1. INTRODUCTION

The role of weapon of Main Battle Tank (MBT) is to fire at the target accurately irrespective of the motion of the tank or the target. The objective is that the weapon must point the target at all times in order to do so. Tanks are meant for cross-country terrains. These rough terrains act as the peaks and valleys of random nature. The weapon disturbances occur mainly due to these terrains. This causes the deviation of gun from the line of eyesight and the target flees away without being hit.

Wani\textsuperscript{1, et al.}, describes the weapon dynamics for azimuth and elevation drives for which the mathematical modelling were presented for the same. The elevation model of the weapon comprises 3 degrees of freedom, arising from the rotational dynamics of the drive, breech, and muzzle, which has sequentially been coupled to the half car model. Thereafter, the backstepping, LQR and PID control techniques have been derived and incorporated into the state space matrix for the coupled dynamics model, in which the control parameters have been arrived at through various iterations. Comparative weapon dynamics response studies have been carried out between that obtained from the above control strategies and the passive model, over standard terrain conditions at specified speeds. The above study would form a very useful framework for implementation of alternate control strategies for weapon stabilisation in the full tracked vehicle.

**Keywords:** Weapon dynamics; Backstepping; PID; LQR; Ride dynamics
Controller designed with nonlinear filter showed the potential to achieve these conflicting control objectives by setting suspension to be soft when suspension travel is small, and it is adjusted to become stiff as it approaches the travel limits. Purdy, showed that stabilising an out of balance gun on a moving platform (tank or ship) was achieved using models of a balanced and out of balance gun on a main battle tank which was very difficult or impossible to achieve. The models of the guns used, included the effect of non-linear friction and out of balance. To improve the stabilisation of the out of balance gun, trunion vertical acceleration feedforward was used. This is used to develop the elevation dynamics for WCS and later incorporating the control technique. Mathworks has described the theoretical background behind Matlab programming in an elaborate manner.

The following points must be obeyed with respect to weapon control system (WCS):

(i) It must respond quickly
(ii) It must hit the target accurately irrespective of whether the target or tank in motion
(iii) It must be capable of withstanding any terrain disturbances.

In order to achieve the above mentioned points, the controllers namely, PID; Linear Quadratic Regulator (LQR) and Backstepping control techniques have been studied and implemented for WCS. The control can be achieved by adopting two strategies as follows:

- Indirect Control
- Direct control

1.1 Indirect Control

The suspension system of tank at present consists of the nitrogen gas and oil, which behave non-linearly. Implementation of control at suspension system minimises the terrain disturbances, which finally leads to minimisation of gun disturbance. The ride dynamics of the vehicle would be controlled which in turn takes care of the weapon stabilisation. In the present paper, a half car model has been considered to demonstrate the control over suspension system.

1.2 Direct Control

The control technique is implemented in the weapon itself to minimise the gun oscillations. In the present paper, elevation dynamics of the WCS has been considered. The location of controller is assumed to be between breech and the elevation drive.

The simulation is performed in MATLAB by prior formulation of the governing differential equations of motion for the half car, which in turn is sequentially coupled with the elevation dynamics, and solved using state space approach.

2. BACKGROUND

2.1 Suspension System

The half car is considered to have linear stiffness and damping characteristics, comprising totally 4 degree of freedom as shown in Fig. 1.

where \( y_{01} \) and \( y_{02} \) are the base excitations from the terrain.

2.2 Weapon System

In the present math model, the barrel is divided into 2 primary sections- muzzle (barrel front portion) and the breech (barrel rear portion), as shown in Fig. 2. The breech section in the weapon model is connected to the turret through the trunion (which is considered to be a hinge joint with a certain amount of torsional damping between the breech and trunion, which results from joint frictional effects), as shown in Fig. 2. The mass of the breech block and recoil systems is lumped to the breech section mass. The mathematical model for the gun barrel is formulated based on the lumped parameter flexible beam technique, in which the barrel length is divided into finite beam sections (for simplicity here 2 sections of the barrel are taken- muzzle and breech). Since the muzzle and breech are structurally coupled to each other, therefore a structural torsional spring and damper with appropriate torsional stiffness and damping is implemented between the muzzle and breech (as shown in Fig. 4). So, the barrel is not considered to be rigid. Considering the above, the muzzle and breech sections of the barrel have independent degrees of freedom.

The half car bounce and pitch responses serve as the input to the weapon dynamics model, in which the vibrations are transmitted to the muzzle through the breech by means of
structural coupling. The reference to the model has been taken from Purdy, based on which further control techniques have been implemented.

The elevation dynamics comprise 3 degree of freedom, arising from the angular displacements of breech, muzzle and drive. The drive in this case is an electric motor, which is having a rotational degree of freedom. This provides the required torque to elevate and depress the gun barrel to the desired angle through a rack and pinion arrangement, as shown in Fig. 2. The gun breech and muzzle, by virtue of being discrete beam sections have both angular and vertical degrees of freedom. The vertical degrees of freedom are eliminated by equations of constraint as described later in the paper.

Since for future, an All-Electric drive is being considered for gun control, therefore, the present model is accordingly developed. Figure 3 shows the elevation dynamics dimensions.

The rest of the paper has been arranged as; Mathematical model, control technique algorithms, Simulation results and comparison.

\[ \begin{align*}
    m_2 \ddot{y}_2 + K_1 (y_2 - y_1 + l_1 \dot{\theta}) + K_2 (y_2 - y_3 + l_2 \dot{\theta}) + C_1 (y_2 - y_1 + l_1 \dot{\theta}) - u_1 &= 0 \\
    m_3 \ddot{y}_3 + K_2 (y_3 - y_1 - l_2 \dot{\theta}) + K_3 (y_3 - y_0) + C_2 (y_3 - y_1 - l_2 \dot{\theta}) - u_2 &= 0
\end{align*} \]

The Eqns (1) to (4) are converted to state-space form. The state vectors are derived for these 4 DOF system.

Figure 3. Elevation dynamics dimensions.

3. MATHEMATICAL MODELS
3.1 Suspension System

The suspension system for half car has been shown in Fig. 1.

The control forces \( u_1 \) and \( u_2 \) are applied at the front and rear sides, respectively see Appendix A.

The governing equation of motion for bounce is given by Eqn (1):

\[ m_1 \ddot{y}_1 + K_1 (y_1 - y_2 - l_1 \dot{\theta}) + K_2 (y_1 - y_3 + l_2 \dot{\theta}) + C_1 (y_1 - y_2 - l_1 \dot{\theta}) + u_1 + u_2 = 0 \]

The pitch dynamics is described by Eqn (2):

\[ I \ddot{\phi} - K_1 \dot{\phi} (y_1 - y_2 - l_1 \dot{\theta}) - C_1 \dot{\phi} (y_1 - y_2 - l_1 \dot{\theta}) + K_2 J (y_1 - y_3 + l_2 \dot{\theta}) - u_1 l_1 + u_2 l_2 = 0 \]

The governing equations of motion for sprung mass are given in Eqns (3) and (4).

Thus, the state equations are given in Eqns from (5) to (8):

\[ \begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= \frac{1}{m_1} \left[ K_1 (x_1 - x_2 - l_1 x_3) + K_2 (x_1 - x_3 + l_2 x_3) + C_1 (x_1 - x_2 - l_1 x_3) + u_1 + u_2 \right] \\
    \dot{x}_3 &= x_4 \\
    \dot{x}_4 &= \frac{1}{I} \left[ -K_2 J (x_1 - x_3 + l_2 x_3) + K_1 J (x_1 - x_2 - l_1 x_3) + u_1 l_1 - u_2 l_2 \right]
\end{align*} \]

The control algorithms are implemented into the Eqns (5) to (8), as discussed later in the paper.

3.2 Elevation Dynamics

The gun elevation dynamics comprise of 3 DOF, namely angular displacements of the drive, breech and muzzle, which are in turn sequentially coupled to the 4 DOF, pertaining to the ride model of the half car.

Equations of constraint for elevation dynamics-

\[ y_i = y + \theta X_i \]

Rotational dynamics for the elevation drive-

\[ I_{de} \ddot{\theta}_{de} + T_{de} + C_{de} \dot{\theta}_{de} + K_{de} \left( \theta_{de} R_{pe} + (\theta - \dot{\theta}) X_{pe} \right) R_{pe} + u R_{pe} = 0 \]
4. PID CONTROL DESIGN

PID is the most widely applied controller. The objective function can be controlled by varying the gains values $K_P$, $K_I$, and $K_D$. If ‘$u$’ is the control variable, $y(t)$ and $f(t)$ are the actual and desirable output, then the tracking error is $e(t) = y(t) - f(t)$. The PID control law is given by:

$$u(t) = K_P e(t) + K_I \int e(t) dt + K_D \frac{d}{dt} e(t)$$

The proportional term of PID controller usually gives the controlled output without other terms. Integral term rejects the disturbance and derivative term provides the damping.  

4.1 Suspension Model

The difference between the sprung and unsprung mass velocities is considered as the control parameter for front side of half car.

4.2 Elevation Dynamics

The control parameter is the breech velocity, which is to be regulated.

$$e(t) = y(t) - l \dot{\theta} = x_2 - x_6 - l \dot{x}_4$$

The control law is obtained as:

$$u_1 = K_P (x_2 - x_6 - l \dot{x}_4) + K_I (x_1 - x_4 - l \dot{x}_3) + K_D (-\ddot{x}_2 + \dot{x}_6 - l \ddot{x}_4)$$

Similar steps are followed to obtain the control parameter for rear side.

$$u_2 = K_P (x_2 - x_6 - l \dot{x}_4) + K_I (x_1 - x_7 - l \dot{x}_3) + K_D (-\ddot{x}_2 + \dot{x}_8 - l \ddot{x}_4)$$

5. LQR CONTROL DESIGN

Considering the state-space form as:

$$\dot{x} = Ax + Bu$$

The control vector ‘$u$’ in case of LQR control is decided based upon the quadratic performance index. The performance index is given by:

$$J_{LQR} = \int L(x,u) dt$$

where $L(x,u)$ is the quadratic or Hermitian function such that,

$$L_{LQR} = -L_{LQR} x$$

The LQR gain matrix, $K_{LQR}$, leads to linear control. The linear quadratic performance index is expressed as:

$$J_{LQR} = \int \left( x^T Q x + u^T R u + 2 x^T N u \right) dt$$

where, the first term on the right hand side signifies the difference in the error between initial and final state and second term signifies the energy expenditure on control signal. The control ‘$u$’ is said to be optimal for:

$$u = -K_{LQR} x$$

where, $K_{LQR} = R^{-1} B^T P$

The $K$ matrix can be obtained from the Algebraic Riccati matrix formula:

$$PA + A^T P - (PB + N)R^{-1}(B^T P + N^T) = Q = 0$$

The $K$ matrix can be obtained by using following code through MATLAB:

$$[K, P] = LQR(A, B, Q, R, N)$$

6. BACKSTEPPING CONTROL DESIGN

The backstepping control design consists of two steps. 

**Elevation dynamics**

We choose $z_1 = \dot{\theta} X_{yp}$ to regulate the breech displacement to minimise the muzzle displacement.

**Step 1:** The derivative of $z_1$ is computed as:

$$\dot{z}_1 = \dot{x}_{11} X_{yp} = x_{12} X_{yp}$$

where, $c_1$ is a positive design constant. The corresponding error
state variable is defined as:
\[ z_2 = x_{12}X_{yp} - \alpha_1, \]
and the resulting error equation is
\[ \dot{z}_1 = -c_1 z_1 + z_2. \]

Step 2: The derivative of \( z_2 \) is given by:
\[ \dot{z}_2 = \dot{x}_{12}X_{yp} - \dot{\alpha}_1 
+ \frac{1}{X_{yp}} \left[ m_3 n_2 l_{el} \dot{x}_{10} + C_{ip} x_{12} - C_{ip} \dot{\theta} + 
(m_3 l_{el} + m_1 l_{el} + m_4 l_{el}) \left( \ddot{X}_r + \ddot{\theta} X_r \right) - K_{12 x_9} + K_{12 x_11} - C_{12 x_10} + C_{12 x_{12}} 
- K_{de} (\dot{x}_{13} R_{pe} + (\theta - x_{11}) X_{yp} + u X_{yp}) + c_1 \left[-c_1 z_1 + z_2 \right] \right]. \] (24)

The actual control inputs occur in the Eqn (24) and therefore the control law is given as in Eqn (25):
\[ u = -\frac{1}{X_{yp}} \left[ \left( c_1^2 - 1 \right) X_{yp} x_{11} - (c_1 + c_2) \left( X_{yp} x_{12} + c_1 X_{yp} x_{11} \right) \right]. \]
\[ \frac{1}{X_{yp}} \left[ \left( l_1 + m_2 l_{el}^2 + m_4 l_{el}^2 \right) + m_3 n_2 l_{el} \dot{x}_{10} + C_{ip} x_{12} - C_{ip} \dot{\theta} 
+ (m_3 l_{el} + m_4 l_{el}) \left( \ddot{X}_r + \ddot{\theta} X_r \right) - K_{12 x_9} + K_{12 x_11} - C_{12 x_{10}} 
+ C_{12 x_{12}} - K_{de} (\dot{x}_{13} R_{pe} + (\theta - x_{11}) X_{yp} + u X_{yp}) \right] \] (25)

where, \( c_1 \) is the positive design constant.

To analyse the system stability, the Lyapunov criteria is considered as:
\[ V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2. \]

The derivative of the Lyapunov function derived is given by Eqn (26):
\[ \dot{V} = z_1 \dot{z}_1 + z_2 \dot{z}_2 
= z_1 \left[ -c_1 z_1 \right] + z_2 \left[ -\frac{X_{yp}}{l_1 + m_2 l_{el}^2 + m_4 l_{el}^2} \right] \left[ m_3 n_2 l_{el} \dot{x}_{10} + C_{ip} x_{12} 
- C_{ip} \dot{\theta} + (m_3 l_{el} + m_4 l_{el}) \left( \ddot{X}_r + \ddot{\theta} X_r \right) - K_{12 x_9} + K_{12 x_11} - C_{12 x_{10}} 
+ C_{12 x_{12}} - K_{de} (\dot{x}_{13} R_{pe} + (\theta - x_{11}) X_{yp} + u X_{yp}) \right] + c_1 \left[-c_1 z_1 + z_2 \right] \]
\[ = -c_1 z_1^2 - c_2 z_2^2. \] (26)

The Eqn (26) shows that the error system is Globally Exponentially Stable according to Lyapunov theorem.

Alternatively, the closed loop error system is given by:
\[ \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 \\ 1 & -c_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \]

The above 2x2 matrix is Hurwitz, and so the error system has a globally exponentially stable equilibrium at \((z_1, z_2) = (0, 0)\). Therefore, the defined objective of breech displacement minimisation is achieved.

7. SIMULATION RESULTS AND COMPARISON

7.1 Suspension System

The passive and controlled half car model equations of motion have been formulated and solved in Matlab using Runge Kutta explicit solver. The road wheels are applied with base excitation with suitable time delay (as shown in Fig. 5), as it negotiates the Aberdeen Proving Ground (APG) terrain at a vehicle speed of 30 kmph. The bounce and pitch dynamics for the passive and controlled half car models are shown in Figs. 6 and 7, respectively. Table 1 in Appendix A provides parameters and values for half car suspension system.

7.2 Elevation Dynamics

Table 2. in Appendix A provides parameters and values for elevation dynamics system. Here, as discussed earlier, the passive half car model is integrated with the elevation dynamics model, in which the control techniques are implemented.

Prior to application of the base excitation (as shown in Fig. 5), the elevation dynamics model is applied with a suitable drive torque input (as shown in Fig. 8) in order to position the gun at the desired angle.

The natural frequencies (Hz) of the half car system are found to be 1.17, 1.44, 9.35 and 9.34. The bounce and pitch acceleration responses are shown in the frequency domain in Figs 9 and 10, respectively.

The bounce and pitch behaviour of the sprung mass serve as inputs to the elevation dynamics model of the weapon system.

The details of inputs to the elevation model are:
- Torque – 2 s to 4.15 s
- APG – 5.54 s to 15.34 s

![Figure 5. Base excitation of APG at speed 30 kmph.](image)
Figure 6. Bounce of sprung mass CG.

Figure 7. Pitch of sprung mass about CG.

Figure 8. Plot of torque input to elevation drive section.

Figure 9. Plot for FFT of bounce acceleration of half car sprung mass at CG.
The comparative magnitude variation between the sprung mass pitch and breech angular displacements is shown in Fig. 11.

With respect to the weapon passive math model, the breech is connected to the turret through the trunnion, which is a hinge joint with a certain amount of torsional damping coefficient in between them (value taken from the Purdy\(^6\)). Due to this, there is a very small amount of relative angular motion of the weapon platform with respect to the vehicle pitch, as observed from Fig. 11. With the same math model, as the value of the torsional damping is increased in between breech and trunnion, it would decrease the relative rotation of the weapon platform with respect to the vehicle motion, as shown in Fig. 12.

Incorporating higher value of torsional damping coefficient between breech and trunnion (300 kNms/rad), providing APG input to the half car model with vehicle moving at 30 kmph speed, it is observed from Fig. 12 that the relative magnitudes of pitch angular displacement response obtained from the half car ride model and breech angular displacement response are closer compared to that obtained with relatively lesser value of torsional damping, as in Fig. 11.

In practice, the actual amount of torsional damping between the breech and trunnion may be determined from the experimental evaluation of energy loss per cycle during elevation and depression of the gun, which can be taken up in future to further fine-tune the inputs to the math model. Since the main focus of the paper is on implementing a suitable control technique for achieving the desired muzzle response over the standard terrain conditions, therefore, as of now, the values of torsional damping coefficient between the breech and trunnion is taken with reference from Purdy\(^6\) (mentioned in Appendix A).

The integrated model for the weapon elevation and half car dynamics are solved in Matlab. The comparative muzzle and breech angular displacements after implementation of the control techniques are represented in Figs. 13 and 15, respectively.
The enlarged plot of the Fig. 13 is shown in Fig. 14. The comparative muzzle angular acceleration responses from the passive and controlled system is shown in both time and frequency domains, as shown in Figs. 16 and 17, respectively.

8. CONCLUSIONS

The present work describes the comparative response analyses for the half car as well as the gun elevation dynamics with and without the implementation of the stated controller algorithms. The controller working range is set by the amount of weight (gain) values provided. By adopting certain tuning techniques such as Zinger-Nichols method in case of PID controller, tuned gain values may be used.

8.1 Suspension System

It is observed from the half car suspension system responses that, LQR control provides more damping characteristics spending lesser control effort than that of PID. This helps in estimating the amount of reduction of muzzle responses, when
the half car controlled model responses are fed as the input to weapon dynamics.

8.2 Weapon Dynamics

With respect to the weapon control system, it is observed that backstepping controller method provides lesser breech and muzzle angular displacement responses, compared to that of PID and LQR techniques. As a further study, it is observed that backstepping controller technique also requires lesser control effort for similar order of reduction, as compared to LQR and PID techniques. This study would form a very useful platform for implementation of alternate control techniques for the gun system. Furthermore, on the same lines the above control techniques can also be extended to non-linear suspension system, which would prove the purpose of weapon control.
REFERENCES


7. MATHWORKS; Help literature; Matlab R2013a.

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In the current study, he provided suggestions to the methodology and in the detailed discussion of the results of the current work.
Appendix A

(A) Suspension System

(i) PID gain values

\[ K_{p1} = 1000 \]
\[ K_{i1} = 5000 \]
\[ K_{d1} = 100 \]
\[ K_{p2} = 1000 \]
\[ K_{i2} = 0 \]
\[ K_{d2} = 0 \]

(ii) The LQR control gain matrices

\[ Q_1 = \text{diag}([1e10, 0, 0, 0, 0, 0, 0, 0, 0]) \]
\[ R_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \]
\[ Q_2 = \text{diag}([0, 0, 1e12, 0, 0, 0, 0, 0]) \]
\[ R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

(B) Elevation Dynamics System

(i) PID gain values

\[ K_{p} = 5000000 \]
\[ K_{i} = 0 \]
\[ K_{d} = 0 \]

(ii) LQR control gains are

\[ Q = \text{diag}([0, 0, 0, 1e12, 0, 0]) \]
\[ R = 0.1 \]

(iii) Backstepping control design constants

\[ \epsilon = 0 \]
\[ c_1 = 25000 \]
\[ c_2 = 0 \]