Tracking the Warhead Among Objects Separation from the Reentry Vehicle in a Clear Environment

Cheng-Yu Liu and Chi-Teh Chen
Lee-Ming Institute of Technology, Taipei, Taiwan, R.O.C.

ABSTRACT

Separating a reentry vehicle into warhead, main body, and debris is a conventional and efficient means of producing a huge decoy and increasing the kinetic energy of the warhead. This procedure causes the radar to track the main body and debris, which radar cross section are large, and ignore the warhead, is the most important part of the reentry vehicle. The warhead is difficult to identify after separation using standard tracking criteria. This study presents a novel tracking algorithm by integrating input estimation and modified probabilistic data association filter to identify warhead among objects separation from the reentry vehicle in a clear environment. The proposed algorithm provides a good tracking capability for the warhead ignoring the radar cross section. Simulation results reveal that the errors between the updated and warhead trajectories are reduced to a small interval in a short time. Therefore, the radar can generate a beam to illuminate the right area and keep tracking the warhead all the time. This algorithm is worthy of further study and application.

Keywords: Input estimation, probabilistic data association filter, extended Kalman filter, tracking algorithm, trajectory estimation

NOMENCLATURE

- $C$: Ballistic coefficient for the RV (kg/m$^2$)
- $C_i$: Ballistic coefficient for the $i$-th object (kg/m$^2$)
- $C_D$: Zero-lift drag coefficient
- $e_i$: Length of the normalized innovation in position for the $i$-th candidate
- $g$: Gravity (m/s$^2$)
- $G(i)$: $6 \times 6$ gain matrix
- $I_{nn}$: $n \times n$ identity matrix
- $K(n+1)$: $6 \times 6$ Kalman gain matrix
- $K'(n+1)$: $6 \times 6$ Kalman gain matrix with the estimated input for the $i$-th object
- $N$: Number of objects separation from the RV
- $P(k+l+1|k+l)$: $6 \times 6$ covariance matrix of the predicted states for the EKF with no input
- $P_i$: $6 \times 6$ variance matrix of $\hat{Z}(k+l)$
- $Q$: $6 \times 6$ covariance matrix of process noise
- $Q_i$: $6 \times 6$ covariance matrix of process noise for the $i$-th object
- $R$: $6 \times 6$ covariance matrix of measurement noise
- $S$: Reference area of the RV (m$^2$)
- $V(i)$: $6 \times 6$ covariance matrix of the estimated input
- $W$: Weight of the RV (kg)
- $\hat{Z}(k+l)$: Unmodeled inputs in $X_R$, $Y_R$ and $Z_R$, respectively (m/s$^2$)
- $u_x, u_y, u_z$: Unmodeled inputs in $X_R$, $Y_R$ and $Z_R$, respectively
- $v, v_i$: Total velocities for the RV and the $i$-th object (m/s)
- $v_x, v_y, v_z$: Velocity components (m/s)
- $x, y, z$: Positions (m)
- $\hat{Z}(n+1)$: Measurements at $t = (n+1)\Delta t$
- $\hat{Z}(k+l)$: Measurement residual for the EKF formation with inputs
- $Z^{n+1}$: Measurement vector collecting the measurements from $t = 0$ to $t = (n+1)\Delta t$
- $\mathbf{0}_{n \times n}$: $n \times n$ zero matrix
- $\Delta t$: Sampling period (s)
- $\rho$: Air density (kg/m$^3$)
- $\zeta$: $6 \times 1$ process noise vector
- $\zeta_i$: $6 \times 1$ process noise vector for the $i$-th object
- $\varepsilon$: $6 \times 1$ measurement noise vector
- $\gamma_1, \gamma_2$: Elevation and flight path angles for the RV, respectively
- $\gamma_{i1}, \gamma_{i2}$: Elevation and flight path angles for the $i$-th object, respectively

1. INTRODUCTION

Separation of the reentry vehicle (RV) into several objects, including the warhead, main body, and debris during the reentry phase, is an effective means of confusing radar by generating many sets of measurements from a radar beam. The radar estimates and predicts the target trajectory at the next sampling...
period from the measurement with the highest signal-to-noise ratio and forms a beam to illuminate it. The body or debris is then tracked and intercepted first, since its radar cross section is inevitably larger than that of the warhead. The warhead, which is the most significant part of the RV, then becomes a new track or is completely disregarded. This problem is hard to solve unless the standard tracking criterion is modified.

Two issues, estimation of warhead trajectory and origination of measurements, have been introduced to solve this problem. If a warhead does not alter its trajectory in separation, then it typically follows the original trajectory closely. However, the body and debris fall down faster than the warhead in random directions. Then, the residual wrt the warhead, given by the difference between the estimated and real warhead trajectories, is smaller than that wrt the other objects. An online precision trajectory estimation approach is required. Data association technique can be adopted to solve the origination problem.

The main problems with the online estimation of the RV trajectory relate to model validation, owing to model error between the mathematical model and the physical system. The model error is normally the result of the simplifying assumptions, manoeuvring and unpredictable external forces during flight or parameter uncertainty. The extended Kalman filter (EKF) is a well-known and helpful state estimation scheme for a nonlinear dynamic system, but fails to reach the required accuracy in a short time. Input estimation (IE) provides a good solution to this problem. IE has been successfully employed to estimate inputs for solving tracking1-4, inverse heat conduction 5-7, and initial leveling problems 8,9. Lee and Liu proposed a filter associating the EKF with IE to handle model validation problems, and provided an accurate trajectory estimation approach for the RV10-14. Their proposed filter has lower estimation errors than the original EKF, and is helpful for this problem.

Bar-Shalom designed a suboptimal Bayesian algorithm, probabilistic data association filter (PDAF)15-16, for tracking a single target in a cluttered environment. It combines the states of all radar returns weighted by a posteriori probability, known as association probability, to form the combined states of the target. The PDAF has been successfully utilised in sonar and radar systems to increase their tracking capacity17-18. A modified PDAF (MPDAF) with a new defined association probability has also been developed. MPDAF links with the EKF and IE to create a tracking algorithm for clarifying warhead between two objects split from the RV in a clear environment19. This algorithm enables the radar to identify the warhead in a few seconds following separation, and could be used to track warhead in a complex situation.

This investigation presents a novel algorithm by inserting an identification procedure on the MPDAF to detect the warhead among more than two objects split from the RV in a clear environment.

2. DYNAMIC EQUATIONS

Define the radar Cartesian coordinate system to be centred at the radar site \( O_R \) with three axes downrange \( X_R \), off-range \( Y_R \), and altitude \( Z_R \).
The nonlinear state equation can be written as

\[
\hat{X}(t) = F(X) + \varphi u + \zeta
\]

(7)

where

\[
F(X) = \begin{bmatrix}
-\rho \frac{C_y}{g} \gamma_1 \cos \gamma_2 \\
-\rho \frac{C_y}{g} \gamma_1 \cos \gamma_2 \\
\rho \frac{C_y}{g} \sin \gamma - g
\end{bmatrix}
\]

\[
\varphi = \begin{bmatrix}
0_{3x3} & 0_{3x3} \\
0_{3x3} & I_{3x3}
\end{bmatrix}
\]

The precision phased array radar, i.e., a digital radar is used in tracking, and is the only instrument in the system for detecting RV. The detected elevation, azimuth, and range are transferred into position in Cartesian frame. Since the detected velocity measured from pulse Doppler did not meet the accuracy needs, the velocity obtained by a specific filter in velocity measured from pulse Doppler did not meet the accuracy needs, the velocity obtained by a specific filter.

3. EXTENDED KALMAN FILTER WITH INPUT ESTIMATION

The input estimation algorithm estimates unknown inputs in state equations from pseudo-residuals. At \( t \leq t_s \), the predicted and updated state vectors of the RV by the EKF from \( t = n\Delta t \) to \( t = (n+1)\Delta t, n = 0,1,2,... \), under input vector \( u(n) \) at \( t = n\Delta t \) are given by, respectively.

\[
\tilde{X}(n+1|n+1) = \tilde{X}(n+1|n) + K(n+1) \quad Z(n+1) - \tilde{X}(n+1|n)
\]

(16)

where \( \varphi = \varphi \Delta t \) and let \( \bar{X} = (n+1|n+1) \) denote the updated state for the EKF with no input at \( t = (n+1)\Delta t \). For simplicity, let \( \tilde{X}(n+1) = \bar{X}(n+1|n+1) \) and \( \bar{X}(n+1) = \bar{X}(n+1|n+1) \).

Define

\[
M(n+1) = [I - K(n+1)]\phi(n)
\]

(17)

\[
N(n+1) = [I - K(n+1)]\phi(a)
\]

(18)

Assume that the abrupt deterministic inputs are applied during \( k\Delta t \leq t \leq (k+s)\Delta t \),

\[
u = \begin{cases}
0 & t < k\Delta t, t > (k+s)\Delta t \quad k,s > 0 \\
u(k+l) & k\Delta t \leq t \leq (k+s)\Delta t \quad l = 0,1,2,....s
\end{cases}
\]

(19)

where \( u(k+l) \) is a constant vector over the sampling interval. Then, \( \tilde{X}(k) = \bar{X}(k) \) during \( t \leq k\Delta t \). The difference induced by the abrupt inputs between these two formations during \( k\Delta t \leq t \leq (k+s)\Delta t \) can then be written as

\[
\Delta X(k+l) = \tilde{X}(k+l) - \bar{X}(k+l)
\]

(20)

Define the measurement residual for the EKF formation without and with inputs to be

\[
\Delta Z(k+l) = Z(k+l) - \bar{X}(k+l) = Z(k+l) - \tilde{X}(k+l)
\]

(20)

respectively. The recursive least-squares input estimator can be derived as

\[
\hat{u}(k+l-1) = \hat{u}(k+l-2) + G(k+l)
\]

(21)

where

\[
\frac{\hat{Y}(k+l)}{\hat{Y}(k+l)} = \hat{Z}(k+l) - M(k+l) \Delta \hat{X}(k+l-1)
\]

(20)

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\[ \Delta \hat{x}(k+l-1) = M(k+l) \Delta \hat{x}(k+l-2) + N(k+l) \hat{u}(k+l-2) \]

\[ G(k+l) = V(k+l-1) N(k+l) P_{z}^{-1} \]

\[ V(k+l-1) = V(k+l-2) - V(k+l-2) N(k+l) ^T \]

\[ \times \left[ N(k+l) V(K+1-2) N(k+l) ^T + P_{z} \right]^{-1} \]

\[ \times N(k+l) V(k+l-2) \]

\[ P_{z} = R + P(k+l+1|k+l) \]

In Eqn. (19), \( k \) and \( k+s \) respectively denote the starting and stopping indices of the system input, which can be obtained by testing. Two hypotheses, existence and absence of inputs, and stopping indices of the system input, which can be obtained from the cumulative normal distribution table for a certain preset confidence level \( 1-\alpha \).

Once the input is estimated, the EKF is corrected with the estimated input at the same time. By incorporating the on-line input estimator into the EKF, the predicted and updated states at time interval \( k \Delta t \leq t \leq (k+s) \Delta t \) are given by

\[ \hat{x}^\ast(k+1|k+l-1) = \phi(k+l-1) \hat{x}^\ast(k+l-1|k+l-1) + \phi \hat{u}(k+l-1) \quad (22) \]

\[ \hat{x}^\ast(k+l|k+l) = \hat{x}^\ast(k+l|k+l-1) + K^\ast(k+l) \]

\[ [Z(k+l) - \hat{x}^\ast(k+l|k+l-1)] \]

The Kalman gain becomes

\[ K^\ast(k+l) = P^\ast(k+l|k+l-1) L(k+l) + R \]

with covariance matrices at \( k \Delta t \leq t \leq (k+s) \Delta t \) being

\[ P^\ast(k+l|k+l-1) = P(k+l|k+l-1) \]

\[ + \phi(k+l-1) L(k+l) \phi^T(k+l-1) \]

\[ + \phi \lambda V(k+l-1) \phi^T \]

\[ P^\ast(k+l|k+l) = [I - K^\ast(k+l)] P^\ast(k+1|k+l-1) \]

where

\[ L(k+l) = 0 \quad L(k+2) = N(k+2) V(k) N^T(k+2) \]

\[ L(k+l) = M(k+l-1) L(k+l-1) M^T(k+l-1) \]

\[ \quad \times [I - K^\ast(k+l)] \]

For time beyond the interval \( t < k \Delta t \) and \( t > (k+s) \Delta t \), state estimation can be also based upon the original EKF. Note that the initial states and covariance matrices at \( t > (k+s) \Delta t \) are reinitiated by \( \hat{x}^\ast(k+s|k+s) \) and \( P^\ast(k+s|k+2) \).

Figure 2 schematically depicts the proposed filter.

\[ \Delta x(k+l-1) = M(k+l) \Delta x(k+l-2) + N(k+l) \hat{u}(k+l-2) \]

\[ G(k+l) = V(k+l-1) N(k+l) P_{z}^{-1} \]

\[ V(k+l-1) = V(k+l-2) - V(k+l-2) N(k+l) ^T \]

\[ \times \left[ N(k+l) V(K+1-2) N(k+l) ^T + P_{z} \right]^{-1} \]

\[ \times N(k+l) V(k+l-2) \]

\[ P_{z} = R + P(k+l+1|k+l) \]

The proposed method comprises two steps. Selecting two candidates from \( N \) objects based on a certain criterion is the first, and then the MPDAF is used to compute the combined updated state from these two candidates.

### 4. IDENTIFICATION ALGORITHM

The MPDAF with the EKF and IE was presented to identify the warhead between two objects separation from the RV in a clear environment\(^{10}\) with clutter free. The major difference between MPDAF and the original PDAF is that the MPDAF considers the two real objects being tracked, but not random clutter. This section defines an algorithm associated with the MPDAF to track the warhead among \( N \) measured objects when \( t > t_c \).

For more accurate choosing candidates, residual based on the updates states is adopted. Define the measurement residual of object \( i \) at \( t = (n+1) \Delta t \) as

\[ r_i(n+1) = Z_i(n+1) - \hat{X}_i(n+1) \]

Increasing the accuracy of the model reduces the distance between the measurement and predicted state in position. Restated, if the \( i \)-th object measurement \( Z_i(n+1) \) is the target-originated measurement, then the length of the normalised residual in position is the smallest among all objects. Two candidates are chosen who are with the smallest values of the length of the normalised residual in position among \( N \) objects. These two candidates are sent to the MPDAF to calculate the association probabilities and combined updated state.

### 4.1 Selection of Candidates

Assume \( N \) objects to be detected by a phased array radar with detection probability 1 during \( t > t_c \). The predicted and updated states of the \( i \)-th object using the EKF with IE are, respectively,

\[ \hat{x}_i^\ast(n+1|n) = \phi(n+1) \hat{x}_i^\ast(n|n) + \phi \hat{u}_i(n) \]

\[ \hat{x}_i^\ast(n+1|n+1) = \hat{x}_i^\ast(n+1|n) + K^\ast_i(n+1) \]

\[ [Z_i(n+1) - \hat{X}_i^\ast(n+1|n+1)] \]

For more accurate choosing candidates, residual based on the updates states is adopted. Define the measurement residual of object \( i \) at \( t = (n+1) \Delta t \) as

\[ r_i(n+1) = Z_i(n+1) - \hat{X}_i(n+1) \]

The proposed method comprises two steps. Selecting two candidates from \( N \) objects based on a certain criterion is the first, and then the MPDAF is used to compute the combined updated state from these two candidates.

### 4.2 The MPDAF

Two candidates are selected based on the normalised residual defined in Eqn. (29). The MPDAF needs to compute the combined state from these two objects which may contain the warhead or just body and debris. Let \( \hat{X}_{i1}^\ast(n+1|n+1) \) and
\( \hat{X}_{\gamma}^{i}(n+1|n+1) \) to be the updated states of two candidates corresponding to measurements \( Z_{\gamma}(n+1) \) and \( Z_{\gamma}(n+1) \), respectively, at time \( t = (n+1)\Delta t \). Define the events to be

\[
\begin{align*}
\theta_{1}(n+1) &= \{ Z_{\gamma}(n+1) \text{ is the target-originated} \} \\
\theta_{2}(n+1) &= \{ Z_{\gamma}(n+1) \text{ is the target-originated} \}
\end{align*}
\]

with association probabilities conditioned on \( Z^{n+1} \)

\[
\beta_{i}(n+1) = \mathbb{P}\{ \theta_{i}(n+1) | Z^{n+1} \} \quad i = 1, 2
\]

The association probabilities should satisfy

\[
\beta_{1}(n+1) + \beta_{2}(n+1) = 1
\]

The updated state for the EKF with input estimation at time \( t = (n+1)\Delta t \) can be regarded as

\[
\hat{X}^{\gamma}(n+1|n+1) = \mathbb{E}[X(n+1)|Z^{n+1}]
\]

Using the total probability theorem, \( \hat{X}^{\gamma}(n+1|n+1) \), known as the combined updated state, becomes

\[
\begin{align*}
E\left[ X(n+1) | Z^{n+1} \right] &= E\left[ X(n+1) | \theta_{1}(n+1)Z^{n+1} \right] \\
&+ E\left[ X(n+1) | \theta_{2}(n+1), Z^{n+1} \right] \\
&+ E\left[ X(n+1) | \theta_{1}(n+1), Z^{n+1} \right] \\
&\mathbb{P}\{ \theta_{1}(n+1) | Z^{n+1} \} + \mathbb{P}\{ \theta_{2}(n+1), Z^{n+1} \} + \mathbb{P}\{ \theta_{1}(n+1), Z^{n+1} \}
\end{align*}
\]

then one gets

\[
\hat{X}^{\gamma}(n+1|n+1) = \beta_{1}(n+1)\hat{X}_{\gamma}^{1}(n+1|n+1) + \beta_{2}(n+1)\hat{X}_{\gamma}^{2}(n+1|n+1)
\]

The problem then focuses on the determination of \( \beta_{i} \) by means of \( Z_{\gamma} \).

As previously mentioned, trajectory of the warhead is closer to the original than of the body and debris. The innovations of the warhead are smaller than of other objects if an accurate estimation method is adopted. The larger association probability should be assigned to an object whose innovation is the smaller estimation error w.r.t. the warhead to ensure that the warhead is constantly tracked. IE plays an important role in the proposed algorithm that provides accurate residuals. To indicate the importance of IE to tracking problem, the simulation results of the proposed algorithm were compared with those of the EKF with identification algorithm only. Denote the proposed algorithm as Method I and the EKF with the identification algorithm but without IE as Method II. The following study cases are with \( N_{MC} = 50 \) Monte Carlo runs.

Consider an RV in the reentry phase with \( C = 2500 \text{ kg/m}^2 \) and initial values of \( x(0) = 300 \text{ m}, y(0) = 300 \text{ m}, z(0) = 30500 \text{ m}, v(0) = 1500 \text{ m/s}, \gamma(0) = 65^\circ \), and \( \psi(0) = 15^\circ \). The RV separates into the warhead, body, and two debris with the ballistic coefficients \( C_{w} = 2000 \text{ kg/m}^2, C_{b} = 3000 \text{ kg/m}^2, C_{1} = 3500 \text{ kg/m}^2 \) and \( C_{2} = 4000 \text{ kg/m}^2 \), respectively, at \( t_{s} = 5 \text{ s} \). Equations (1) ~ (9) simulate the trajectories of the warhead, body, two debris, and the original RV without separation. Measurement noises are generated by following the standard normal distribution. Figures 4 to 9 display the measured trajectories with measurement noises. The

Figure 3. Flowchart of the proposed algorithm.

by inspecting the estimation error w.r.t the warhead, which signifies the difference between the estimated and actual warhead trajectories. The proposed algorithm should have a small estimation error w.r.t. the warhead to ensure that the warhead is constantly tracked. IE plays an important role in the proposed algorithm that provides accurate residuals. To indicate the importance of IE to tracking problem, the simulation results of the proposed algorithm were compared with those of the EKF with identification algorithm only. Denote the proposed algorithm as Method I and the EKF with the identification algorithm but without IE as Method II. The following study cases are with \( N_{MC} = 50 \) Monte Carlo runs.

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5. SIMULATION ANALYSIS

The performance of the proposed algorithm was evaluated
The sampling period of the radar is $\Delta t = 0.1s$. The flight times of the original RV, warhead, body, and two other debris are 36 s, 42 s, 33 s, 31 s, and 29 s, respectively. The body and two debris are falling down faster than the warhead. The purpose of the proposed algorithm is to identify the warhead among these four objects.

The ballistic coefficient was set to a constant of $C = 2500$ kg/m$^2$ in entire estimation. Let the confidence level $1-\alpha$ to be 0.95. The initial conditions of $R$, $Q$, $V$, and $P_v(0|0)$ were $I_{6x6}$, $I_{6x6}$, 500$I_{6x6}$, and 500$I_{6x6}$, respectively. $R$ and $Q$ remain constants in entire estimation. The first detection $Z(0)$ is taken as initial state $X^v(0|0)$.

Figures 10 and 11 show the relative association probability evolution by Methods I and II, respectively. The association probability of Method I for the warhead reached 1 within 2s. Restated, the estimated trajectory followed the warhead, and was never affected by the measurements of the body and debris in 2s after separation. However, Method II assigned the same probabilities to the warhead and another object. The radar beam
thus denoted the location with the same distance to warhead as to another object. This implies that the radar lost the track if all objects have a large enough separating distance. Figure 12 shows the norms of estimation error for Method I wrt. the warhead. The error norms induced by Method I were limited to 5m in position and 9 m/s in velocity. The small estimation error ensures that the warhead located in the radar beam and was tracked all the time. The body and two debris were thus no longer in the same radar beam. Figure 13 displays the norm of the estimation error generated by Method II w.r.t. the warhead that were almost up to 1500m in position and 450m/s in velocity and larger than those induced by Method I. Figures 14-17 depicts the standard deviations (SD's) of the estimated position and velocity for Methods I and II that converged to an acceptable range in a certain time. The SD of Method I was slight greater than those of method II because of involvement of IE.

Case 1: Maneuvering Warhead:
Maneuvering is generally undertaken after separation to

Figure 9. The measured velocity of objects in $Z_n$.

Figure 10. Association probabilities of warhead and other objects for Method I.

Figure 11. Association probabilities of warhead and other objects for Method II.

Figure 12. Norm of estimation errors wrt the warhead for Method I.

Figure 13. Norm of estimation errors wrt the warhead for Method II.
change warhead trajectory. Consider the case that 3G, 3G, and -3G lateral accelerations with duration 3 s along three axes at 2 s after separation are undertaken by the warhead. Figures 18 and 19 show the corresponding association probability evolution formed by methods I and II, respectively. The association probability of method I to warhead reached 1 in 2 s as in the nonmanoeuvring case, but fluctuated after maneuvering, and died out rapidly after termination. Nevertheless, method II gives the same probabilities to the warhead and another object as the nonmanoeuvring case. The proposed algorithm assigned higher probabilities to the warhead than to another object that led the radar to track the warhead as well as the nonmanoeuvring case. The IE estimates the unknown input terms and helps identification algorithm to select the right target and keep in track. Figure 20 demonstrates the norms of estimation error for Method I that were restricted to 27 m in position and 5 m/s in velocity. The small error made the radar to track warhead well. Figure 21 shows the norms produced by method II that were much larger than those produced by method I. Figures 22~25 depict the SD's of these two methods that both converged to a certain interval. This case measures the robustness property of Method I to model error generated by maneuvering.

Case 2: Parameter Uncertainty:

This case evaluates the robustness of the proposed algorithm to ballistic coefficient, initial values of state and its covariance, and confidence level by inspecting the root mean square (RMS) of estimation error for real time. The RMS is defined as

$$\text{RMS} = \frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} \left\{ \frac{1}{N_p} \sum_{i=1}^{N_p} [v_j(i) - \overline{v}_j(i)]^T [v_j(i) - \overline{v}_j(i)] \right\}$$

(39)

where $v_j(i)$ and $\overline{v}_j(i)$ denote the estimation error and its mean, respectively, at the $j$-th run and $N_p$ is the total number of samples.

The ballistic coefficient is the critical parameter for the RV dynamic equations and is unknown for the defender. One
Figure 18. Association probabilities for Method I under manoeuvring case.

Figure 19. Association probabilities for Method II under manoeuvring case.

Figure 20. Norm of estimation errors wrt the warhead for Method I under manoeuvring case.

Figure 21. Norm of estimation errors wrt the warhead for Method II under manoeuvring case.

Figure 22. SD in position wrt the warhead for Method I under manoeuvring case.

Figure 23. SD in velocity wrt the warhead for Method I under manoeuvring case.
should assign a constant to the ballistic coefficient before estimating and tracking. If the assigned ballistic coefficient is far away from the true value, the model error increases and the estimated trajectory might become unreliable. Table 1 lists the RMS of estimation errors for non-manoeuvring and manoeuvring cases in different assigned ballistic coefficients. This indicates that only a little change for method I was achieved and confirms that the proposed approach is robust to the ballistic coefficient.

Tables 2 and 3 summarise the RMS of estimation error induced by varying initial state and covariance, respectively. For initial state, the estimation errors produced by Method I were larger than the original especially in $Z_R$. Taking $Z(0)$ as an initial is simple and is suggested. From Table 3, Method I provided a slight difference and an acceptable errors due to $P(0|0)$ variation. Although Method II also had a slight change, its estimation errors were unacceptable. Covariance of process noise, $Q$, is another critical parameter for EKF. $Q$ is assigned to be a constant matrix in entire estimation procedure for both non-manoeuvring and manoeuvring cases. Table 4 lists the RMS for methods I and II in different $Q$. The RMS's for method I were small and almost the same. However, the RMS's offered by method II were much greater than those by method I. According to these two tables, robustness of the proposed algorithm to $P(0|0)$ and $Q$ was validated.

Confidence level relates input duration $l$ and confidence interval means a threshold to detect inputs during estimation. Lower confidence level, implying larger $l$, allows more estimated inputs to join into the estimation procedure. Table 5 summarises the RMS of estimation error generated by Method I for different confidence levels or window lengths. The RMS's were almost the same for cases of confidence levels 0.9, 0.85, and 0.8. This reveals that the proposed method is robust to confidence level because errors induced by unreal inputs can be corrected by the estimated input at next sampling period.

### 6. CONCLUSIONS

This study presents an accurate algorithm for tracking a warhead that has separated from a reentry vehicle in the reentry

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**Table 1. RMS of the estimation error for non-manoeuvring (above) and manoeuvring (below) cases in different assigned C**

<table>
<thead>
<tr>
<th>Assigned C (kg/m²)</th>
<th>Position in $X_a$ (m)</th>
<th>Position in $Y_a$ (m)</th>
<th>Position in $Z_a$ (m)</th>
<th>Velocity in $X_a$ (m/s)</th>
<th>Velocity in $Y_a$ (m/s)</th>
<th>Velocity in $Z_a$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1.09 (754.16)*</td>
<td>1.12 (227.03)</td>
<td>1.4 (1677.2)</td>
<td>0.91 (188.8)</td>
<td>1.98 (123.5)</td>
<td>4.14 (180.09)</td>
</tr>
<tr>
<td>3500</td>
<td>3.33 (830.53)</td>
<td>9.66 (352.49)</td>
<td>13.02 (1522.6)</td>
<td>1.83 (255.91)</td>
<td>1.52 (147.15)</td>
<td>4.42 (323.89)</td>
</tr>
<tr>
<td>5000</td>
<td>0.56 (300.07)</td>
<td>0.53 (27.63)</td>
<td>0.85 (567.75)</td>
<td>0.54 (105.72)</td>
<td>1.17 (100.46)</td>
<td>2.28 (125.79)</td>
</tr>
<tr>
<td></td>
<td>3.64 (301.31)</td>
<td>8.54 (235.52)</td>
<td>12.86 (573.62)</td>
<td>1.24 (95.31)</td>
<td>1.06 (83.87)</td>
<td>2.93 (187.4)</td>
</tr>
<tr>
<td></td>
<td>0.46 (346.73)</td>
<td>0.46 (506.42)</td>
<td>0.79 (1587.8)</td>
<td>0.48 (89.3)</td>
<td>1.04 (108.71)</td>
<td>1.97 (163.55)</td>
</tr>
<tr>
<td></td>
<td>6.16 (309.94)</td>
<td>10.64 (136.73)</td>
<td>10.15 (1704.1)</td>
<td>0.92 (106.22)</td>
<td>1.24 (56.64)</td>
<td>2.23 (169.02)</td>
</tr>
</tbody>
</table>

* Value in (.) means the RMS from Method II.
phase. The proposed method comprises an extended Kalman filter, an input estimator, and an identification algorithm. The extended Kalman filter associated with input estimation can accurately predict the trajectory. The identification algorithm selects candidates and offers a precision combined updated state for the warhead from all measured objects. Simulation results monitor the performance of the recommended method by inspecting the estimation error corresponding to the warhead.

Table 2. RMS of the estimation error for non-manoeuvring (above) and manoeuvring (below) in initial state variation

| \( \hat{X}'(0|0) \) | Position in \( X_R \) (m) | Position in \( Y_R \) (m) | Position in \( Z_R \) (m) | Velocity in \( X_R \) (m/s) | Velocity in \( Y_R \) (m/s) | Velocity in \( Z_R \) (m/s) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.5Z(1) | 8.94 (96.02) | 8.94 (1078.4) | 890.33 (148.21) | 4.91 (121.1) | 18.23 (201.15) | 40.4 |
| | 10.51 (234.92) | 10.27 (1898.6) | 890.44 (105.46) | 5.05 (43.14) | 18.25 (151.46) | 40.44 |
| 2Z(1) | 17.83 (457.39) | 17.83 (1882.2) | 1780.7 (148.66) | 9.75 (125.29) | 36.37 (168.65) | 80.75 |
| | 19.3 (192.69) | 18.81 (1800.7) | 1780.8 (114.58) | 9.83 (98.24) | 36.42 (166.9) | 80.77 |

Value in (.) means the RMS from Method II.

Table 3. RMS of the estimation error for nonmanoeuvring (above) and manoeuvring (below) in different initial covariance

| \( P(0|0) \) | Position in \( X_R \) (m) | Position in \( Y_R \) (m) | Position in \( Z_R \) (m) | Velocity in \( X_R \) (m/s) | Velocity in \( Y_R \) (m/s) | Velocity in \( Z_R \) (m/s) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 10I_{e=6} | 0.69 (95.95) | 0.67 (609.63) | 0.95 (1522.6) | 0.64 (255.91) | 1.08 (147.15) | 1.05 (323.89) |
| | 2.93 (352.49) | 9.86 (1522.6) | 12.42 (255.91) | 1.08 (147.15) | 1.05 (323.89) | 2.97 |
| 100I_{e=6} | 0.67 (95.61) | 0.67 (609.01) | 0.95 (1522.6) | 0.65 (119.82) | 1.05 (323.89) | 2.97 |
| | 3.22 (210.17) | 8.97 (1522.6) | 12.76 (255.91) | 1.08 (147.15) | 1.05 (323.89) | 2.97 |
| 1000I_{e=6} | 0.69 (95.58) | 0.67 (608.9) | 0.95 (1522.6) | 0.64 (119.79) | 1.05 (323.89) | 2.97 |
| | 10.87 (210.05) | 7.51 (1677.0) | 28.4 (105.62) | 1.02 (38.79) | 2.79 (197.65) | 3.38 |

Value in (.) means the RMS from Method II.

Table 4. RMS of the estimation error for non-manoeuvring (above) and manoeuvring (below) in different \( Q \)

<table>
<thead>
<tr>
<th>( Q )</th>
<th>Position in ( X_R ) (m)</th>
<th>Position in ( Y_R ) (m)</th>
<th>Position in ( Z_R ) (m)</th>
<th>Velocity in ( X_R ) (m/s)</th>
<th>Velocity in ( Y_R ) (m/s)</th>
<th>Velocity in ( Z_R ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3I_{e=6}</td>
<td>0.7 (213.98)</td>
<td>0.68 (1292.9)</td>
<td>0.92 (135.51)</td>
<td>0.65 (112.55)</td>
<td>1.39 (155.14)</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>2.93 (207.82)</td>
<td>8.96 (1670.3)</td>
<td>12.42 (105.43)</td>
<td>1.08 (38.92)</td>
<td>1.05 (193.58)</td>
<td>2.97</td>
</tr>
<tr>
<td>3I_{e=6}</td>
<td>0.69 (132.59)</td>
<td>0.67 (453.38)</td>
<td>0.96 (105.01)</td>
<td>0.64 (105.58)</td>
<td>1.39 (139.45)</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>3.22 (259.71)</td>
<td>8.97 (1375.3)</td>
<td>12.76 (78.92)</td>
<td>1.08 (63.22)</td>
<td>1.21 (179.12)</td>
<td>2.62</td>
</tr>
<tr>
<td>10I_{e=6}</td>
<td>0.69 (558.22)</td>
<td>0.67 (1380.4)</td>
<td>0.95 (81.26)</td>
<td>0.64 (93.38)</td>
<td>1.39 (140.72)</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>10.87 (133.14)</td>
<td>7.51 (692.76)</td>
<td>28.4 (94.62)</td>
<td>1.02 (39.45)</td>
<td>2.79 (219.07)</td>
<td>3.38</td>
</tr>
</tbody>
</table>

Value in (.) means the RMS from Method II.
which should be minimised. Robustness of the proposed algorithm to critical parameters in the model is also evaluated by simulation. This investigation thus concludes that the proposed algorithm is worthy of further study and applications. The effects of a nonlinear measurement equation, when a filter for estimating velocity is involved, will be concerned. Ballistic coefficient estimation is another future study issue for increasing the accuracy more.

ACKNOWLEDGMENT

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REFERENCES


Table 5. RMS of estimation error induced by method I for non-manoeuvring (above) and manoeuvring (below) in different confidence levels

<table>
<thead>
<tr>
<th>$1-\alpha$</th>
<th>Position in $X_m$ (m)</th>
<th>Position in $Y_m$ (m)</th>
<th>Position in $Z_m$ (m)</th>
<th>Velocity in $X_m$ (m/sec)</th>
<th>Velocity in $Y_m$ (m/sec)</th>
<th>Velocity in $Z_m$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.7</td>
<td>0.68</td>
<td>0.96</td>
<td>0.64</td>
<td>1.39</td>
<td>2.78</td>
</tr>
<tr>
<td>(1.64)</td>
<td>3.12</td>
<td>8.99</td>
<td>12.76</td>
<td>1.07</td>
<td>1.11</td>
<td>2.79</td>
</tr>
<tr>
<td>0.85</td>
<td>0.7</td>
<td>0.68</td>
<td>0.96</td>
<td>0.64</td>
<td>1.39</td>
<td>2.8</td>
</tr>
<tr>
<td>(1.44)</td>
<td>3.46</td>
<td>8.91</td>
<td>13.09</td>
<td>1.21</td>
<td>1.2</td>
<td>2.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>0.68</td>
<td>0.96</td>
<td>0.64</td>
<td>1.39</td>
<td>2.8</td>
</tr>
<tr>
<td>(1.28)</td>
<td>3.63</td>
<td>8.96</td>
<td>12.83</td>
<td>1.25</td>
<td>1.29</td>
<td>2.54</td>
</tr>
</tbody>
</table>
Contributors

Dr Cheng-Yu Liu received his MS and PhD both from Chung Cheng Institute of Technology Taiwan, in 1983 and 1998, respectively. He worked at the Chung Shan Institute of Science and Technology during 1985-2002. Presently, he is Associate Professor in the Department of Electronic Engineering of the Lee-Ming Institute of Technology. His research areas include system identification, control theory, and tracking system.

Dr Chi-Teh Chen received his MS and PhD both from Chung Cheng Institute of Technology, Taiwan, in 1985 and 1999, respectively. He worked at the 202 Arsenal of Combined Service Forces ROC from 1985 to 2001. Presently, he is Assistant Professor in the Department of Electronic Engineering of the Lee-Ming Institute of Technology. His research area includes signal processing, guidance and control, and estimation theory.