1. INTRODUCTION

The problem of initiation of detonation in explosives has been studied extensively. Walker and Wasley\(^1\) developed a critical energy criterion for shock initiation of heterogeneous explosives. According to this criterion, a plane shock wave initiates an explosive only when the shock energy entering per unit area of the explosive exceeds a critical amount of energy, \(E_c\) which is given by the relation:

\[
E_c = P U \tau
\]

where \(P\), \(U\) and \(\tau\) represent the pressure, particle velocity, and duration of shock wave, respectively in the explosive.

Longueville\(^2\), et al. produced uniform shock wave of controlled duration and pressure by impacting flyer plates of different thicknesses and velocities on the explosive, and determined the minimum amount of energy which can initiate detonation in the explosive. It was observed that the explosives having binders, like composition \(B\), RDX/TNT (60:40), HMX/nylon and RDX-polybutadiene detonate according to critical energy criterion, in restricted range of shock duration. The homogeneous explosives like molten TNT and granular explosives like RDX powder, however, do not follow the energy criterion in any range of shock duration.

Moulord\(^3\) further found that if the shock wave in explosive is produced by impacting a flat-ended long rod, then the value of \(E_c\) is considerably higher than the value of \(E_c\) obtained from flyer-plate impacts.
Recently, James has modified the critical energy criterion to make it more general and applicable to different types of projectiles. The modified energy criterion replaces the shock duration by the time of maximum shock energy, \( T \), in the explosive. In case of flyer-plate impact, these two timings are equal because

\[
T = \tau = \frac{2d}{U_s}\tag{2}
\]

where \( d \) is the plate thickness and \( U_s \) is the velocity of shock wave in the flyer plate. In case of flat-ended rod projectiles, the time of maximum shock energy is obtained as

\[
T = \frac{D}{6C}\tag{3}
\]

where \( D \) denotes the diameter of the projectile and \( C \) is the velocity of sound wave in the shocked explosive. In case of round-nosed rod and shaped-charge jet, however, the expression for time \( T \) was obtained from experimental data as

\[
T = \frac{D}{18C}\tag{4}
\]

The modified critical energy criterion for shock initiation of explosives is thus expressed as

\[
E_c = PUT\tag{5}
\]

where \( T \) for different projectiles is given by the Eqns (2)-(4). Held investigated experimentally the initiation of explosive by impact of shaped-charge jets and found a surprisingly simple jet-initiation criterion as

\[
V_j^2D = K_0\tag{6}
\]

where \( V_j \) and \( D \) denote the velocity and diameter of the jet. James also determined the value of \( V_j^2D \) using the Eqn (4) and critical energy obtained from flyer-plate impact and found that the value of \( V_j^2D \), obtained from the modified energy criterion, is in fairly good agreement with those obtained experimentally.

The present study attempts to derive the critical energy criterion by considering the kinetic energy of the projectile before and after its impact on the explosive and to theoretically obtain a relation for constant \( K_0 \) of Eqn (6), which is generally determined experimentally.

2. THEORETICAL DEVELOPMENT

2.1 Relations for Impact-generated Shock Wave

To determine the energy in a shock wave which initiates detonation in the explosive, it is essential to obtain appropriate relations for impact-generated shock wave. Suppose, a flyer plate of density \( \rho_p \) strikes an explosive of density, \( \rho_e \), as shown in Fig.1, with velocity \( V \). The resistance of the explosive reduces the velocity of the projectile from \( V \) to \( U_p \). The interface of the explosive and the flyer plate moves in the direction of plate motion with velocity,
The sudden change of momentum of the projectile produces two shock waves, one of pressure $P_1$ which travels into the explosive in the direction of plate motion and another of pressure $P_2$ which travels into the flyer plate in the opposite direction. The condition of continuity of pressure and particle velocity at the interface gives $P_1 = P_2$

and

$$U_p = V - U_r \tag{7}$$

where $U_r$ is the change in the velocity of flyer plate due to its impact on the explosive. The pressure of shock waves $P_1$ and $P_2$ are obtained from conservation of mass and momentum across the shock front as

$$P_1 = \rho_s U_s U_p \tag{8}$$

in the explosive, and

$$P_2 = \rho_p U_s U_r \tag{9}$$

in the projectile. Here, $\rho_s$ and $\rho_p$ denote the original density of the explosive and the flyer plate, respectively, and $U_s$ and $U_r$ represent the shock velocities in the explosive and the plate, respectively. Expressing Eqns (8) and (9) in terms of shock-impedance, $Z_s = \rho_s U_s$ and $Z_p = \rho_p U_r$, one gets:

$$P_1 = Z_s U_p \tag{10}$$

$$P_2 = Z_p U_r \tag{11}$$

Using Eqn (7) and the interface condition $P_1 = P_2$ in Eqns (10) and (11), one gets:

$$Z_s U_p = Z_p (V - U_p) \tag{12}$$

which on simplification yields:

$$U_p = \frac{V}{1 + \frac{Z_s}{Z_p}} \tag{13}$$

In the case of solids, the shock and particle velocity are related linearly as

$$U_s = a + b U_p \tag{14}$$

where $a$ and $b$ are the Hugoniot constants of the materials. The shock-impedance of the explosive, $Z_s$, and that of the projectile, $Z_p$ are therefore obtained as

$$Z_s = \rho_s (a_s + b_s U_p) \tag{15}$$

$$Z_p = \rho_p (a_p + b_p U_p) \tag{16}$$

where $s$ and $p$ subscripts refer to the explosive and the plate, respectively. Substituting Eqns (15) and (16) into Eqn (13) and solving the resulting equation, one gets:

$$A U_p^2 + B U_p - C = 0 \tag{17}$$

where

$$A = \rho_s b_s - \rho_p b_p$$

$$B = \rho_s a_s + \rho_s a_s + 2 \rho_p b_p V$$

$$C = \rho_p (a_p + b_p V) \frac{V}{1 + \frac{Z_s}{Z_p}}$$

The constants of Eqn (17) are known from the Hugoniot data of the explosive and the flyer plate, it can, therefore, be solved to determine $U_p$ for a particular velocity of the projectile.

2. **Sound Velocity in a Shock-Compressed Explosive**

The sound velocity, $C$, in a shocked explosive is obtained from the relation:

$$C^2 = C_H^2 (1 - \Delta) \tag{18}$$

where

$$C_H = \left( \frac{dp}{dP} \right)_{Hug}^{1/2}$$

and

$$\Delta = \frac{\gamma}{2V_1} \left( V_0 - V_1 \right) \left[ 1 - \left( \frac{U_s - U_{p,press}}{C_H} \right)^2 \right] \tag{19}$$
In this relation, $\gamma$ denotes the Mie-Gruneisen's constant and $V_i$ and $V_o$ denote the specific volume of shocked and unshocked explosive. The slope of Hugoniot, $C_H$ at its $P_1, V_1$ point is obtained by expressing shock parameters $U_p, U_s, P$ in terms of a single parameter as

$$n = 1 - \frac{\rho_o}{\rho}$$

(20)

Using law of conservation of mass across the shock front, one gets:

$$\rho_o U_s = \rho_1 (U_r - U_p)$$  

(21)

where $\rho_1$ and $\rho_o$ denote the density of the medium behind and ahead of shock front. Equation (21) yields:

$$n = \frac{U_p}{U_s}$$

(22)

Substituting Eqn (22) in Eqn (14), one gets:

$$U_p = \frac{an}{1-nb}$$

(23)

$$U_s = \frac{a}{1-nb}$$

(24)

and by substituting Eqns (23) and (24) in Eqn (8), one gets:

$$P = \frac{\rho_o a^2 n}{(1-nb)^2}$$

(25)

Differentiating Eqn (20), one gets:

$$dn = \rho_o \rho_i^{-2} d\rho$$

(26)

and therefore

$$C_H^2 = \left( \frac{dP}{d\rho} \right)_{Hug} = \rho_o \rho_i^{-2} \left( \frac{dP}{dn} \right)_{Hug}$$

(27)

Substituting $(dP/dn)$ from Eqn (25) in the above equation, one gets:

$$C_H^2 = \frac{a^2 (1-n)^2 (1+nb)}{(1-nb)^3}$$

(28)

Substituting the above equation in Eqn (18), one determines the value of the sound velocity in a shocked explosive.

3. SHOCK WAVE ENERGY IN EXPLOSIVE

Neglecting the heat and frictional losses, one can assume that the total energy in one-dimensional shock wave, produced by impact of the flyer plate, is equal to the difference in kinetic energy of the plate before and after the impact, and this difference is readily obtained if the velocity of the flyer plate after the impact is determined. To achieve this, it is essential to know the pressure and particle velocity at the interface of the plate and the explosive. When the flyer plate makes an impact on the explosive, shocks of pressure $P_1$ and $P_2$ are produced in the explosive and the flyer plate, respectively and the mass velocity of the flyer plate behind its shock front is reduced by $U_r$ as shown in Fig. 1. The pressure $P_2$, on getting released on the rear face of the plate, imparts an additional velocity, approximately equal to $U_r$, to the free surface in the direction opposite to the original direction of the plate motion. The complete thickness of the plate attains the additional velocity at the time when the rarefaction wave, originated at rear free-surface of the plate, reaches the interface of the plate and the explosive. At this time, the velocity of the flyer plate in the direction of its initial motion becomes:

$$V_r = V - 2 U_r$$

(29)

where $U_r$ is obtained from Eqn (7) as

$$U_r = V - U_p$$

(30)

Substituting the value of $U_p$ from Eqn (13), one gets:

$$U_r = \left( \frac{f}{1+f} \right) V$$

(31)

and
\[ V_r = \left( \frac{1 - f}{1 + f} \right) V \]  

(32)  

where \( f = \frac{Z_s}{Z_p} \).

It is clearly seen from Eqns (31) and (32) that if \( Z_s = Z_p \), then \( U_r = \frac{V}{2} \) and \( V_r = 0 \).

If \( E_1 \) and \( E_2 \) represent the kinetic energies of the flyer plate before and after the impact, respectively, then

\[ E_1 - E_2 = \frac{1}{2} A \rho d \left( V^2 - V_r^2 \right) \]  

(33)

where \( d, \rho \) and \( A \) denote the thickness, density, and area of the flyer plate. The energy transferred to shock wave per unit area of the explosive is thus obtained as

\[ E_c = \frac{E_1 - E_2}{A} \]  

(34)

or

\[ E_c = \frac{1}{2} \rho d \left( V^2 - V_r^2 \right) \]  

(35)

Substituting the value of \( V_r \) from Eqn (32), one gets:

\[ E_c = \frac{2d \rho V^2 f}{(1 + f)^2} \]  

(36)

Since \( U_p = \frac{V}{1 + f} \) and \( U_r = \frac{f}{1 + f} V \), therefore Eqn (36) can be written as

\[ E_c = 2d \rho U_p U_r \]  

(37)

The law of conservation of mass and momentum across the shock front in the flyer plate yields:

\[ P = \rho U_{sr} U_r \]  

(38)

Eliminating \( U_r \) from Eqns (37) and (30), one gets:

\[ E_c = \frac{2d \rho U_p}{\rho U_{sr}} \]  

(39)

or

\[ E_c = \frac{2d U_p \cdot 2d}{U_{sr}} \]  

(40)

Since, duration of the shock in the explosive, \( \tau = \frac{2d}{U_{sr}} \), and the pressures of shock waves in the plate and the explosive are equal, therefore

\[ E_c = PU_p \tau \]  

(41)

represents the energy of shock wave in the explosive, threshold value of which initiates detonation in the explosive.

When the shock wave in the explosive is produced by the impact of a flat-ended rod-type of projectile, then the duration of shock wave is not equal to the time of maximum shock energy in the explosive as it was in the case of flyer plate impact. It is basically due to the fact that the rarefaction wave, in the case of a flyer plate impact, which originates from the rear face of the flyer plate and travels through the thickness of the plate controls the duration of shock wave as well as the time of maximum shock energy. But in flat-ended/rod-type or round-nosed rod-type projectiles, the rarefaction wave, which controls the time of shock wave, originates from the lateral surface of the shock wave and travels through the diameter of the shock wave which is initially equal to the diameter of the projectile, controls the time of maximum shock energy, \( T \), in the explosive.

Recently, James\textsuperscript{4} has found that the time \( T \) for flat-ended rod-type of projectile is given by Eqn (3) and for round-nosed rod-type projectile and shaped-charge jet impact, the time, \( T \), is given by the Eqn (4).
3.1 Jet-initiation Criterion

Combining Eqns (2) and (4) in Eqn (5), the jet-initiation criterion is obtained as

$$E_c = \frac{P U_p D}{18 C}$$  \hspace{1cm} (42)

Substituting the value of $P = \rho_s U_s U_p$ in Eqn (42), one gets:

$$E_c = \frac{\rho_s U_s U_p^2 D}{18 C}$$  \hspace{1cm} (43)

Since, $U_p = \frac{V_j}{1+f}$, therefore, above equation yields:

$$E_c = \frac{\rho_s U_s V_j^2 D}{18 C (1+f)^2}$$  \hspace{1cm} (44)

where $V_j$ and $D$ denote the velocity and diameter of the jet and $f = Z_x/Z_p$, denote the ratio of shock impedances of the explosive and the shaped-charge jet. If one assumes that $U_s/C$ and $Z_x/Z_p$ are the constants in the limited range of shock-pressures, which is generally required for initiation of detonation in most of the explosives, then $V_j^2 D$ represents a constant value which is given as

$$V_j^2 D = \frac{18 (1+f)^2 E_c}{\rho_s U_s}$$  \hspace{1cm} (45)

where $E_c$ is the critical shock energy for the initiation of the explosive as obtained from the flyer plate impact experiments. Denoting the ratio $U_s/C$ by a constant $K$, one gets the expression for jet-initiation criterion as

$$V_j^2 D = \frac{18 (1+f)^2 E_c}{K \rho_s}$$  \hspace{1cm} (46)

The relation for $K_0$ is obtained as

$$K_0 = \frac{18 (1+f)^2 E_c}{K \rho_s}$$  \hspace{1cm} (47)

4. RESULTS & DISCUSSION

To examine the validity of the assumptions that the ratio of shock velocity and sound velocity, $U_s/C$ and the ratio of shock-impedances of the explosive and the projectile, $Z_x/Z_p$, are reasonably constant in the range of shock pressures sufficient to initiate detonation in most of the explosives, the shock parameter and the sound velocities in a shocked explosive have been computed for different impact velocities of the shaped-charge jet in the two typical explosives, RDX/TNT (60:40) and TNT using the material constants of Table 1 and the Eqns (17) to (30). The particle velocity $U_p$ initially generated by the impact of the jet has been obtained by solving Eqn (17) for a particular velocity of the jet. The shock velocity is then obtained using linear relation between shock velocity and particle velocity for the explosive. Equations (22), (25), and (28) yield the values of $n, p$ and $C_H$ parameters of the shock wave. Equations (17) and (18) then give the value of sound velocity in the shocked explosive.

Computed values of shock parameters are given in Tables 2 and 3 for RDX/TNT (60:40) and TNT, respectively. These tables also include the values of ratios of $U_s$ and $C$, and $Z_x$ and $Z_p$. It is clearly seen from these tables that value of $U_s/C$ and $Z_x/Z_p$ varies very slowly. In fact, both the ratios can be approximated by their constant values, in limited range of shock pressures. The graphical depiction...
Table 2. Parameters of shock wave produced by impact of copper jet of density, $\rho = 8.9$ g/cc in RDX/TNT (60:40) explosive of density, $\rho = 1.7$ g/cc. Shock hugoniot of copper and explosive (RDX/TNT) are: $U_s = 3.94 + 1.489 U_p$ and $U_s = 2.71 + 1.86 U_p$

<table>
<thead>
<tr>
<th>Jet velocity ($V_j$) (mm/µs)</th>
<th>Particle velocity ($U_p$) (mm/µs)</th>
<th>Shock velocity ($U_s$) (mm/µs)</th>
<th>Shock pressure ($P$) (Kbar)</th>
<th>Sound velocity ($C$) (mm/µs)</th>
<th>$Z_x 10^{-5}$</th>
<th>$Z_p 10^{-5}$</th>
<th>$Z_x 10^{-5}/Z_p$</th>
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</table>

Average $U_s = 0.7986$, Average $f = 0.2570$.

Table 3. Parameters of shock wave produced by impact of copper jet of density, $\rho = 8.9$ g/cc in TNT of density, $\rho = 1.5$ g/cc. Shock hugoniot of copper, $U_s = 3.94 + 1.489 U_p$. Shock hugoniot of TNT, $U_s = 2.08 + 2.33 U_p$

<table>
<thead>
<tr>
<th>Jet velocity ($V_j$) (mm/µs)</th>
<th>Particle velocity ($U_p$) (mm/µs)</th>
<th>Shock velocity ($U_s$) (mm/µs)</th>
<th>Shock pressure ($P$) (Kbar)</th>
<th>Sound velocity ($C$) (mm/µs)</th>
<th>$U_s = K$</th>
<th>$Z_x 10^{-5}$</th>
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</table>

Average of all values $U_s = 0.652$; Average of initial three values, $U_s = 0.699$; Average $f = 0.249$.

of $U_s$ versus $C$ and $Z_x$ versus $Z_p$ in Figs (2) to (4) also shows that a straight line fits well in the middle values data of both these explosives within a variation of 3 per cent to 6 per cent. The assumptions used in derivation of Eqn (46) for jet-initiation criterion are therefore reasonable.

The value of $V_j^2 D$ has been computed by taking average values of $K$ and $f$ as constants from Tables 2 and 3, respectively and $E_c$ from Table 4 for RDX. TNT (60:40) and TNT. The
average value of $K$ for calculation of $V_j^2 D$ for TNT explosive is obtained as average of the first three values of Table 3 as the assumption of linearity between shock velocity and sound velocity holds better in a limited range of shock pressures. These values of $V_j^2 D$ are given in Table 4. It is found that the computed values are in agreement with those obtained by James\(^5\).

It is interesting to note that if one uses the average of $K$ and $f$ values of RDX/TNT and TNT as $K = 0.75$ and $f = 0.25$ in Eqn (46) to calculate the value of $V_j^2 D$ for these two explosives, then, as shown in Table 4, these values are within a variation of 3 per cent only. It suggests that a rough approximation of $V_j^2 D$ can be obtained for other explosives also by just calculating the right hand side of Eqn (46) by taking same values of $K$ and $f$. For example, the value of $V_j^2 D$ for PBX explosive of density 1.04 g/cc and critical energy, $E_c = 0.70$ MJ/m\(^2\), is obtained as 14.22 mm\(^3\)/μs\(^2\) against a value of 16 mm\(^3\)/μs\(^2\) reported in literature and for cyclotol (75:25) of density 1.76 g/cm\(^3\) and critical energy, $E_c = 1.98$ MJ/m\(^2\), the value of $V_j^2 D$ is obtained as 42.2 mm\(^3\)/μs\(^2\) against literature value of 41 mm\(^3\)/μs\(^2\).

### 5. CONCLUSION

The fact that the values of $V_j^2 D$, computed in this study for RDX/TNT (60:40) and TNT explosive are in agreement with those reported in the literature, leads to the conclusion that even averaged constants, $K$ and $f$, which appear in jet-initiation criterion, give fairly accurate values of $V_j^2 D$. Since the present jet-initiation criterion is based on the relation for critical energy, it is, therefore, concluded that the jet-initiation criterion is an approximation of the critical energy criterion.

With the present theoretical development for getting the expression for the constant of jet-initiation criterion, it is possible now to calculate the value of this constant for any explosive where density, $\rho_c$, critical energy, $E_c$, Hugoniot parameters $a$ and $b$, and Mie Gruneisen Constant, $\Gamma_e$, are known.

### REFERENCE


**Contributor**

**Dr H.S. Yadav** obtained his PhD (Shock Physics) from the Punjab University in 1984. He served as Scientist G at the High Energy Materials Research Laboratory (HEMRL), Pune, till 2001. His areas of research include: Detonics, shock waves, explosives metal welding, explosive reactive armour, etc. He received *Man of Achievements Award* (1988) from the International Biographic Centre, UK, *DRDO Technology Award* (2000) for the development of advanced explosive reactive armour. He visited California Institute of Technology in 1997 and delivered a series of lectures on shock wave physics and also visited RAFAEL, Israel, in 1997 as the Leader of DRDO delegation. He has published about 50 papers in national/international journals.