>
</tr>
<tr>
<td>(s_3)</td>
<td>Moderate Severity</td>
</tr>
<tr>
<td>(s_4)</td>
<td>High Severity</td>
</tr>
</tbody>
</table>
On the basis of the symptoms, one has to determine a diagnosis for this system from among the three possible faults $f_1$, $f_2$, and $f_3$.

The normal range of severity of each of the four symptoms, that can be expected in a system with the fault, is given in terms of trapezoidal fuzzy numbers (Table 3).

**Table 3. Normal range of severity of different symptoms**

<table>
<thead>
<tr>
<th>Fault</th>
<th>Symptoms</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>(0,0,0,0.2)</td>
<td>(0.6,0.7,0.7,1)</td>
<td>(0.5,0.6,0.6,0.7)</td>
<td>(0,0,0,0)</td>
<td></td>
</tr>
<tr>
<td>$f_2$</td>
<td>(0,0,0,0)</td>
<td>(0.9,0.95,0.95,1)</td>
<td>(0.3,0.7,0.7,1)</td>
<td>(0.2,0.3,0.3,0.4)</td>
<td></td>
</tr>
<tr>
<td>$f_3$</td>
<td>(0,0,0,0.3)</td>
<td>(0,0,0,0)</td>
<td>(0.7,0.8,0.8,0.9)</td>
<td>(0,0,0,0)</td>
<td></td>
</tr>
</tbody>
</table>

The importance given to different weights of the symptoms in the detection of fault $f$ is given by the following matrix $W$:

$$W = \begin{bmatrix}
0.4 & 0.8 & 1.0 \\
0.5 & 0.6 & 0.3 \\
0.7 & 0.1 & 0.9 \\
0.9 & 0.6 & 0.3
\end{bmatrix}$$

Writing down the linguistic phrases, almost absent, very high severity, moderate severity, and high severity in terms of trapezoidal fuzzy numbers, one has the symptoms of $x$ as follows:

<table>
<thead>
<tr>
<th>System</th>
<th>Symptoms</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>(0,0,1,0,0.2)</td>
<td>(0.5,0.7,0.7,1)</td>
<td>(0.2,0.4,0.4,0.6)</td>
<td>(0.4,0.6,0.6,0.8)</td>
<td></td>
</tr>
</tbody>
</table>

The intervals of confidence for the trapezoidal fuzzy numbers are given as follows:

For $x$

$$S_{x_1} = (0.1\alpha, -0.1\alpha + 0.2)$$

$$S_{x_2} = (0.2\alpha + 0.5, -0.3\alpha + 1)$$

$$S_{x_3} = (0.2\alpha + 0.2, -0.2\alpha + 0.6)$$

$$S_{x_4} = (0.2\alpha + 0.4, -0.2\alpha + 0.8)$$

For $f_1$

$$S_{y_{1f_1}} = (0, -0.2\alpha + 0.2)$$

$$S_{y_{2f_1}} = (0.1\alpha + 0.6, -0.3\alpha + 1)$$

$$S_{y_{3f_1}} = (0.1\alpha + 0.5, -0.1\alpha + 0.7)$$

$$S_{y_{4f_1}} = (0, 0)$$

For $f_2$

$$S_{y_{1f_2}} = (0, 0)$$

$$S_{y_{2f_2}} = (0.05\alpha + 0.9, -0.5\alpha + 1)$$

$$S_{y_{3f_2}} = (0.4\alpha + 0.3, -0.3\alpha + 1)$$

$$S_{y_{4f_2}} = (0.1\alpha + 0.2, -0.1\alpha + 0.4)$$

For $f_3$

$$S_{y_{1f_3}} = (0, -0.3\alpha + 0.3)$$

$$S_{y_{2f_3}} = (0, 0)$$

$$S_{y_{3f_3}} = (0.1\alpha + 0.7, -0.1\alpha + 0.9)$$

$$S_{y_{4f_3}} = (0, 0)$$

Taking $\beta_1$ and $\beta_2$ as the two extreme values of the trapezoidal fuzzy numbers, ie, $\beta_1 = 0$ and $\beta_2 = 1$, one finds the normalised distance between the normalised symptom values of $s_i$ of $x$ and $s_i$ of $f_1$, $s_2$ of $x$ and $s_2$ of $f_1$ and so on, for all the three types of faults.

$$\delta_{x_{s_1}} f_{y_{1f_1}} = \frac{0.2\alpha}{2}$$

$$\delta_{x_{s_2}} f_{y_{2f_1}} = \frac{0.1\alpha - 0.1}{2}$$

$$\delta_{x_{s_3}} f_{y_{3f_1}} = 0.2$$

$$\delta_{x_{s_4}} f_{y_{4f_1}} = 0.6$$

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For $\alpha = 0$, using formula 3.1, one gets the modified distance as

$$D_{\eta_1} = [0.05 + (0.2 \times 0.7)^2 + (0.6 \times 0.9)^2]^{1/2}$$

$$= 0.558$$

$$\sim 0.56$$

In the same manner, the distance between symptoms of system $x$ and symptoms of faults $f_2$ and $f_3$ can be worked out as

$$D_{f_2} = 0.25$$

$$D_{f_3} = 0.465$$

Since $D_{f_2}$ is minimum, one can conclude that the system's symptoms are most similar to those of fault $f_2$, one will get same conclusion if one takes any other value of $\alpha$ (say $\alpha = 0.5$ or $\alpha = 1$).

5. CONCLUSION

Methods presented earlier for determining the most likely fault in system/systems use conventional quantitative analysis. However in practice, documentation relating different symptoms to different systems are available in linguistic terms. Also, when an observer makes note of the symptoms in a particular system, one may find it easier to express one's views in linguistic terms. In such cases, the usual conventional methods cannot be applied. In this paper, such problems have been dealt with by expressing the linguistic terms as trapezoidal fuzzy numbers. Since fuzzy numbers are easy to deal with, the proposed methods can provide useful ways of detecting the faults.

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